

Sequences and recurrence relations



Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to generate a sequence of terms recursively.
- To be able to generate a sequence of numbers from a worded description using a calculator.
- To be able to generate a sequence from a recurrence relation.
- To be able to generate a sequence of numbers from a recurrence relation using a calculator.
- To be able to number and name terms in a sequence.



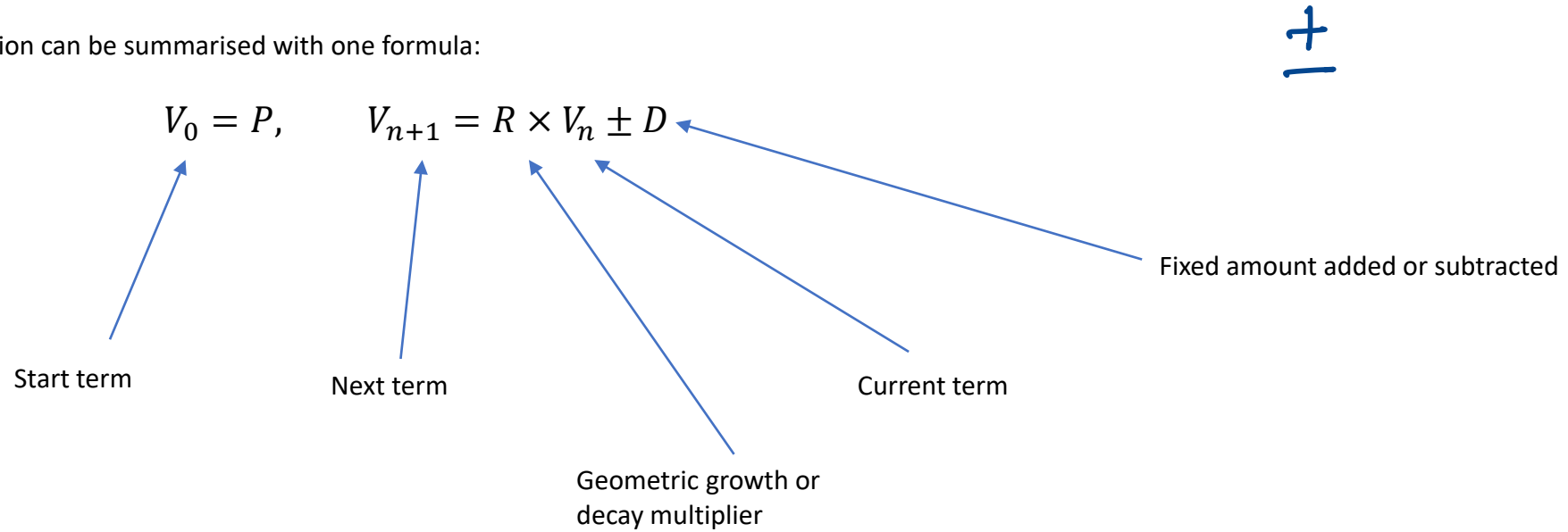
Recap

As this is the first lesson in the topic for Year 12, there really isn't much to recap excepting that this topic was covered in detail during Year 11. I will be building on that knowledge and would advise the use of a summary book.



Fast Forward

The whole topic is recursion can be summarised with one formula:



Growth and decay can be **linear** or **geometric**.
When R is 1 (one) the growth will be linear.

When there is no value for D :

When R is between 0 and 1 (not including) it will be geometric decay
When R is greater than 1 (one) it will be geometric growth..

If there is a value for D then the growth will not be geometric.



What is a sequence?

This is a list of numbers:

3, 5, 2, 7, 10, 2, 15, 19, -5, 0.7 ← Random sequence

This is also a list of numbers, but they seem to follow a pattern or sequence:

4, 7, 10, 13, 16, 19, 22, ...

Each number in the sequence is called a term and each term has a position.

4, 7, 10, 13, 16, 19, 22, ... ← Follows a pattern

Term Term



Recursion and rule

The following numbers were created by using a rule.

I started at 4 and added three each time:

4, 7, 10, 13, 16, 19, 22, ...

The rule would be “add three to each term”



Recursion and rule

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In questions, the start number and the rule will generally be given.

How they give it to you is what this lesson is about ...



Example: Generating a sequence

Write down the first five terms of the sequence with a starting value of 6 and the rule 'add 4 to the previous term'.

6 10 14 18 22

Common VCE mistakes:

Not writing the correct number of terms.



Example 2: Generating a sequence

Write down the first five terms of the sequence with a starting value of 5 and the rule 'double the number and then subtract 3'.

5 7 11 19 35

$\times 2 - 3$

Common VCE mistakes:

Not writing the correct number of terms.



Using the CAS to generate a sequence of numbers

The CAS is amazing and can help you do so much in General Maths really quickly. One of those things is to generate sequences.

Know how to use the ANS button and start thinking of this as “my current number” or “my current term”.

Example:

Use a calculator to generate the first five terms of the sequence with a starting value of 5 and the rule ‘double and then subtract 3’.

Common VCE mistakes:

Not knowing how to use the CAS properly to do simple and complex things.

5 7 11 19 35

Ans



A recurrence relation

A recurrence relation is a mathematical way of writing a worded rule.

It has a very defined format.

$$V_0 = 10, \quad V_{n+1} = V_n + 5 \quad 10$$

$$V_0 = -3, \quad V_{n+1} = V_n - 6 \quad -3$$

$$V_0 = 17, \quad V_{n+1} = 2V_n + 1 \quad 17$$

$$V_0 = 0.5, \quad V_{n+1} = 0.8V_n - 4 \quad 0.5$$

$$V_0 = \textit{start num}, \quad V_{n+1} = V_n \pm \textit{number}$$



Example: A recurrence relation

Write down the first five terms of the sequence defined by the recurrence relation

$$V_0 = 29, \quad V_{n+1} = V_n - 4$$

29 25 21 17 13



Example: A recurrence relation

Write down the first five terms of the sequence defined by the recurrence relation.

$$V_0 = 300, \quad V_{n+1} = 0.5V_n - 9$$

Use your calculator to generate this sequence and determine how many terms at the start of the sequence are positive.

300 141 61.5 21.75 1.875

5 terms



Example: A recurrence relation

Consider the recurrence relation

$$V_0 = 3, \quad V_{n+1} = V_n + 6$$

State the values of:

- V_1
- V_4
- V_5

$$V_0 = 3$$

$$V_1 = 9$$

$$V_2 = 15$$

$$V_3 = 21$$

$$V_4 = 27$$

$$V_5 = 33$$



VCAA Questions

VCAA 2022 Further Maths
Exam 1

Question 17

A sequence of numbers is generated by the recurrence relation shown below.

$$R_0 = 2, \quad R_{n+1} = 2 - R_n$$

The value of R_2 is

- A. -4
- B. -2
- C. 0
- D. 2
- E. 4

$$R_0 = 2$$

$$R_1 = 0$$

$$R_2 = 2$$



VCAA Questions

VCAA 2022 Further Maths
Exam 1

The balance of a loan, V_n , in dollars, after n months is modelled by the recurrence relation

$$V_0 = 400\,000, \quad V_{n+1} = 1.003V_n - 2024$$

Question 18

The balance of the loan first falls below \$398 000 after how many months?

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5



VCAA Questions

VCAA 2021 Further Maths
Exam 1

Question 17

The following recurrence relation can generate a sequence of numbers.

$$L_0 = 37, \quad L_{n+1} = L_n + C$$

The value of L_2 is 25.

The value of C is

- A. -6
- B. -4
- C. 4
- D. 6
- E. 37

$$L_0 = 37$$

$$L_1 =$$

$$L_2 = 25$$

$$-12$$

$$-6.$$



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