

Modelling linear growth and decay



Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

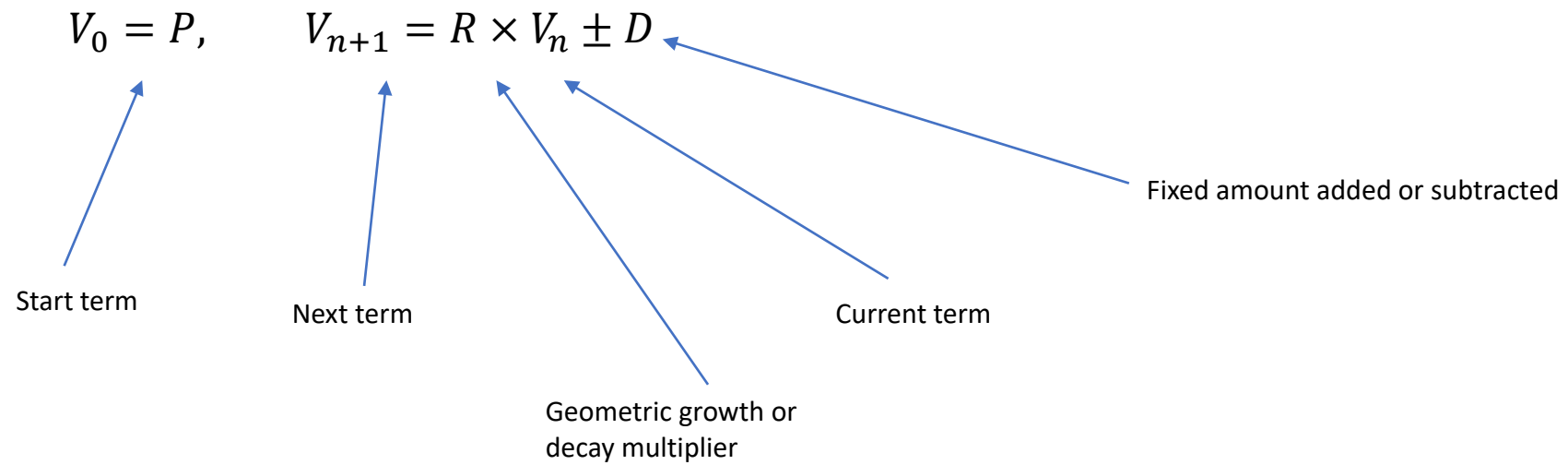
- To be able to graph the terms of a linear growth/decay sequence.
- To be able to model simple interest loans and investments using recurrence relations.
- To be able to use a recurrence relation to analyse a simple interest investment.
- To be able to model and analyse flat rate depreciation using a recurrence relation.
- To be able to model and analyse unit cost depreciation using a recurrence relation.



Recap

In the previous lesson we looked at how to write sequences using recurrence relations. These were a specific form of mathematical language which, if we understand it, can create sequences.

All they need give us is the start number and the recurrence relation for us to create any number of terms. We can do this using pencil and paper and using the CAS.



Linear growth and decay

The following formula, when the value of R becomes 1 (one) models linear growth and decay:

$$V_0 = P, \quad V_{n+1} = R \times V_n \pm D$$

Linear growth or decay formula:

$$V_0 = P, \quad V_{n+1} = V_n \pm D$$

$R = 1$

We can see that we are growing or shrinking by the value of D .

The following are examples of linear growth and decay.

$$V_0 = 10, \quad V_{n+1} = V_n + 5$$

10

$$V_0 = -3, \quad V_{n+1} = V_n - 6$$

-3

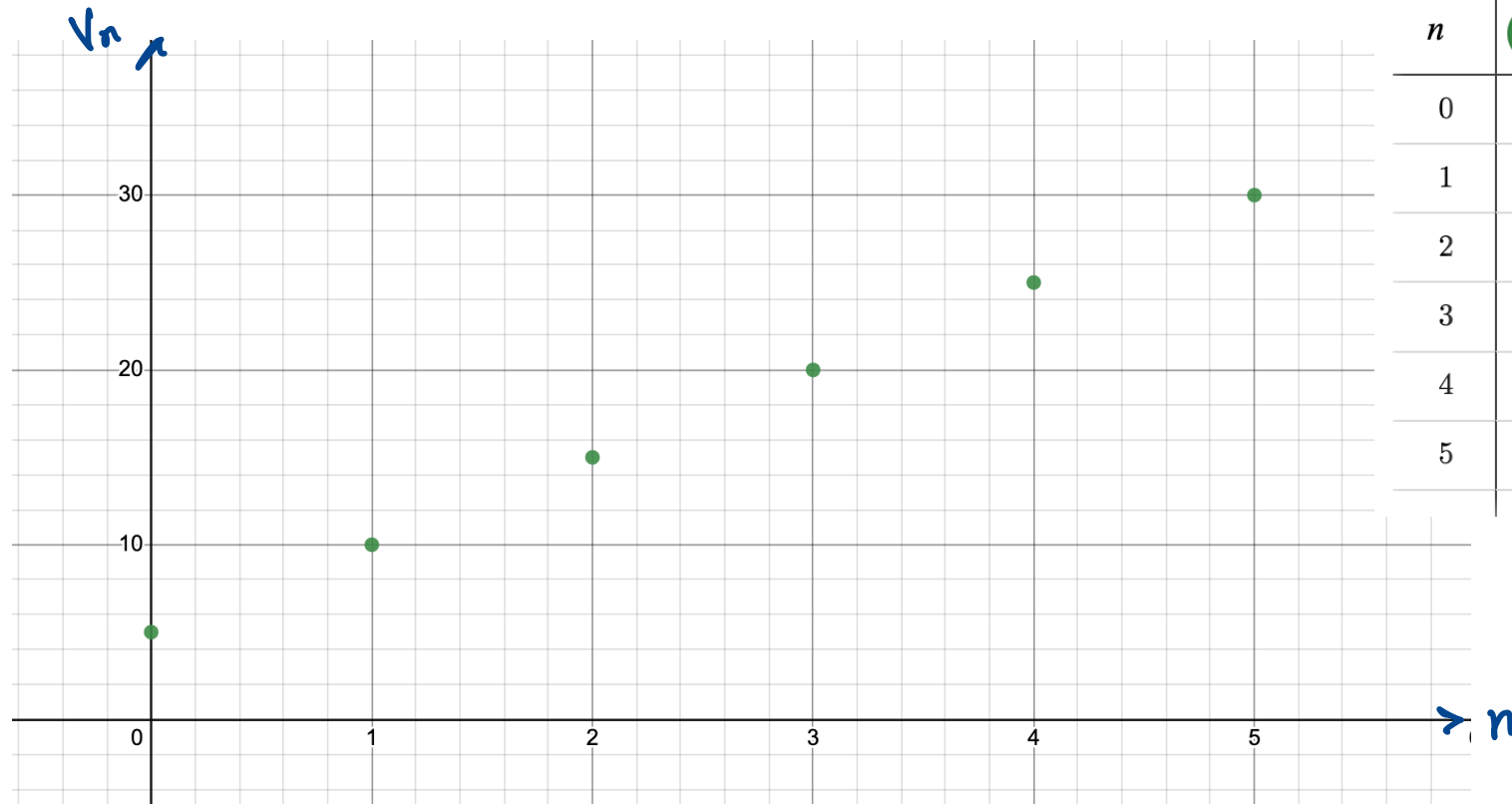


How do we know it's linear?

When we use the technology to plot the numbers on a set of axes, this is what can be seen:

$$V_0 = 10, \quad V_{n+1} = V_n + 5$$

desmos.com



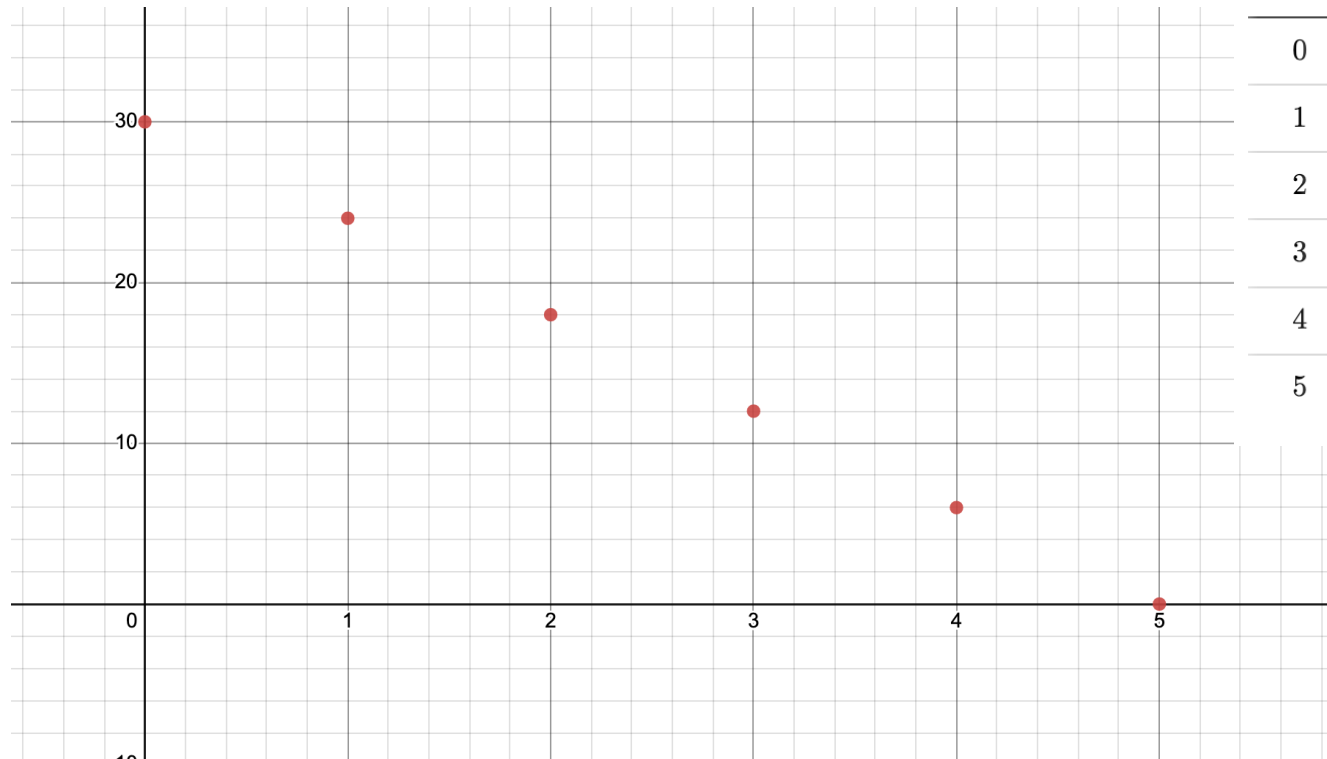
n	 v
0	5
1	10
2	15
3	20
4	25
5	30



How do we know it's linear?

When we use the technology to plot the numbers on a set of axes, this is what can be seen:

$$V_0 = 30, \quad V_{n+1} = V_n - 6$$



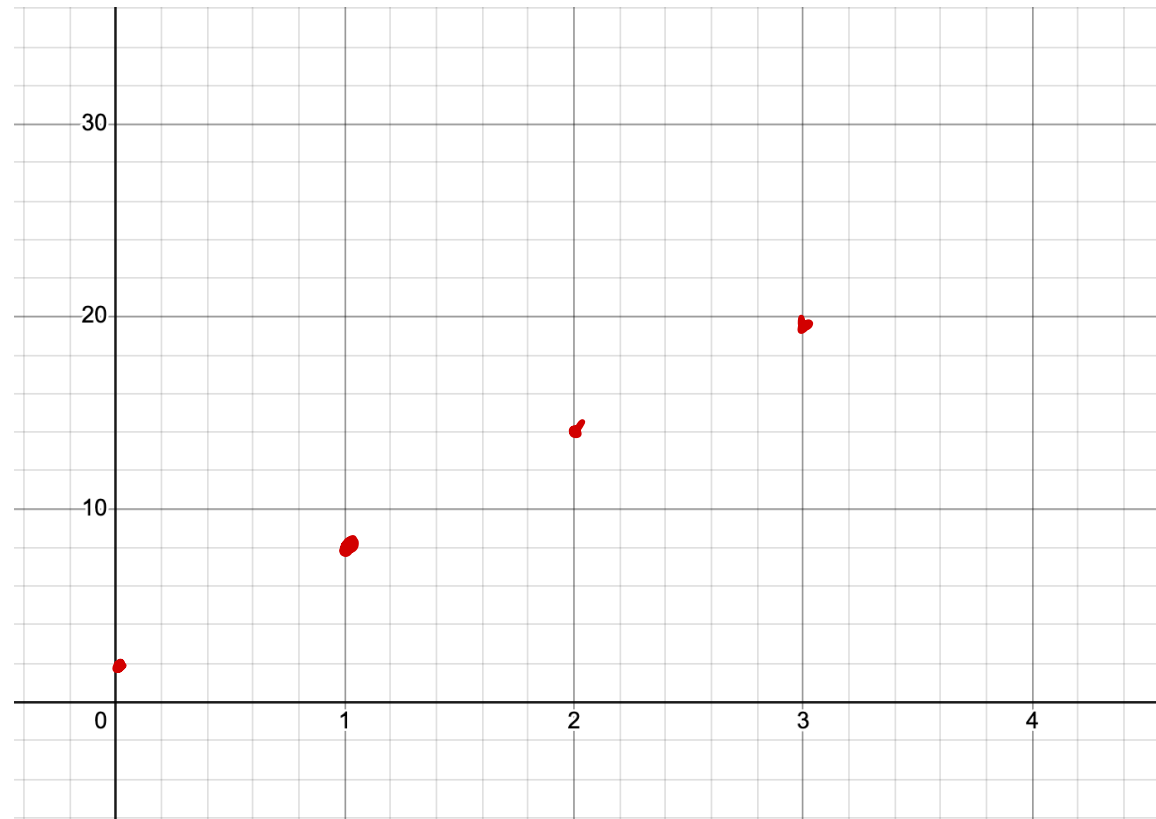
n	 v
0	30
1	24
2	18
3	12
4	6
5	0



Example: Graphing linear growth and decay

For each of the following recurrence relations, list the first four terms and graph the corresponding points.

$$V_0 = 2, \quad V_{n+1} = V_n + 6$$



Common VCE mistakes:

- Not using a pencil
- Not using a ruler
- Not plotting the points accurately
- Not reading the scale properly

2
8
14
20



Making Maths Real: Simply interest

The best way to confuse a Maths student is to make something “real world”.

Interest is paid, by the banks, as a reward for you giving them your money. This will be paid on a savings account.

It will be quoted as a **percentage per annum** which means per year.

Simple interest (which is really bad) is a fixed rate of interest paid to you based on the opening amount in your account.

We still use the following but adapt it as **interest is added on to my principal**.

$$V_0 = P, \quad V_{n+1} = V_n \pm D$$

So it becomes:

$$V_0 = P, \quad V_{n+1} = V_n + D$$

10% p.a.

\$ 1000 ↑ 10%

10% of 1000

$$\frac{10}{100} \times 1000 = 100$$



Finding the value of D

If interest is paid at 8% **per annum** it means we take the original amount and find what 8% of that is.

So, If I open an account with \$200, 8% of this will be:

$$\frac{8}{100} \times 200 = \$16$$

So, my value of D, for this one specific question would be 16.

But can we write it more generally for any values given in a question?

$$\frac{8}{100} \times 200$$

$$D = \frac{r}{100} \times V_0$$

8% p.a.

\$200

8% of 200

'r' is the **percentage value** of interest per annum

The principal or opening amount



Example: Modelling simple interest

Cheryl invests \$5000 in an investment account that pays 4.8% per annum **simple interest**.

Model this simple investment using a recurrence relation of the form:

$$V_0 = \text{the principal}, \quad V_{n+1} = V_n + D, \quad \text{where } D = \frac{r}{100} \times V_0$$

Let V_n be the value of the investment after n years.

$$V_0 = 5000, \quad V_{n+1} = V_n + 240$$

$$D = \frac{4.8}{100} \times 5000 = 240$$

Common VCE mistakes:

Not taking a note of what the value of n stands for.

These can vary and make a difference to the questions.



Example: Modelling simple interest

Cheryl's simple interest investment is modelled by

$$V_0 = 5000, \quad V_{n+1} = V_n + 240$$

Let V_n be the value of the investment after n years.

- Use the recurrence relation to **show that** the value of Cheryl's investment after 3 years is \$5720.
- When will Cheryl's investment first exceed \$6000, and what will its value be then?

$$V_0 = 5000$$

$$V_1 = 5000 + 240 = 5240$$

$$V_2 = 5240 + 240 = 5480$$

$$V_3 = 5480 + 240 = \underline{\underline{5720}}$$

At the end of the 5th year $\cdot \underline{\underline{\$6200}}$

Common VCE mistakes:

Not understanding what the 240 means at the end of the recurrence relation.

Not understanding that V_0 stands for the opening amount and not the amount at the end of the first year.



Depreciation

When something depreciates, it goes down in value. Driving a car off the forecourt will see it lose value. It then depreciates each year. The cost of anything you buy will depreciate (except perhaps houses!).

This means we will not be adding on the value of D but will be subtracting it instead. The rest of the maths stays the same really.

There are two main ways we look at things losing value:

Flat Rate: It loses the same amount per time period

Unit Cost: It loses the same amount per unit of use.

Note: These are the same but dressed up to look different!



Flat Rate Depreciation

The value of D is negative. So, we will use the following:

$$V_0 = \text{the principal}, \quad V_{n+1} = V_n - D, \quad \text{where } D = \frac{r}{100} \times V_0$$

Why are we still using the same equation for D ?

Well, with flat rate depreciation, like simple interest, it reduces in value at a rate fixed to its starting value. This rate will also be given in terms of a percentage.



Example: Flat Rate Depreciation

A new car was purchased for \$24 000 in 2014. The car depreciates by 20% of its purchase price each year. Model the depreciating value of this car using a recurrence relation of the form:

$$V_0 = \text{the principal}, \quad V_{n+1} = V_n - D, \quad \text{where } D = \frac{r}{100} \times V_0$$

Let V_n be the value of the car after n years depreciation.

$$20\% \text{ of } 24000$$

$$\frac{20}{100} \times 24000 = 4800$$

$$V_0 = 24\,000, \quad V_{n+1} = V_n - 4800$$

Common VCE mistakes:

Not writing the equation for the recurrence relation.

Getting the recurrence relation and rule mixed up.



Example: Flat Rate Depreciation

The flat rate depreciation of a car is modelled by where V_n is the value of the car after n years.

$$V_0 = 24000, \quad V_{n+1} = V_n - 4800$$

- Use the model to determine the value of the car after 2 years.
- If the car was purchased in 2023, in what year will the car's value depreciate to zero? **2028**
- What was the percentage depreciation rate?

$$2023 \quad V_0 = 24000$$

$$V_1 = 24000 - 4800 = 19200$$

$$V_2 = 19200 - 4800 = 14400$$

$$\therefore \frac{4800}{24000} \times 100 = \underline{\underline{20\%}}$$

Common VCE mistakes:

Not understanding what the final number means.



Unit cost depreciation

This will use the same formula but we need to remember that D will stand for the amount the value will decrease with each use. **This will be given in the question.**

In this situation we will use:

$$V_0 = \text{the principal}, \quad V_{n+1} = V_n - D$$



Example: Unit cost depreciation

A professional gardener purchased a lawn mower for \$270. The mower depreciates in value by \$3.50 each time it is used.

- Model the depreciating value of this mower using a recurrence relation of the form:

$$V_0 = \text{the principal}, \quad V_{n+1} = V_n - D$$

where D is the depreciation in value per use and V_n is the value of the mower after being used to mow n lawns.

- Use the model to determine the value of the mower after it has been used three times.
- How many times can the mower be used until its depreciated value is first less than \$250?

$$V_0 = 270, \quad V_{n+1} = V_n - 3.5$$

$$V_1 = 270 - 3.5 = 266.50$$

$$V_2 = 266.50 - 3.5 = 263$$

$$V_3 = 263 - 3.50 = \$259.50$$

$\therefore 6$ times



VCAA Questions

VCAA 2020 Further Maths
Exam 2

Question 7 (4 marks)

Samuel owns a printing machine.

The printing machine is depreciated in value by Samuel using flat rate depreciation.

The value of the machine, in dollars, after n years, V_n , can be modelled by the recurrence relation

$$V_0 = 120\,000, \quad V_{n+1} = V_n - 15\,000$$

- a. By what amount, in dollars, does the value of the machine decrease each year?

1 mark

\$15 000



VCAA Questions

VCAA 2021 Further Maths
Exam 2

Question 7 (3 marks)

Sienna owns a coffee shop.

A coffee machine, purchased for \$12 000, is depreciated in value using the unit cost method.

The rate of depreciation is \$0.05 per cup of coffee made.

The recurrence relation that models the year-to-year value, in dollars, of the coffee machine is

$$M_0 = 12\,000, \quad M_{n+1} = M_n - 1440$$

- a. Calculate the number of cups of coffee that the machine produces per year.

1 mark

28 800

$$\frac{\$1440}{0.05}$$



VCAA Questions

VCAA 2022 Further Maths
Exam 1

Question 20

Nidhi owns equipment that is used for 10 hours per day for all 365 days of the year.

The value of the equipment is depreciated by Nidhi using the unit cost method.

The value of the equipment, E_n , in dollars, after n years can be modelled by the recurrence relation

$$E_0 = 100\,000, \quad E_{n+1} = E_n - 5475$$

3650

The value of the equipment is depreciated by

- A. \$1.50 per hour.
- B. \$10 per hour.
- C. \$15 per hour.
- D. \$1.50 per day.
- E. \$10 per day.

$$\begin{array}{r} \$5475 \\ \hline 365 \text{ days} \\ \hline = \$15 / \text{day} \end{array}$$

$$\begin{array}{r} 5475 \\ \hline \$5475 \\ \hline 3650 \text{ hrs} \end{array} \quad \$1.5 / \text{hr}$$



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