Combinations of transformations

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Be able to combine the transformations from other lessons to create (and find) the equations of images (and their base graphs) using:
    - Translations
    - Dilations, and
    - Reflections

RECAP

In the last two lessons we have looked at how we can apply substitution algebra (and direct substitution) to transform graphs of base (and other functions). We did this one transformation at a time. But what if we need to combine transformations?

Remember: The transformed function (or graph) would be called the image. Being able to do this is critical in the Methods course and will be used a lot. Hence it's important to understand how to do it now.

Cambridge has a really good summary for the substitution algebra which I replicate here.

For the graph of $y = f(x)$, we have the following four pairs of equivalent processes:

1
- Applying the **dilation from the x-axis** $(x, y) \to (x, by)$ to the graph of $y = f(x)$.
- Replacing y with $\frac{y}{b}$ in the equation to obtain $y = bf(x)$ and graphing the result.

2
- Applying the **dilation from the y-axis** $(x, y) \to (ax, y)$ to the graph of $y = f(x)$.
- Replacing x with $\frac{x}{a}$ in the equation to obtain $y = f\left(\frac{x}{a}\right)$ and graphing the result.

3
- Applying the **reflection in the x-axis** $(x, y) \to (x, -y)$ to the graph of $y = f(x)$.
- Replacing y with $-y$ in the equation to obtain $y = -f(x)$ and graphing the result.

4
- Applying the **reflection in the y-axis** $(x, y) \to (-x, y)$ to the graph of $y = f(x)$.
- Replacing x with $-x$ in the equation to obtain $y = f(-x)$ and graphing the result.

Order of transformations when combining them

Really important: The order in which the transformations are applied becomes really important! There is a rule of thumb which need to be learned:

Dr T

This stands for:

- Dilations
- Reflections
- Translations

Unless otherwise stated the transformations should be applied in the order given above.

Example 1a

Find the equation of the image of $y = \sqrt{x}$ under:

a a dilation of factor 2 from the x-axis followed by a reflection in the x-axis

b a dilation of factor 2 from the x-axis followed by a translation of 2 units in the positive direction of the x-axis and 3 units in the negative direction of the y-axis.

We will look at doing this in two ways:
1. Substitution algebra
2. Direct substitution

Note with these two questions the order of the transformations is given.

$(x, y) \to (x, -2y)$

x' y'

$x = x'$ $\quad -2y = y'$

$y = \frac{y'}{-2}$

$y = \sqrt{x}$

$\frac{y'}{-2} = \sqrt{x'}$

$\therefore y = -2\sqrt{x}$

$y = \sqrt{x}$

$\frac{y}{2} = \sqrt{x}$

$y = 2\sqrt{x}$

$-y = 2\sqrt{x}$

$\therefore y = -2\sqrt{x}$

$y \to \frac{y}{a}$

$y \to -y$

Example 1b

Find the equation of the image of $y = \sqrt{x}$ under:

a a dilation of factor 2 from the x-axis followed by a reflection in the x-axis

b a dilation of factor 2 from the x-axis followed by a translation of 2 units in the positive direction of the x-axis and 3 units in the negative direction of the y-axis.

We will look at doing this in two ways:
1. Substitution algebra
2. Direct substitution

Note with these two questions the order of the transformations is given.

$(x, y) \to (x+2, 2y-3)$

$x' \quad y'$

$x + 2 = x' \qquad 2y - 3 = y'$

$x = x' - 2 \qquad 2y = y' + 3$

$y = \frac{y'+3}{2}$

$y = \sqrt{x}$

$\frac{y'+3}{2} = \sqrt{x'-2}$

$y' + 3 = 2\sqrt{x'-2}$

$y = 2\sqrt{x-2} - 3$

$y = \sqrt{x}$

$\frac{y}{2} = \sqrt{x}$

$y = 2\sqrt{x-2}$

$y + 3 = 2\sqrt{x-2}$

$y = 2\sqrt{x-2} - 3$

Example 1a: Using the CAS

Find the equation of the image of $y = \sqrt{x}$ under:

a a dilation of factor 2 from the x-axis followed by a reflection in the x-axis

b a dilation of factor 2 from the x-axis followed by a translation of 2 units in the positive direction of the x-axis and 3 units in the negative direction of the y-axis.

We can also use the CAS to help us find the image, but we need to be very clear of the substitution algebra!

Example 1b: Using the CAS

Find the equation of the image of $y = \sqrt{x}$ under:

a a dilation of factor 2 from the x-axis followed by a reflection in the x-axis

b a dilation of factor 2 from the x-axis followed by a translation of 2 units in the positive direction of the x-axis and 3 units in the negative direction of the y-axis.

We can also use the CAS to help us find the image, but we need to be very clear of the substitution algebra!

Thinking example

The graph of $y = x^2$ is transformed by:

- a translation of a units in the positive direction of the x-axis
- followed by a dilation of factor k from the x-axis.

a If the image of the graph passes through the points $(1, 1)$ and $(2, 4)$, find the possible values of a and k.

b If the image of the graph passes through the points $(1, 1)$ and $(2, 2)$, find the possible values of a and k.

$$y = (x - a)^2$$

$$\frac{y}{k} = (x - a)^2$$

$$y = k(x - a)^2$$

How would we approach the question shown on the left? Does it build on the knowledge we already have?

Is it combining learning areas we have met before?

$x \quad y$
$(1, 1)$

$x \quad y$
$(2, 4)$

Learning Objectives: Revisited

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