Dilations and reflections

Year 11

Mathematical Methods

www.maffsguru.com

Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand how to dilate from:
    - The x-axis
    - The y-axis
- Understand how to reflect in the:
    - x-axis
    - y-axis
- Apply dilations and reflections to sketch graphs

RECAP

In the last lesson we looked at how we can translate graphs using some pretty interested algebra techniques. We looked at what it meant to have a base graph and what it means to translate a graph by a certain number of units horizontally and vertically.

Whilst we can do it using algebra, we can also do it using some direct algebra substitutions which I put into a table shown below.

This video is going to continue to look at transformations but dilations and reflections.

Positive x translation by h units Replace every instance of $\mathrm{x} \ {-}{>} \ \mathrm{x}-\mathrm{h}$	**Negative x translation by h units** Replace every instance of $\mathrm{x} \ {-}{>} \ \mathrm{x}+\mathrm{h}$
Positive y translation by k units Replace every instance of $\mathrm{y} \ {-}{>} \ \mathrm{y}-\mathrm{k}$	**Negative y translation by k units** Replace every instance of $\mathrm{y} \ {-}{>} \ \mathrm{y}+\mathrm{k}$

Dilating from the x-axis

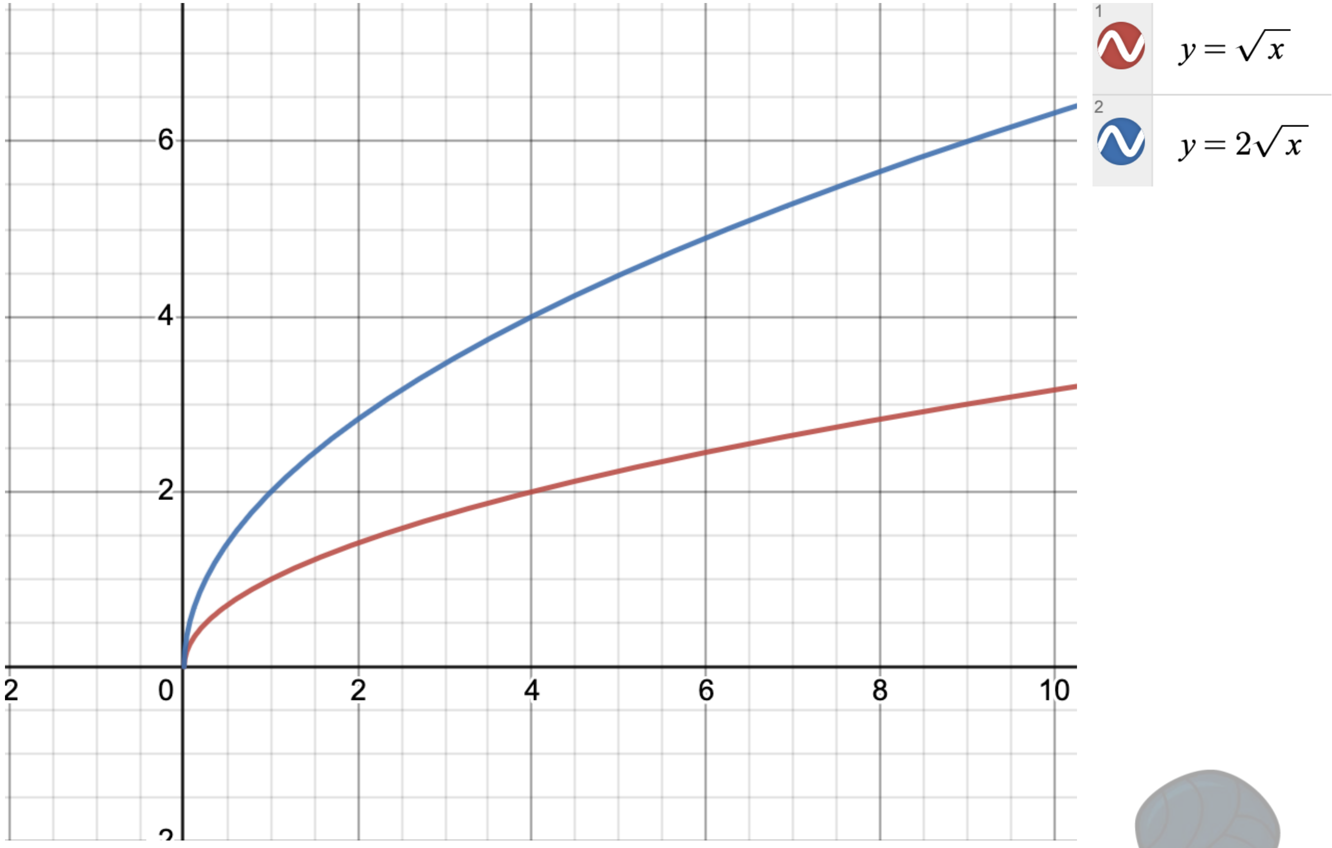
When your eyes dilate, they either expand or contract. This allows the eye to let more or less light into the eye. If we take dilate to mean grow or shrink (contract), then we can use the same idea for a graph.

One of the more common graphs we use to show this is the graph of $y = \sqrt{x}$ which is shown on the right.

When we dilate we look to dlate **away from an axis**. In this case we're going to stretch the graph away from the x-axis.

To do this, we note that the only values we are changing are the y-values. The x-axis values stay the same.

$y = 2\sqrt{x}$



Dilating from the x-axis: Substitution algebra

We can use the ideas from the last video to show how we can go from the equation of the base function to the transformed one.

$(x, y) \rightarrow (x, 2y)$

$\qquad\qquad\quad x' \quad y'$

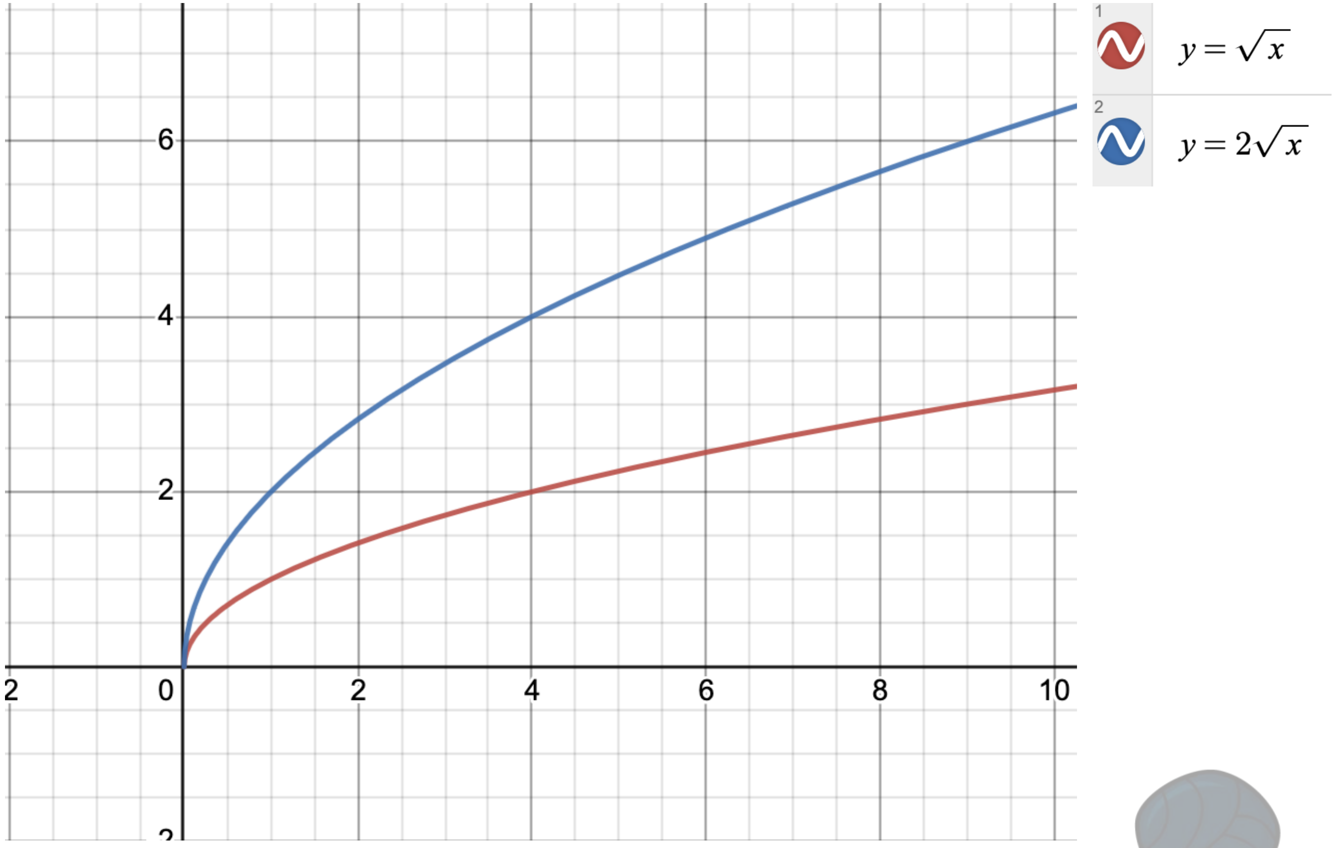
$x' = x$

$y' = 2y \nearrow y = \frac{y'}{2}$

$y = \sqrt{x}$

$\frac{y'}{2} = \sqrt{x'}$

$y = \underline{\underline{2\sqrt{x}}}$



Dilating from the x-axis: Direct substitution

Alternatively, we can use the "short cut" and do the following:

Replace the y with $\frac{y}{b}$ where b is the scale factor dilation from the x-axis.

Notice that it is, once again counter intuitive. We are scaling away from the x-axis but we are changing the y.

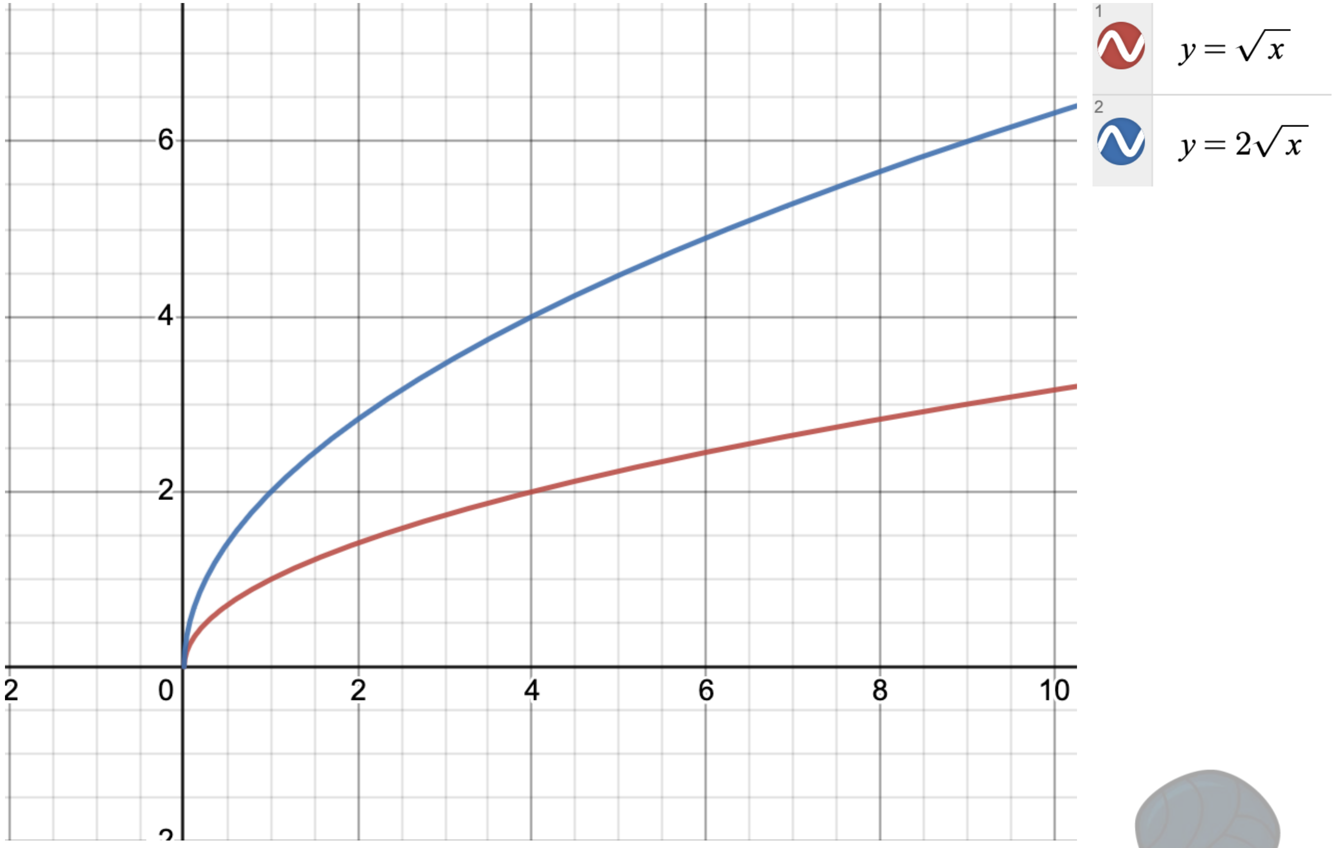
$$y \rightarrow \frac{y}{b}$$

$$y \rightarrow \frac{y}{2}$$

$$y = \sqrt{x}$$

$$\frac{y}{2} = \sqrt{x}$$

$$y = \underline{\underline{2\sqrt{x}}}$$



Dilating from the y-axis

$\rightarrow 2$

This time we are going to look to dilate away from the y-axis.

When this occurs, we leave the y-values the same and we alter the x-values. An example is shown, once again, on the right.

$$(x, y) \rightarrow (2x, y)$$

x' $\quad$ y'

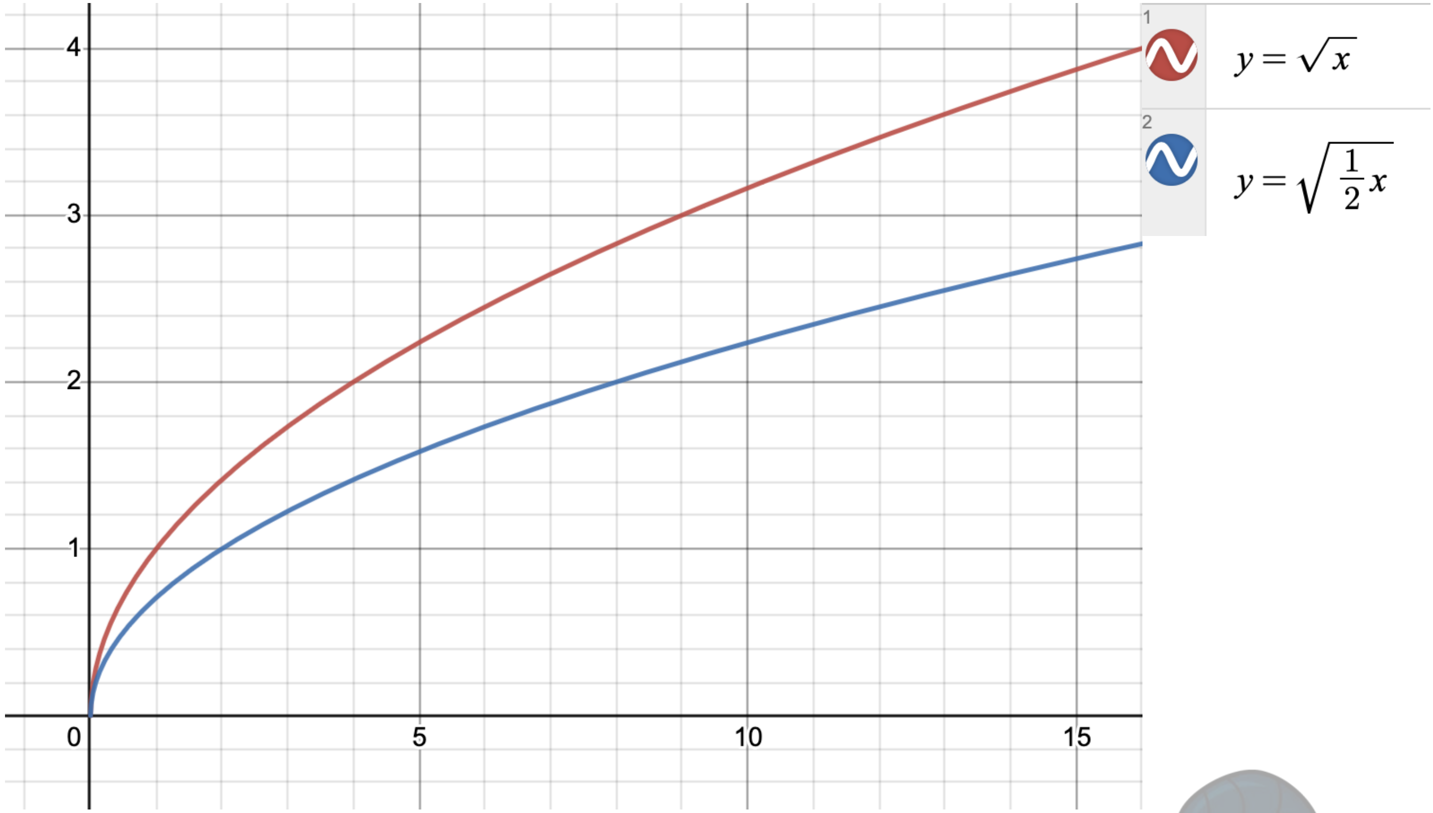
$$x' = 2x \nearrow x = \frac{x'}{2}$$

$$y' = y$$

$$y = \sqrt{x}$$

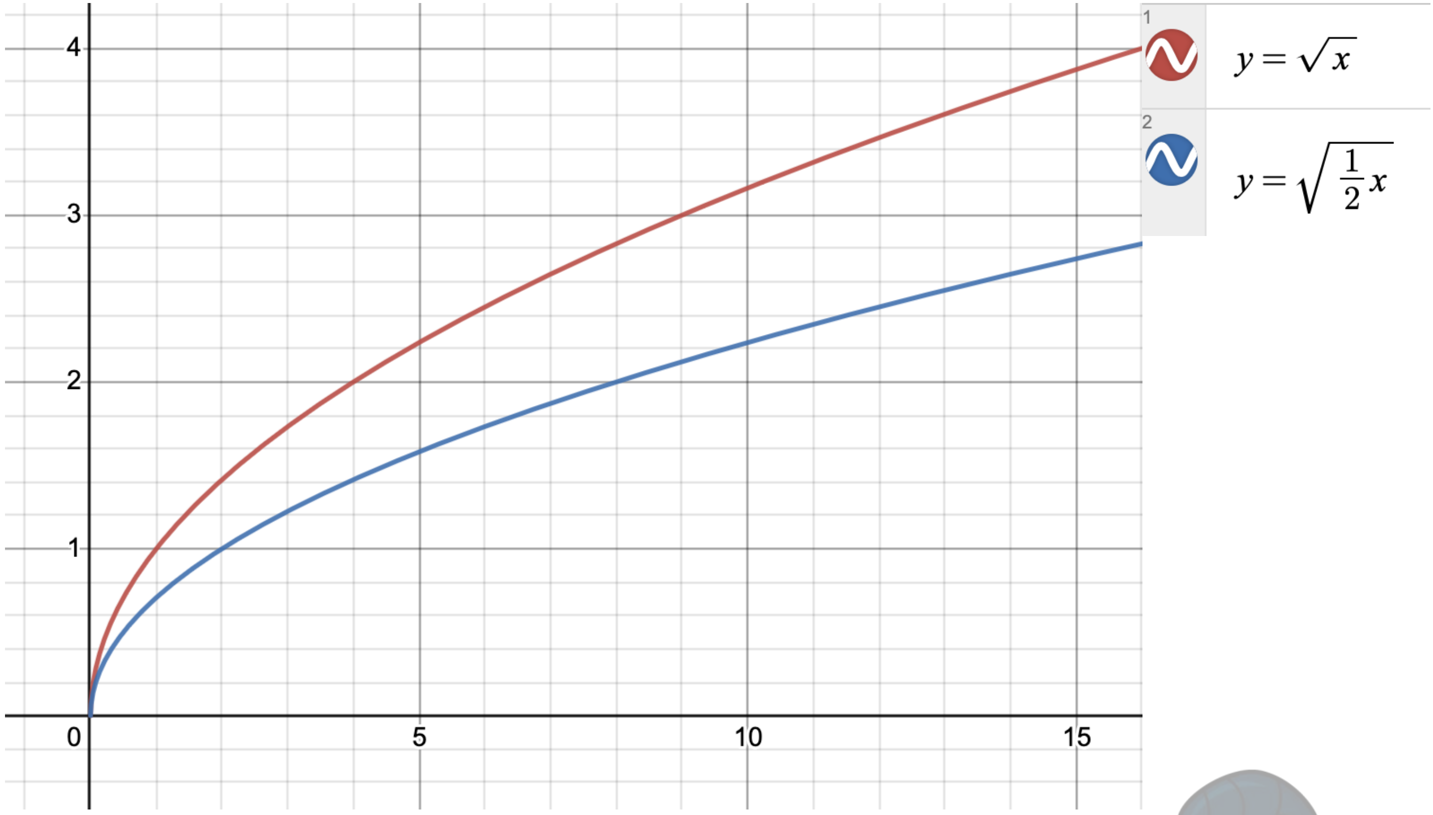
$$y' = \sqrt{\frac{x'}{2}}$$

$$y = \sqrt{\frac{1}{2}x}$$



Dilating from the y-axis: Substitution algebra

In the same way as we could for the dilation away from the x-axis, we can apply the same algebra ideas to dilations away from the y-axis.



Dilating from the y-axis: Direct substitution

$\longrightarrow 2$

Knowing that the rules seem to be counter intuitive, we can assume that, in this case, when we dilate away from the y-axis, we will replace the *x* with something!

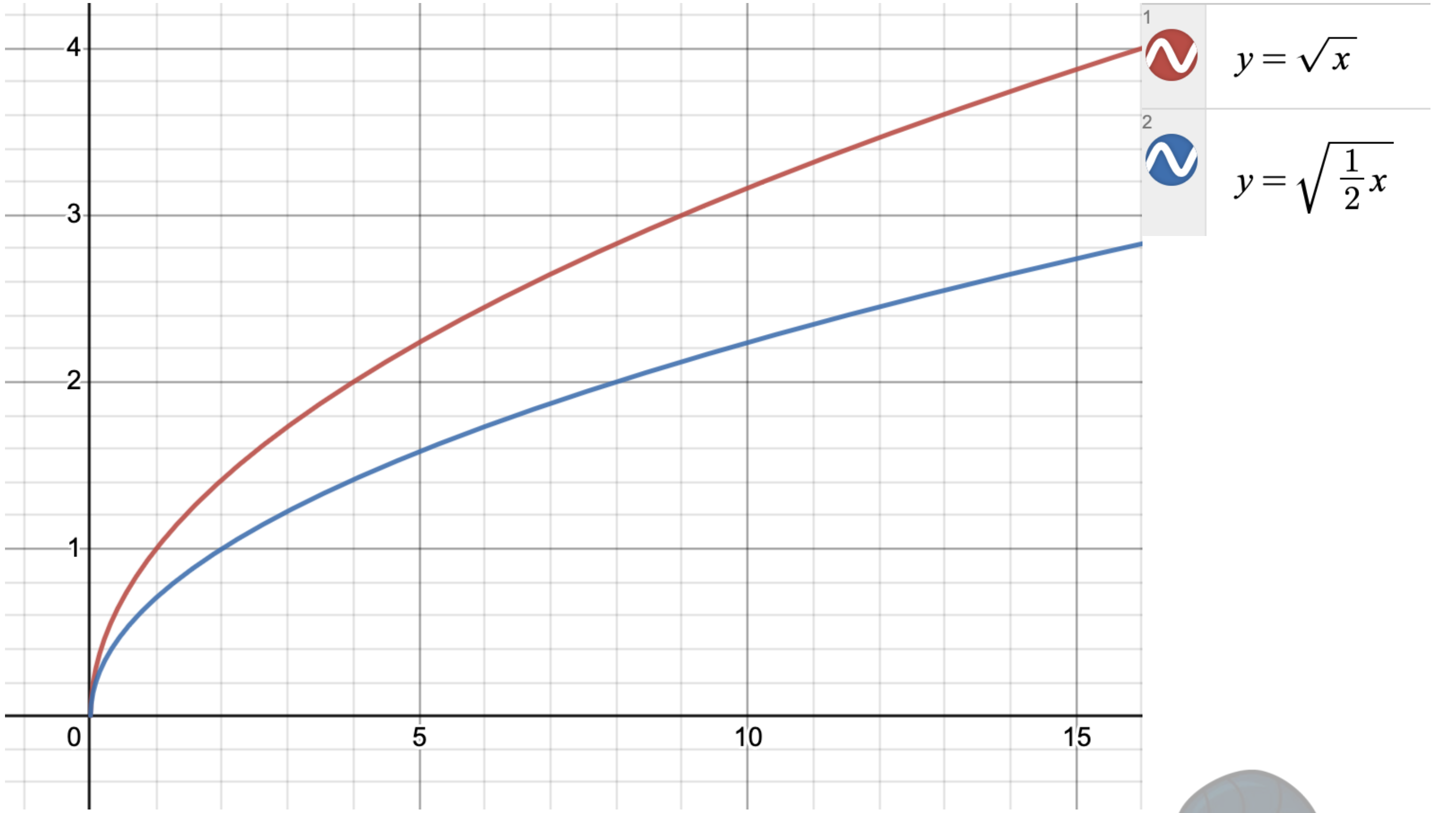
When dilating away from the y-axis with a scale factor of *a* we can **replace *x* with $\frac{x}{a}$**.

$$x \rightarrow \frac{x}{\cancel{a}\,2}$$

$$y = \sqrt{x}$$

$$y = \sqrt{\frac{x}{2}}$$

$$y = \sqrt{\tfrac{1}{2}x}$$



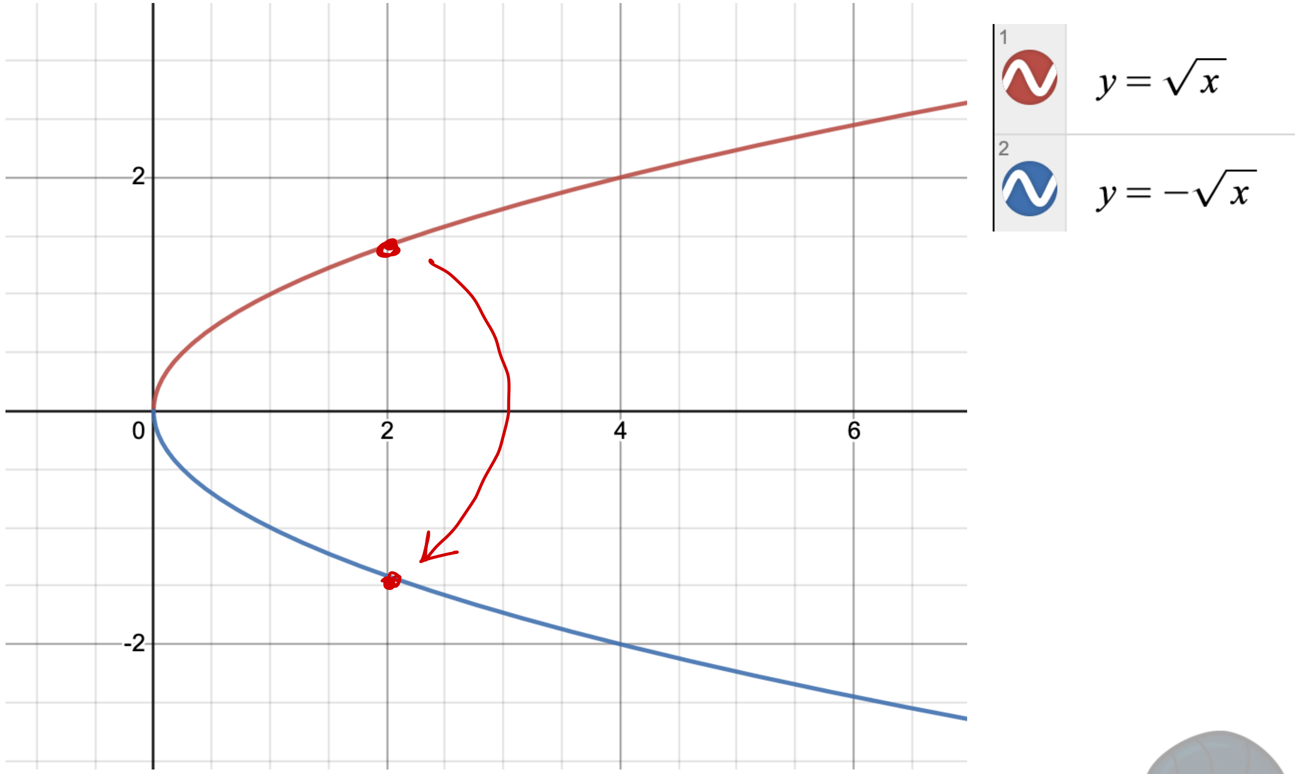
Reflecting in the x-axis

When I look in the mirror I see my reflection. It's a carbon copy of me but (generally) the reverse of me. When I was teaching Maths in the UK, we spent a LOT of time doing reflections in Year 7 and 8. Drawing mirror lines and working out where the reflection would be. Sigh. I wish we still did that …

Oh … hold on … we do! In Methods.

On the right is an example of a reflection in the x-axis.

We notice that the x-values stay the same, but the y-values all negate (multiply by negative one).



Reflecting in the x-axis: Substitution algebra

As I am sure you've come to expect now, we can do find the equation of the image of our base graph by using some substitution algrebra!

$(x, y) \rightarrow (x, -y)$

$x' \quad y'$

$x' = x$

$y' = -y$

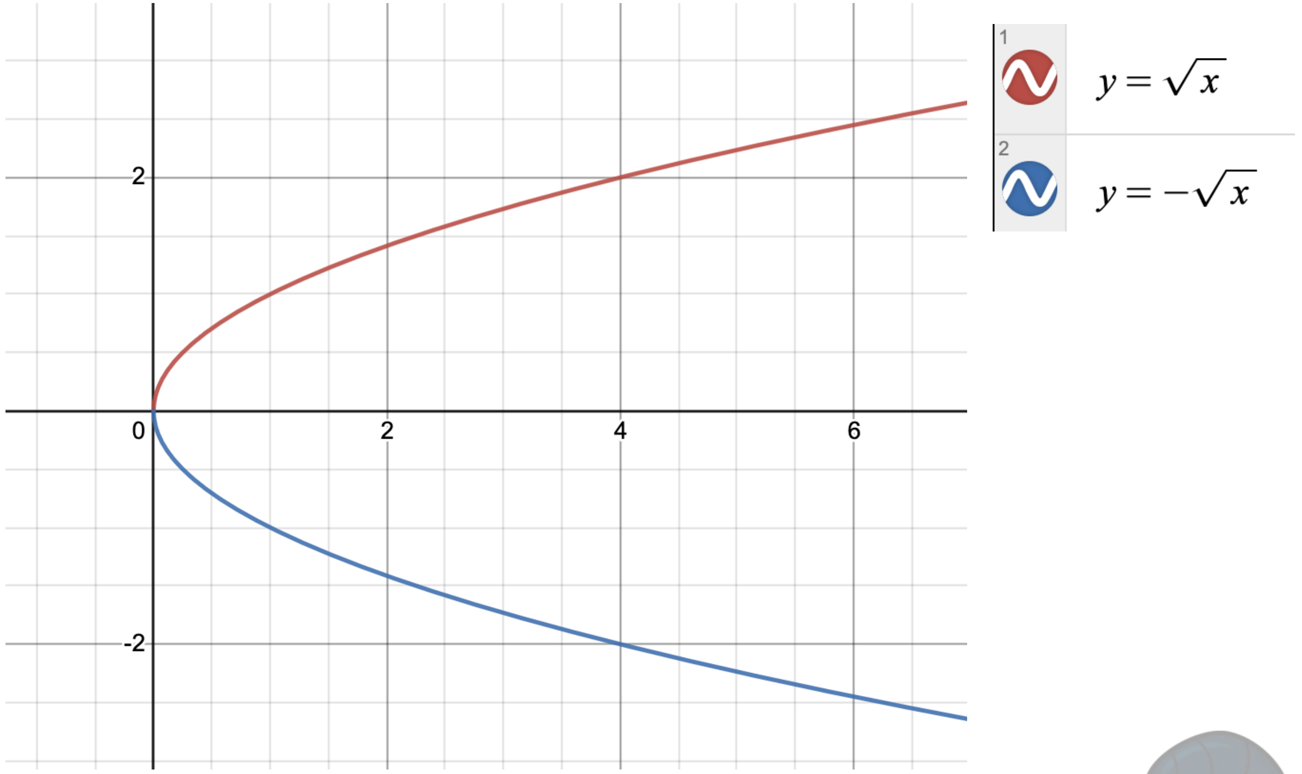
$y = -y'$

$y = \sqrt{x}$

$-y' = \sqrt{x'}$

$-y = \sqrt{x}$

$y = \underline{-\sqrt{x}}$



Reflecting in the x-axis: Direct substitution

Alternatively, we can just use the short cut and do direct substitution.

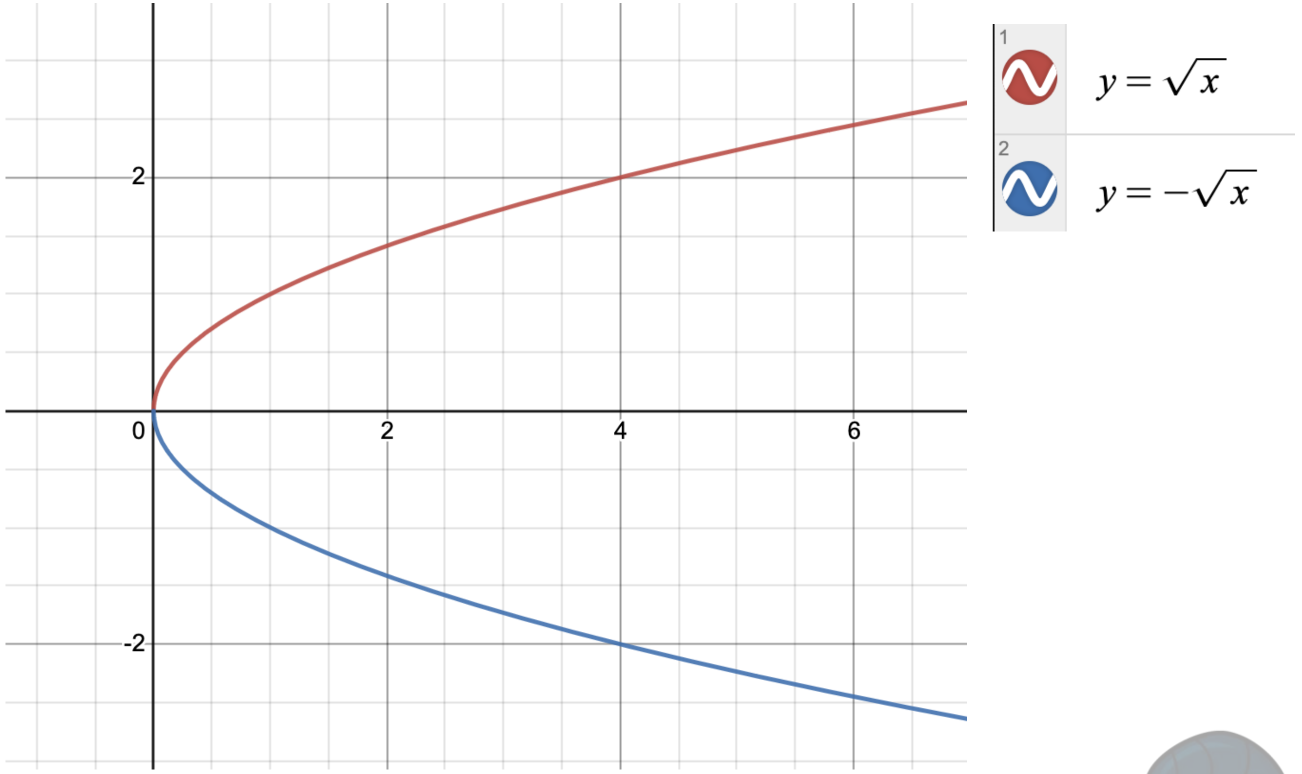
As this is, once again, counter intuitive, as we are reflecting in the x-axis, we need to **replace the *y* with *–y*.**

$$y \to -y$$

$$y = \sqrt{x}$$

$$-y = \sqrt{x}$$

$$y = \underline{\underline{-\sqrt{x}}}$$

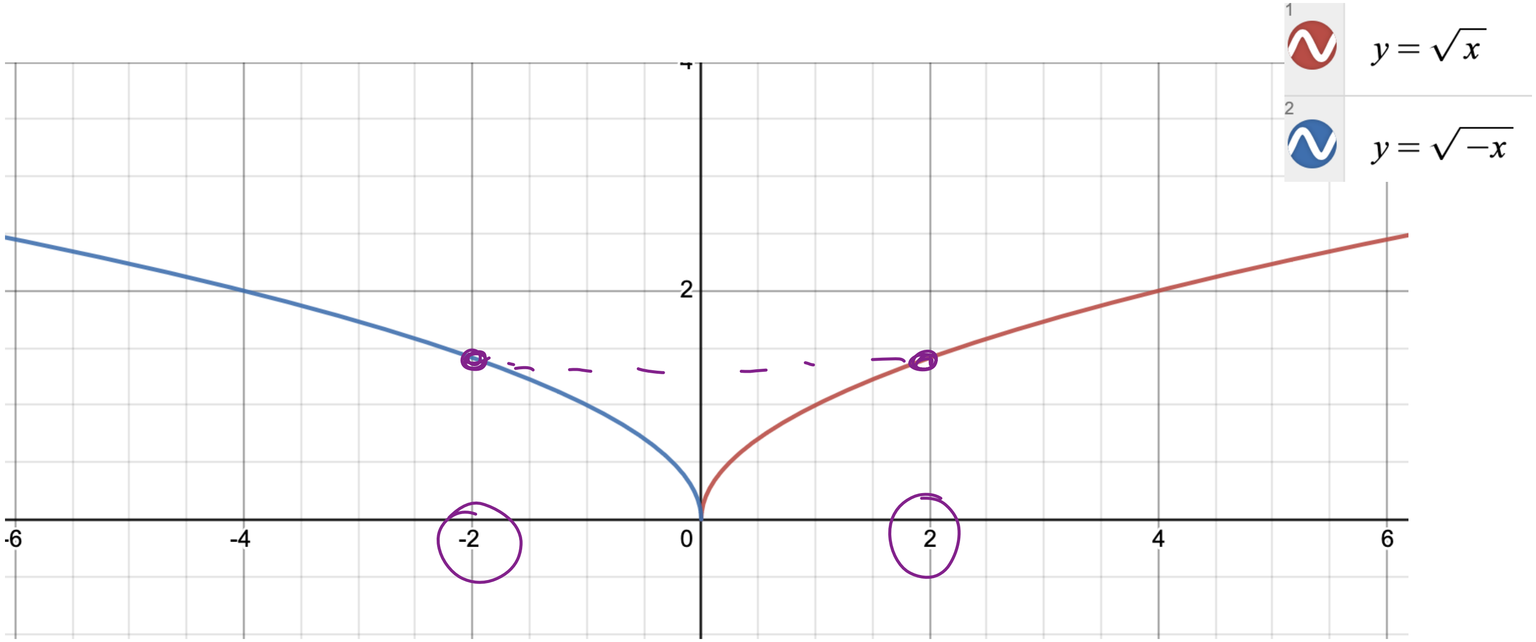


Reflecting in the y-axis

This is the last section of this video and looks at what happens when we reflect in the y-axis.

As can be seen from the picture, when we reflect in the y-axis we keep the y-values the same but negate the x-values.

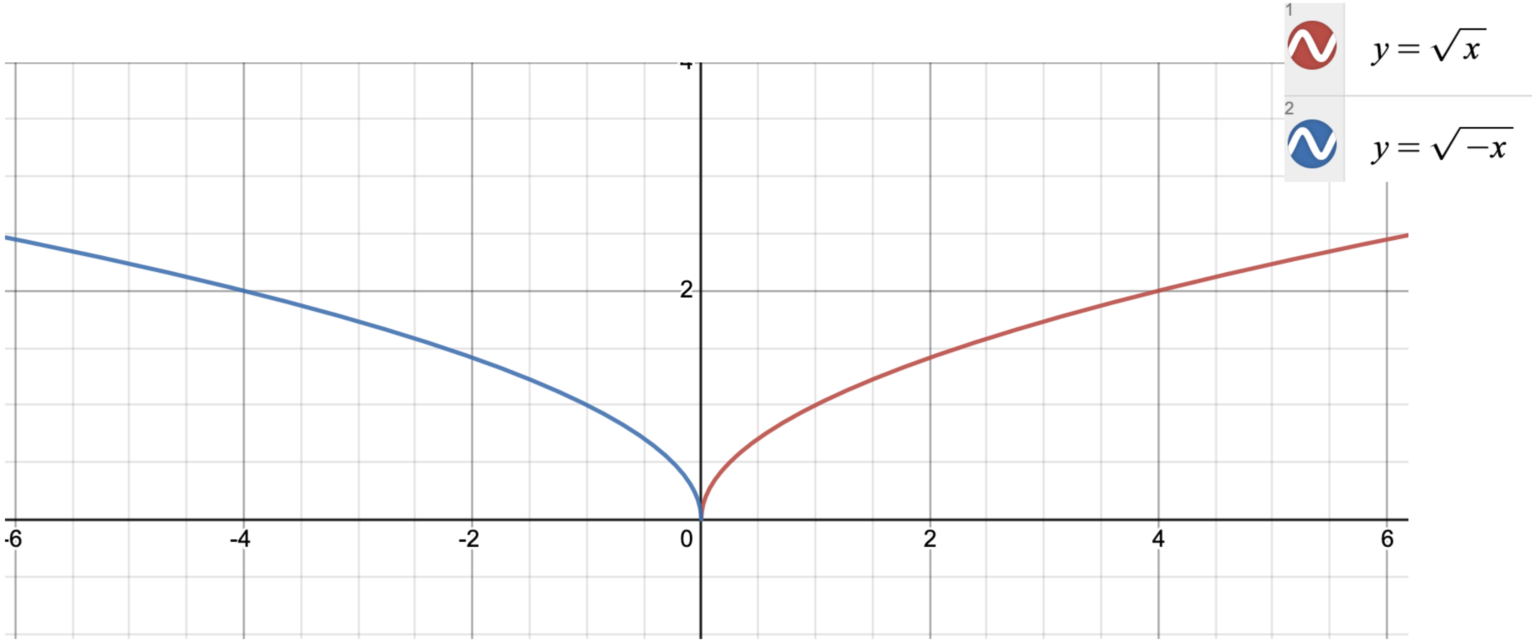
Can you see, from the equations, what I have done in terms of direct substitution?



$(x, y) \rightarrow (-x, y)$

Reflecting in the y-axis: Substitution algebra

Let's look at how we can use substitution algebra to find the equation of the image if we know the equation of the base graph.



$(x, y) \rightarrow (-x, y)$

$x' \quad y'$

$x' = -x \qquad y' = y$

$x = -x' \qquad y = y'$

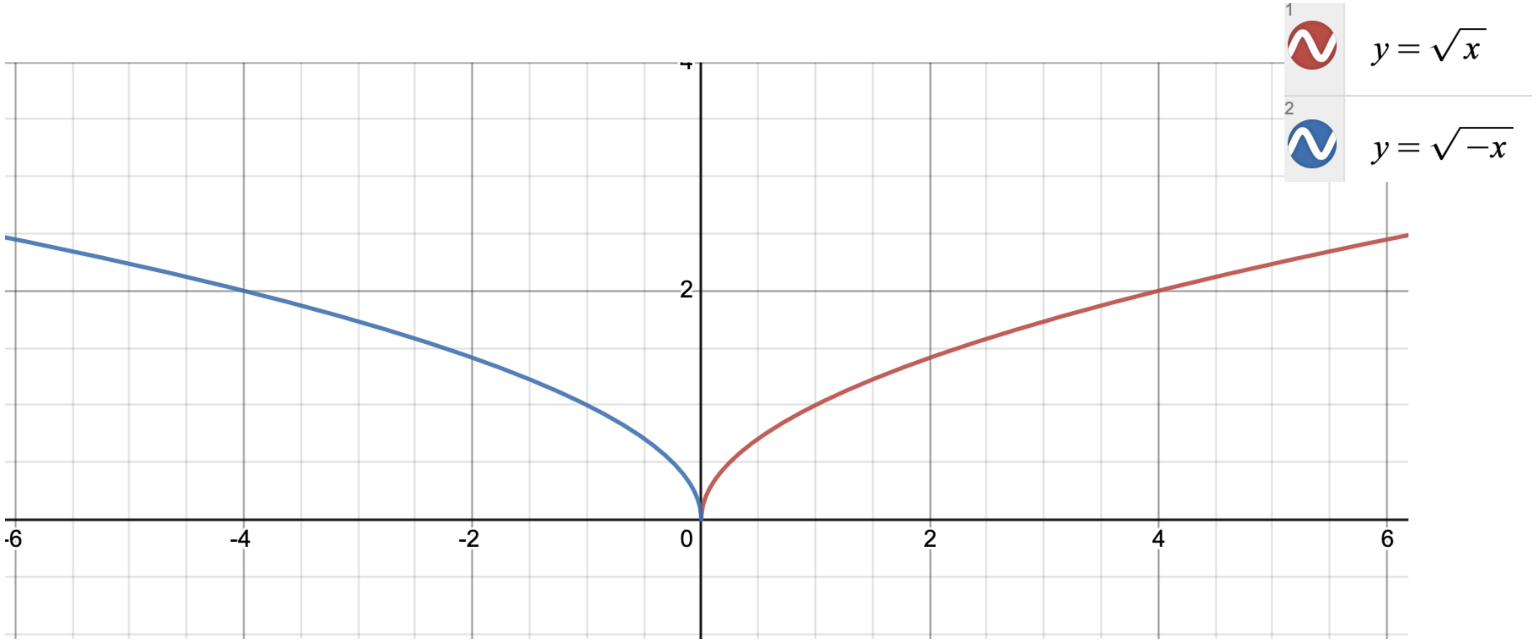
$y = \sqrt{x} \qquad y' = \sqrt{-x'}$

$\therefore \; y = \sqrt{-x}$

Reflecting in the y-axis: Direct substitution

We can use the following short cut to substitute into the base equation.

Replace the *x* with –*x*.



$y = \sqrt{x}$

$y = \sqrt{-x}$

Example

Determine the rule of the image when the graph of $y = \dfrac{1}{x^2}$ is dilated by a factor of 4:

a from the y-axis **b** from the x-axis.

$x \rightarrow \dfrac{x}{4}$

$y = \dfrac{1}{x^2}$

$y = \dfrac{1}{\left(\frac{x}{4}\right)^2}$

$= \dfrac{1}{\frac{x^2}{16}}$

$= \dfrac{16}{x^2}$

$y \rightarrow \dfrac{y}{4}$

$y = \dfrac{1}{x^2}$

$\dfrac{y}{4} = \dfrac{1}{x^2}$

$y = \dfrac{4}{x^2}$

Applying the theory to sketching some graphs

It is really important to note that the reason we are doing all this is so we can sketch graphs.

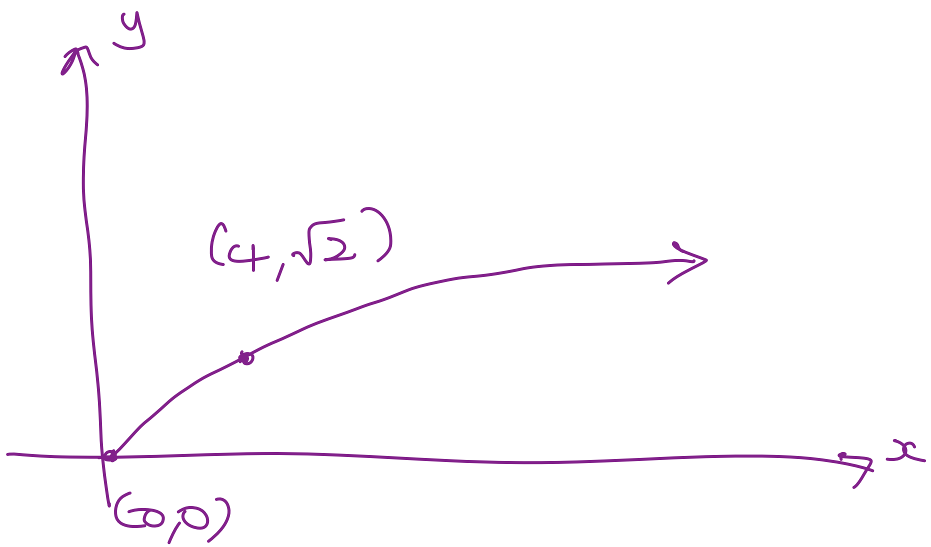
The Methods course is primarily about graphs and doing things with them.

Example:
How would we sketch the graph of

$$y = \sqrt{\frac{x}{2}}$$

$$y = \sqrt{\frac{1}{2}x}$$

$$x \rightarrow \frac{x}{2}$$



Applying the theory to sketching some graphs

What about other graphs?

$$y = \frac{x^2}{4}$$

$$y = \frac{2}{x}$$

$y = \dfrac{x^2}{4}$ $\quad y = \frac{1}{4}x^2$

$\dfrac{y}{1/4} = x^2$ $\quad$ dilation factor $\frac{1}{4}$ from x

$y = \left(\dfrac{x}{2}\right)^2$ $\quad x \rightarrow \dfrac{x}{2}$

dilation factor 2 from y. axis

$y = \dfrac{2}{x}$ $\quad \dfrac{y}{2} = \dfrac{1}{x}$ $\quad$ dilation factor 2 from x axis

Summary: From Cambridge Textbook

I am loathe to reinvent the wheel and the summary from Cambridge, shown below is the best I have seen.

This is for direct substitution.

I know a lot of teachers prefer the substitution algebra and so you need to use what works the best for you.

For the graph of $y = f(x)$, we have the following four pairs of equivalent processes:

1
- Applying the **dilation from the x-axis** $(x, y) \to (x, by)$ to the graph of $y = f(x)$.
- Replacing y with $\frac{y}{b}$ in the equation to obtain $y = bf(x)$ and graphing the result.

2
- Applying the **dilation from the y-axis** $(x, y) \to (ax, y)$ to the graph of $y = f(x)$.
- Replacing x with $\frac{x}{a}$ in the equation to obtain $y = f\left(\frac{x}{a}\right)$ and graphing the result.

3
- Applying the **reflection in the x-axis** $(x, y) \to (x, -y)$ to the graph of $y = f(x)$.
- Replacing y with $-y$ in the equation to obtain $y = -f(x)$ and graphing the result.

4
- Applying the **reflection in the y-axis** $(x, y) \to (-x, y)$ to the graph of $y = f(x)$.
- Replacing x with $-x$ in the equation to obtain $y = f(-x)$ and graphing the result.

Learning Objectives: Reviewed

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand how to dilate from:
  - The x-axis
  - The y-axis
- Understand how to reflect in the:
  - x-axis
  - y-axis
- Apply dilations and reflections to sketch graphs

www.maffsguru.com