Negative indices

Year 9
Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- Understand what a negative index is and what it means
- Remember what a reciprocal is
- How to express a negative index as a positive index by taking its reciprocal

RECAP

We have already met the following laws for indices:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

Anything to the power of zero is one (1).

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

These rules allow us to manipulate terms written in index notation. Now we're going to use indices for some pretty interesting things.

$$x^5 \div x^7$$

$$= x^{-2}$$

Negative indices

We have already heard about Mrs Hirst. Who was my Year 9 teacher and who I thought was pretty special. She was the first person who made me believe I could do Maths.

She left.

I was heartbroken.

Maths was never the same again.

If you remember, Mrs Hirst taught me that when an index is on the bottom of a fraction, we can move it to the top by putting a **negative sign** by the power.

This **negative sign** actually has significance! It really means that we have or can take the reciprocal of a term. Sound confusing? It really isn't.

$$\frac{1}{2} \Rightarrow \frac{2}{1}$$

$$\frac{2}{3} = \frac{3}{2}$$

$$\frac{1\boxed{x^{-2}}}{1} = \frac{1}{x^2}$$

$$\frac{\boxed{2}\boxed{x^{-3}}}{1} = \frac{2}{x^3}$$

Recap of negative indices

We know that when we follow index rules the following gives us an negative index.

$$x^4 \div x^6$$

We know this can be expressed as:

$$\frac{x \times x \times x \times x}{x \times x \times x \times x \times x \times x} = \frac{1}{x^2}$$

Or we can simply go with:

$$x^4 \div x^6 = x^{-2}$$

These must be the same.

So, we know the following are the same:

$$x^{-2} = \frac{1}{x^2}$$

$$x^4 \div x^6 = x^{-2}$$

$$\frac{1}{\boxed{x^2}} = \frac{\boxed{x^{-2}}}{1}$$

Negative index notation

We notice something interesting.

$$x^{-2} = \frac{1}{x^2}$$

The fraction expresses the index as a positive power.

By moving the x^2 to the denominator, it can be expressed as a positive power.

The meaning hasn't changed, but we can see that the negative power can be written as a positive power.

Interesting.

What about these examples:

If we look at more examples we can see the same is true:

$$\frac{b^{-3}c^{-2}}{1} = \frac{1}{b^3c^2}$$

$$x^{-3} = \frac{1}{x^3}$$

$$x^{-10} = \frac{1}{x^{10}}$$

$$b^{-3}c^{-2} = \frac{1}{b^3c^2}$$

$$\boxed{x^{-3}} = \frac{1}{\boxed{x^3}}$$

$$x^{-10} = \frac{1}{x^{10}}$$

That last example might take a little time to think about!

So, the shortcut is to move a negative index from the numerator and make it a denominator.

Mrs Hirst told the negative sign meant to take the reciprocal. This effectively switched the position in a fraction.

Negative index on the denominator

Using Mrs Hirst's logic, what will happen to the following:

$$\frac{1}{x^{-2}}$$

Remember, the **minus sign** in the power means to switch the location in the fraction.

$$\frac{1}{x^{-2}} = 1 \cdot x^{2} = \underline{\underline{x^{2}}}$$

Other examples

Here are some more examples:

$$\frac{1}{b^{-3}} = 1.b^3 = \underline{\underline{b^3}}$$

$$\frac{1}{y^{-10}} = 1.y^{10} = \underline{\underline{y^{10}}}$$

$$\frac{a^2}{b^{-3}}$$

Notice the last example!

$$\frac{a^2}{b^{-3}} = \underline{\underline{a^2 b^3}}$$

Formal "rule"

As is normal in Maths we can write what we have been talking about as a formal "rule"

$$a^{-m} = \frac{1}{a^m}$$

$$a^m = \frac{1}{a^{-m}}$$

$$a^{-m} = \frac{1}{a^m} \qquad \frac{\boxed{a^m}}{1} = \frac{1}{\boxed{a^{-m}}}$$

Other examples

Express the following with positive indices only:

$$\frac{3a^{-2}b^{4}}{1} = \frac{3b^{4}}{a^{2}}$$

Examples have been extracted, with permission, from the Cambridge Essential Series (Year 9)

Other examples

Express the following with positive indices only:

$$\frac{5}{x^3y^{-4}} = \frac{5y^4}{x^3}$$

$$= \frac{5y^4}{x^3}$$

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Evaluating examples

Express the following using positive powers only, then evaluate without using a calculator

3^{-4}

$-\frac{5}{3^{-2}}$

$\left(\frac{2}{3}\right)^{-4}$

x^{-4}

$\frac{3 \times 3 \times 3 \times 3}{9 \times 9}$

$\left(\frac{2}{3}\right)^{-4}$

$= \left(\frac{3}{2}\right)^{4}$

$= \frac{3^4}{2^4}$

$\frac{3^{-4}}{1} = \frac{1}{3^4} = \frac{1}{81}$

$\frac{5}{3^{-2}} = 5 \times 3^2$

$= 5 \times 9$

$= 45$

$\left(\frac{2}{3}\right)^{-4} = \frac{2^{-4}}{3^{-4}} = \frac{3^4}{2^4} = \frac{81}{16}$

Examples have been extracted, with permission, from the Cambridge Essential Series (Year 9)

Last one!

Evaluate without the use of a calculator:

$$\frac{(3^{-2})^3}{3^{-5}}$$

$-6 - -5$

$-6+5$

$$\frac{(3^{-2})^3}{3^{-5}} = \frac{3^{-6}}{3^{-5}} = \frac{\cancel{3^{-6}}\ 1}{3^{\cancel{-5}}\ 1} = \frac{1}{3}$$

$$3^{\boxed{-6}} \div 3^{\boxed{-5}} = 3^{-1} = \frac{1}{3}$$

Work to complete for this topic

The questions I am asking you to complete from this section of the course is shown asked to answer all parts of each question.

They are quick to complete, but it's important you master the Indices component of

Exercise 6E

2bdfhjmo, 3cgjlop, 4aceg, 5bdfh, 6adgjmpsv, 8, 9, 11