

Solving cubic inequalities

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand how we can use the tools in our chest to solve cubic inequalities



RECAP

We have, in the previous lessons, looked a how to use a range of algebraic tools to factorise polynomials. Once we know how to do this, we can then equate the linear factors to zero and solve said polynomials.

Everything we have looked at so far has been a definite equation, but what happens when we are asked to solve inequalities?



RECAP: Above, on and below the axis

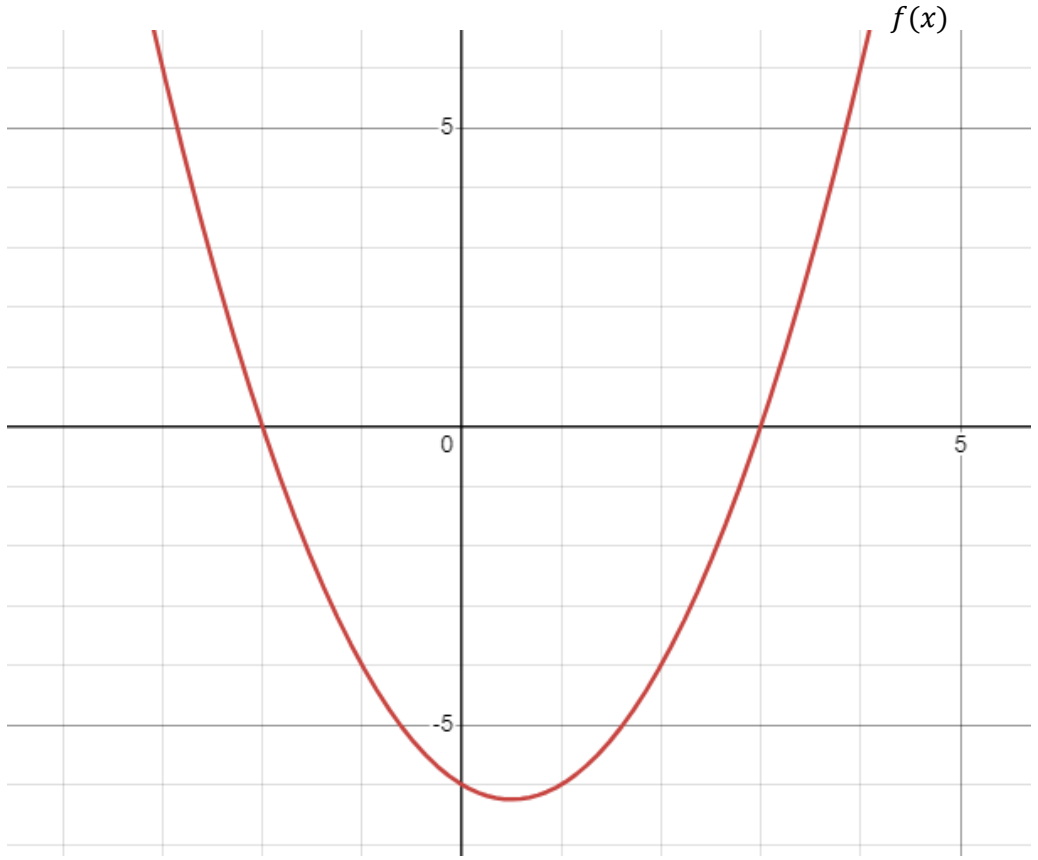
The language used in Methods can confuse a lot of students. For example:

$f(x) > 0$
$f(x) \geq 0$
$f(x) = 0$
$f(x) \leq 0$
$f(x) < 0$

These simply refer to sections of a function f(x) when plotted on a graph. For now, it is easier to remember them as being above the x-axis, on the x-axis or below the x-axis.

$f(x) > 0$

$y > 0$



Remember, the notation f(x) just stands for "y-values" created from various x-values.



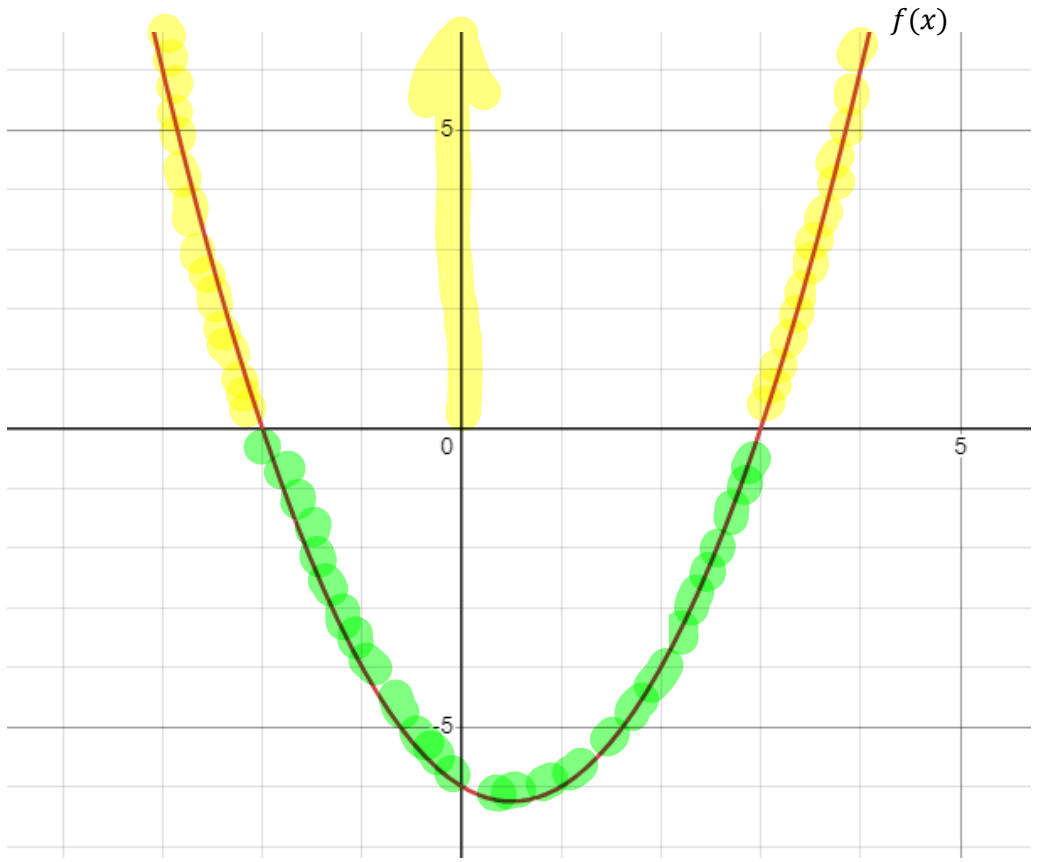
RECAP: Above, on and below the axis

So, the following simply means, for what values of x will y-values be positive

$$f(x) > 0$$

The trick here is that the y-values cannot sit on the x-axis as the y-values all have to be positive.

Note: zero is neither positive or negative.



$y = x^2 - 6$

$f(x) < 0$

$(-2, 3)$

$f(x) > 0$

$y > 0$

$(-\infty, -2) \cup (3, \infty)$



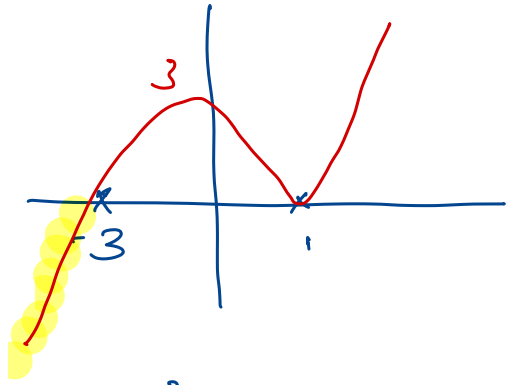
Example 1: Using Pencil and Paper

Find $\{ x : x^3 + x^2 - 5x + 3 \leq 0 \}$.

$f(x) = x^3 + x^2 - 5x + 3$ $\quad (1,0)$

$f(1) = 1^3 + 1^2 - 5(1) + 3$ $\quad \therefore (x-1)$

$= 0$



$x^2 + 2x - 3$

-3

$1 \quad 3$

$-3 \quad 1$

$x^2(x-1) + 2x(x-1) - 3(x-1)$

$x^3 - x^2 + 2x^2 - 2x - 3x + 3$

$\therefore (x-1)(x^2 + 2x - 3)$

$(x-1)(x-1)(x+3) = (x-1)^2(x+3)$

$\therefore x = 1$ or $x \in (-\infty, -3]$



Example 1: Using Technology

Find $\{ x : x^3 + x^2 - 5x + 3 \leq 0 \}$.



Example 2

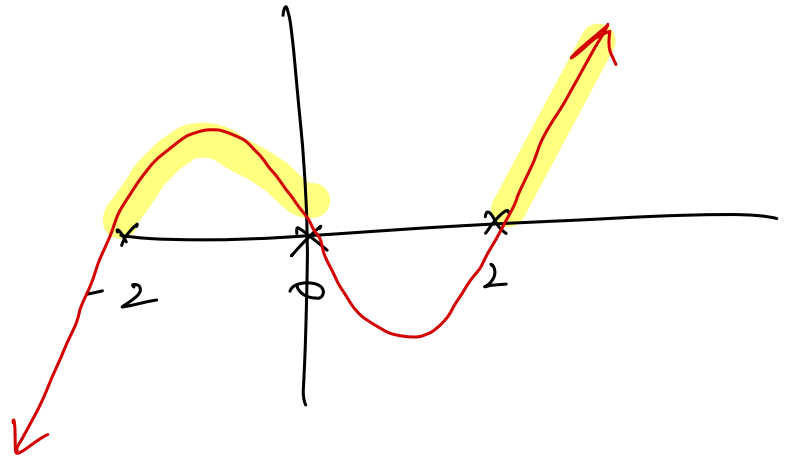
Solve the following cubic inequality:

$x^3 > 4x$

$x^3 - 4x > 0$

$x(x^2 - 4) > 0$

$x(x-2)(x+2) > 0$



$\therefore$

$x \in (-2, 0) \cup (2, \infty)$



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