

Cubic functions of the form

$$y = a(x - h)^3 + k$$

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand how we can transform graphs of the form $y = x^3$ using
  - Dilations from the x- and y-axes
  - Reflections in the x- and y-axes
  - Translations both vertically and horizontally
- Know what the general form of transformations of a cubic looks like
- Understand what a point of inflection is



RECAP

We have already met (a number of times) the ideas of transformations. These fall into three main categories:

- Dilations
- Reflections, and
- Transformations

One or more of the above can be applied to any base graph to create a new graph.

This is a massive part of the course and comprises the first 9 chapters of the Mathematical Methods Units 3 and 4 course.



One rule to bind them all

Let's get to the very core of the learning which is about to take place; one of the general forms of a cubic function.

When it is written in the following way, we can find the point of inflection and so much more!

$$y = a(x - h)^3 + k$$

Dilation from the x-axis

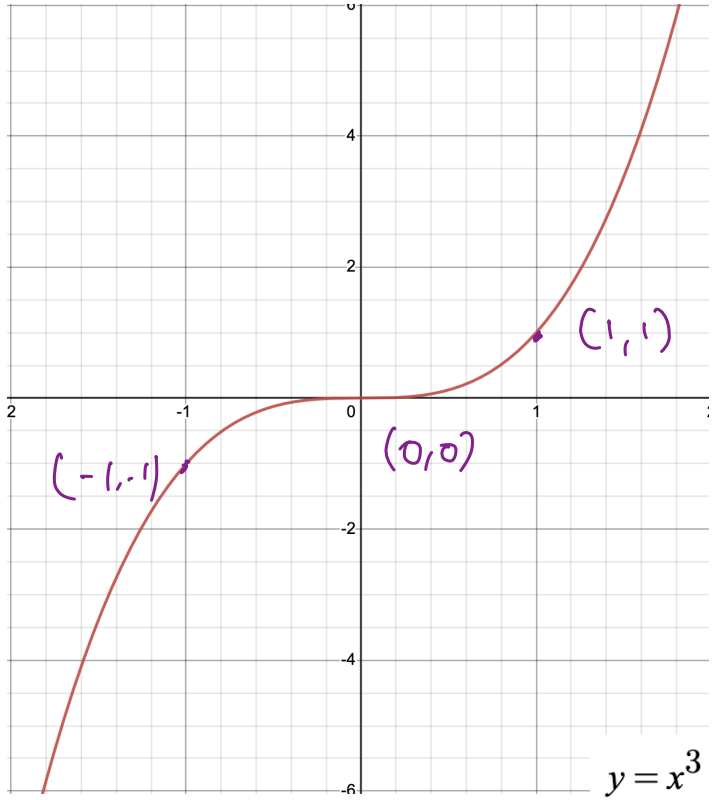
Horizontal translation

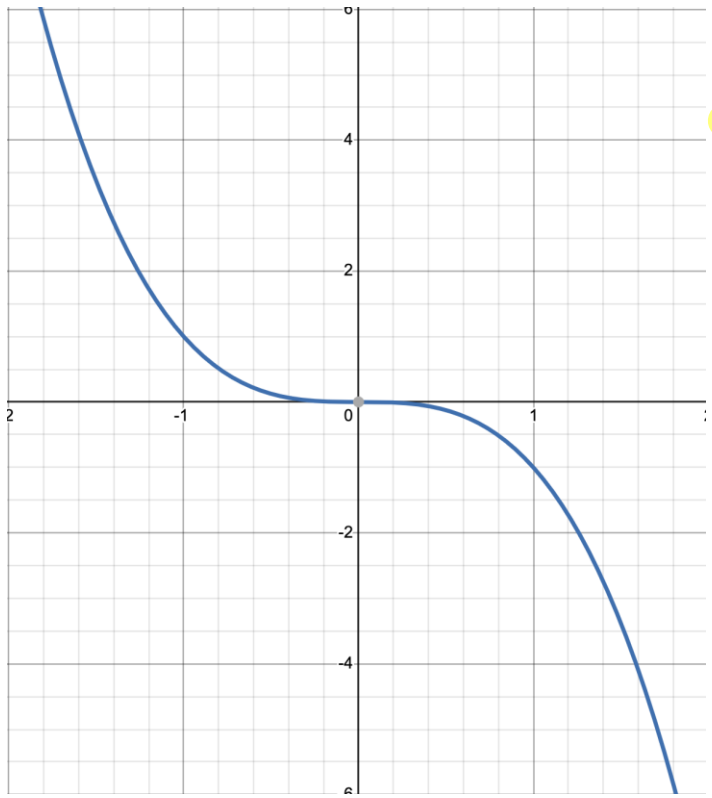
Vertical translation



Reflecting in the x- and y-axes

When we reflect the standard graph of $y = x^3$ in the x- and y-axis something interesting happens.





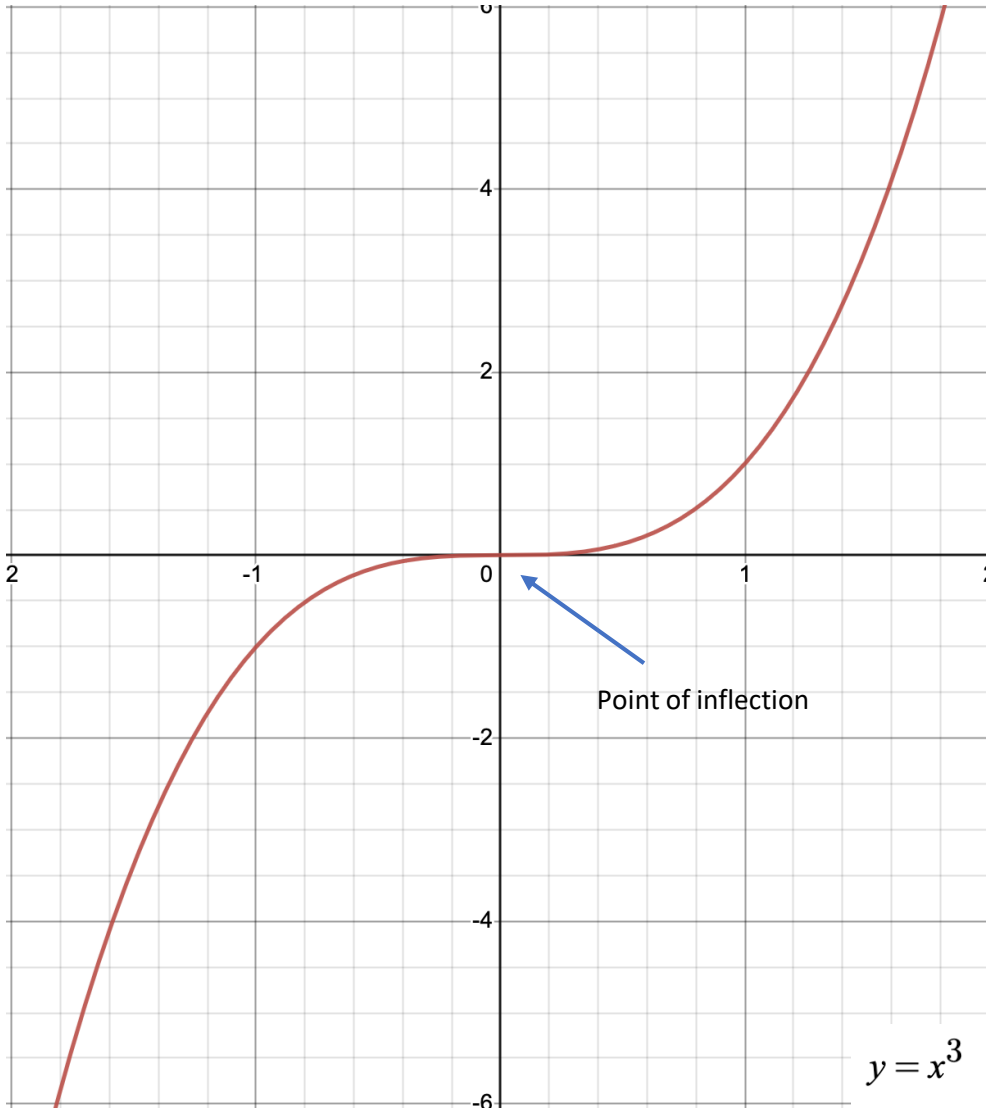
$y = -x^3$

$y = (-x)^3$



Point of inflection

A point of inflection is found where the graph of a function changes concavity. This is beyond the scope of this lesson (for the time being) but for simplicity it's where the gradient (either side of a zero gradient) is the same sign.



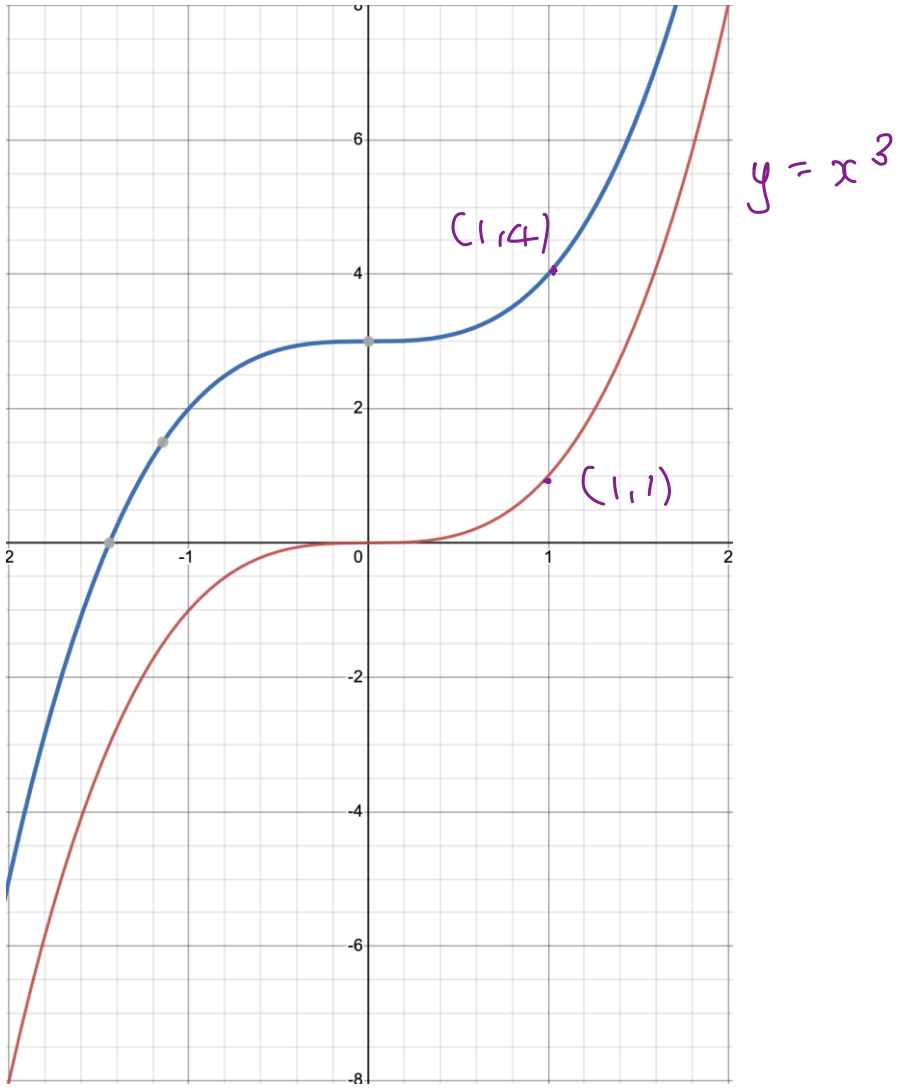


Vertical translations

This is where every point on the base graph is moved up or down by the same amount.

This is the value of *k* in the formula on the previous page.

The graph shows $y = x^3 + 3$. The blue graph is simply the red graph moved up by 3 units.





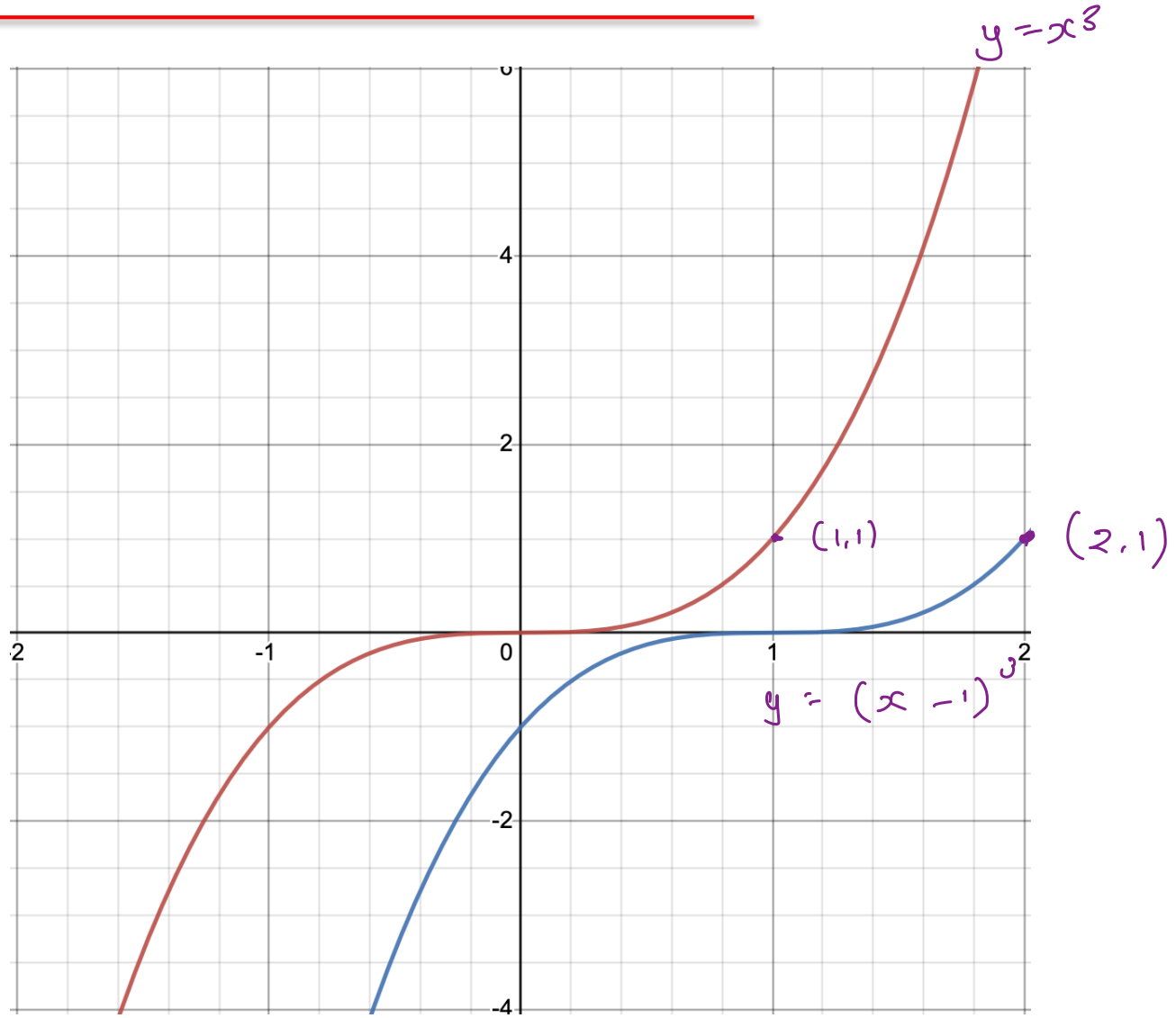
Horizontal translations

This is where every point on the base graph is moved left or right by the same amount.

This is the value of *h* in the formula on the previous page.

The graph shows $y = (x - 1)^3$. The blue graph is simply the red graph moved right by 1 unit.

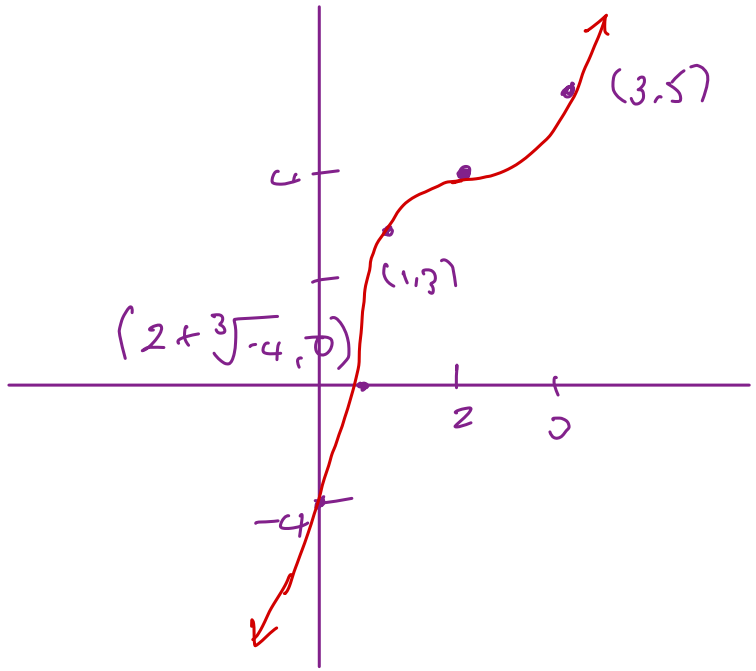
Note that the sign inside is negative but the graph translates in the positive direction.

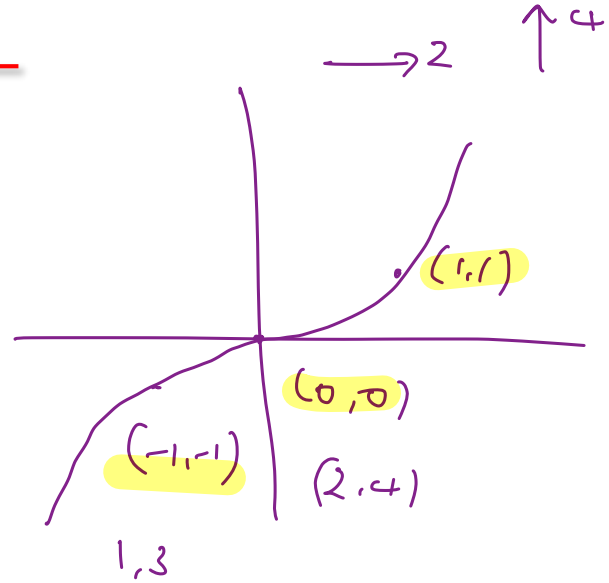




Example 1

Sketch the graph of the function $y = (x-2)^3 + 4$.





y int $x = 0$ $\therefore y = -4$

$y = 0$ $\quad 0 = (x-2)^3 + 4$

$-4 = (x-2)^3$

$\sqrt[3]{-4} = x - 2$

$x = 2 + \sqrt[3]{-4}$

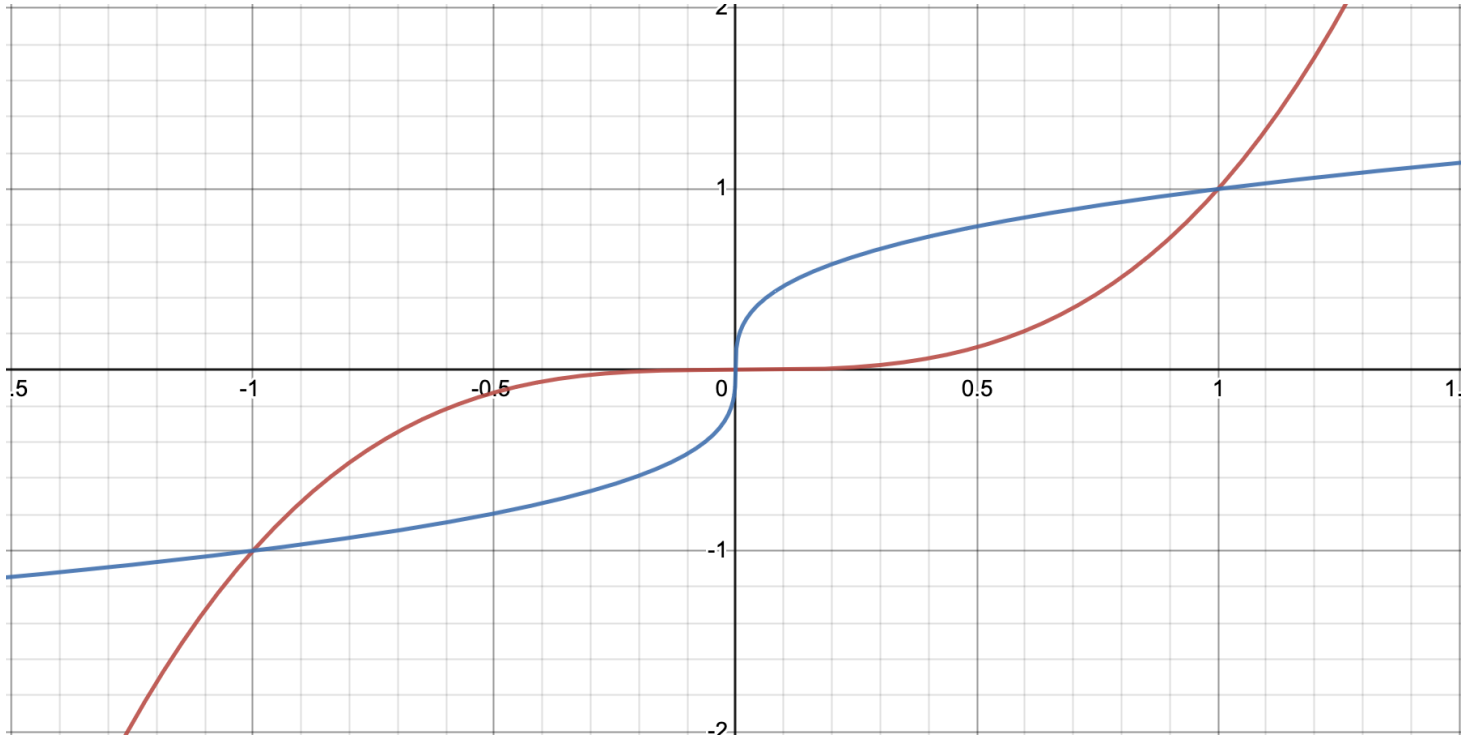


Comparing the graphs of $y = x^3$ and $y = x^{\frac{1}{3}}$ $y = \sqrt[3]{x}$

The more we look at graphs and their properties the more we see that much of the learning is repeated.

There are certain patterns to the learning in Mathematics which can be applied to all areas (whether they are new or not).

Let's look at the two graphs stated in the title.





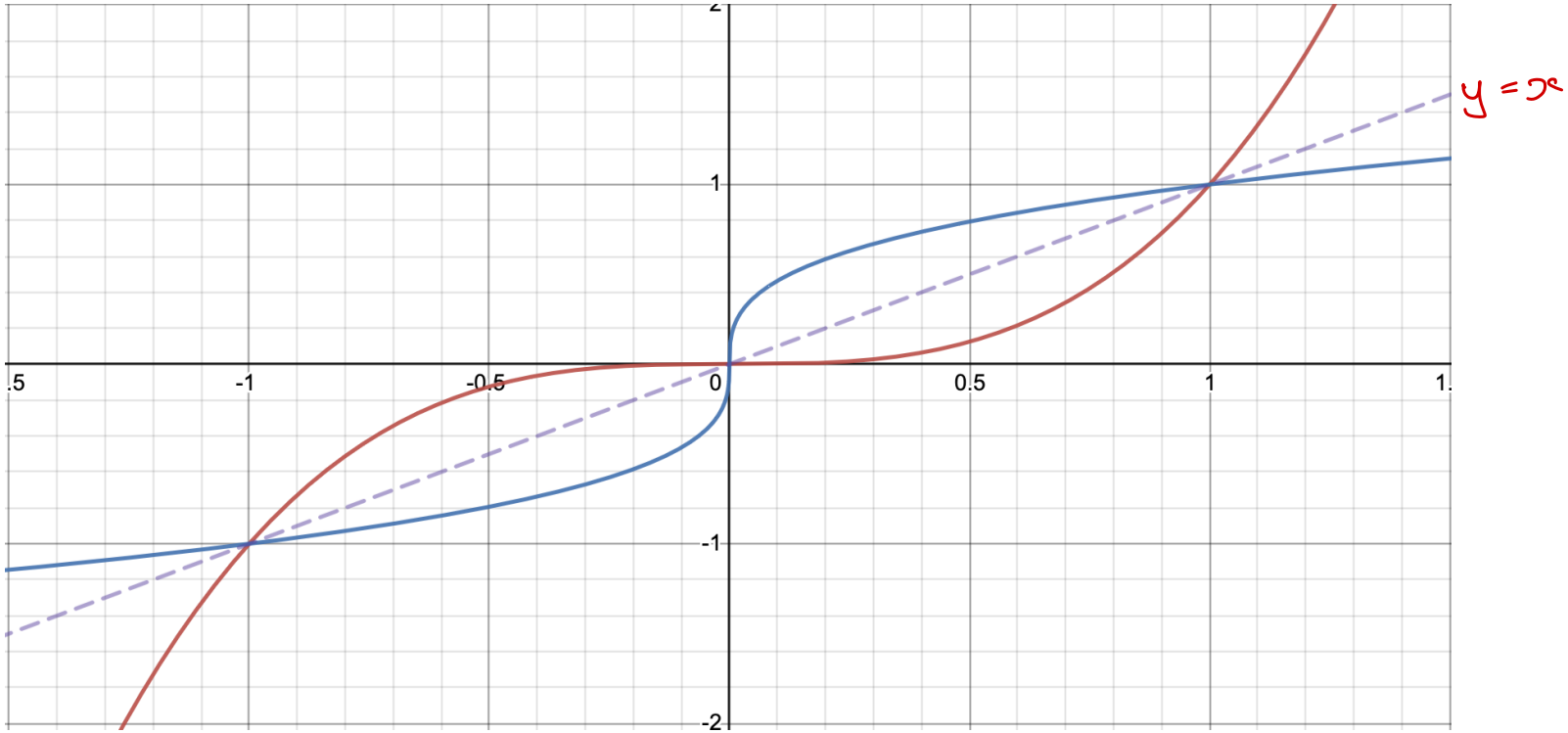
Comparing the graphs of $y = x^3$ and $y = x^{\frac{1}{3}}$

Now I am going to add the line $y = x$.

What you might notice is that the graph of $y = x^3$ seems to have been reflected in the line $y = x$ to become $y = x^{\frac{1}{3}}$

In fact the graph of $y = x^{\frac{1}{3}}$ is the inverse of the graph of $y = x^3$.

Knowing this becomes incredibly important later in the course, but for now, it's useful to know how to draw the graph of $y = x^{\frac{1}{3}}$



It's important to note that $y = x^3$ is a one-to-one function.
As a result, this function will have an inverse.



Example

Sketch the graph of $y = (x-1)^{\frac{1}{3}} - 2$.

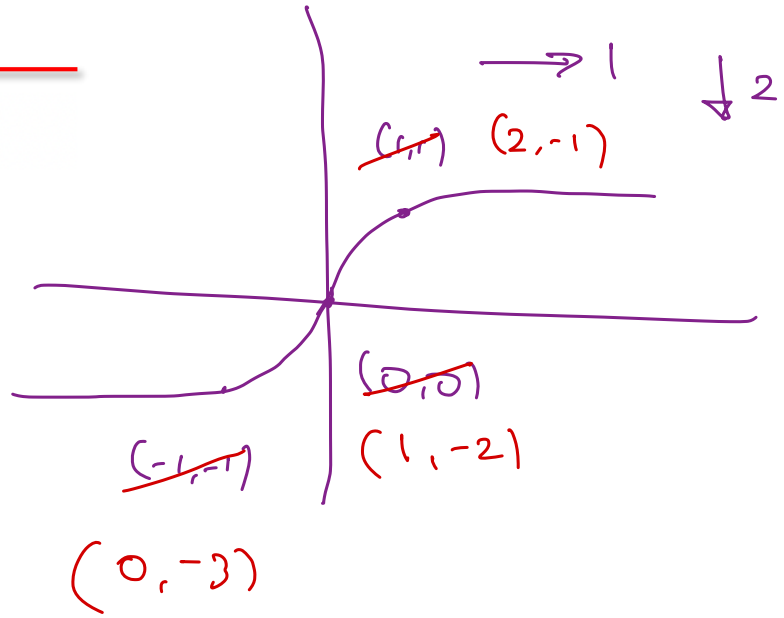
$(9,0)$

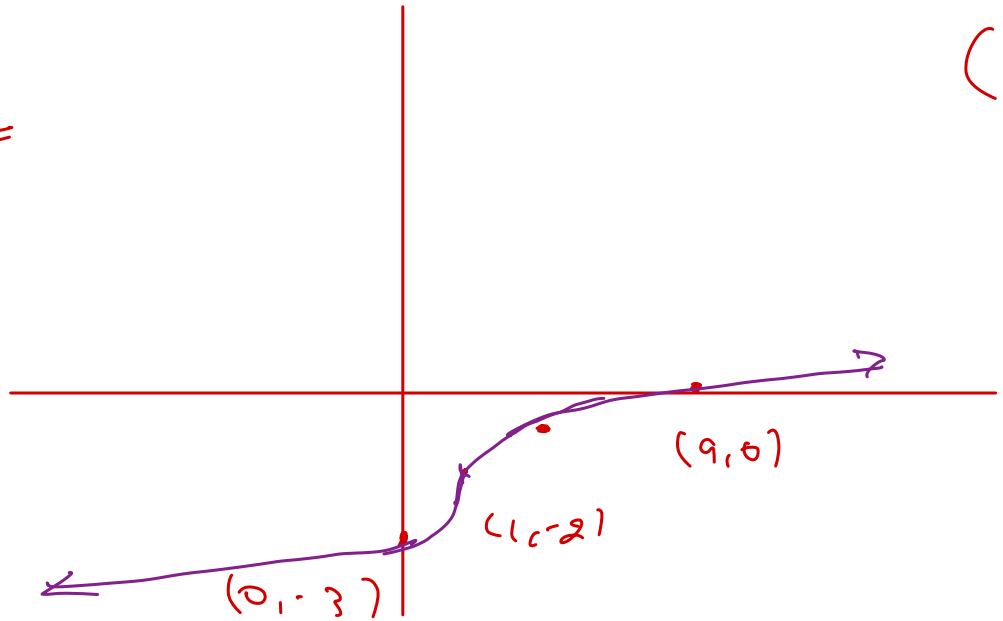
$0 = (x-1)^{1/3} - 2$

$2 = (x-1)^{1/3}$

$8 = x - 1$

$x = 9$







Example

Find the inverse function f^{-1} of $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 2(x-4)^3 + 3$.

$y = 2(x-4)^3 + 3$

$x = 2(y-4)^3 + 3$

$x - 3 = 2(y-4)^3$

$\frac{x-3}{2} = (y-4)^3$

$\sqrt[3]{\frac{x-3}{2}} = y - 4$

$y = 4 + \sqrt[3]{\frac{x-3}{2}}$

$\therefore f^{-1}: \mathbb{R} \to \mathbb{R}, f^{-1}(x) = 4 + \sqrt[3]{\frac{x-3}{2}}$



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