Set notation and sets of numbers

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means by set notation
- Be able to use and understand the following terms
    - Set
    - Element
    - Subset
    - Venn Diagram
    - Intersection
    - Disjoint
    - Empty set
    - Union
- Understand the difference between:
    - Natural numbers
    - Integers
    - Rational numbers
    - Irrational numbers
- Understand what is meant by Interval notation
- Be able to convert between different set notation

RECAP

This is the first part of this section of the course but is arguably one of the most important as the foundations provided from this chapter span the whole of Year 11 and 12.

One of the most confusing parts of Methods 1 and 2 is the Mathematical language used. I have seen, time and again, that when my students understand the different words and notation they can see through and answer any question with confidence.

OK. They need strong algebra skills too … but the notation is the key.

This section shows you some of the notation and explains how it is going to be used.

Set notation

A set is nothing more than a collection of objects. We normally use a letter to stand for the set. The letter we choose is arbitrary:

$$A = \{1, 3, 5, 7, 9\}$$

Note the use of the curly brackets!

Each object is called an element (or member) of the set. We use the $\in$ sign to stand for an element of the set.

$$3 \in A$$

We can also use $\notin$ to mean that something is NOT an element of a set.

$$2 \notin A$$

A subset would contain numbers in the original set. We use the $\subset$ to mean contained within.

$$\{1, 3\} \subset A$$

If B contains all numbers in A (they are the same) we can say:

$$B \subseteq A$$

Set notation continued

When we have two sets, we can look at overlapping as being a possibility (or intersection).

$$A = \{1, 3, 5, 7, 9\}$$
$$B = \{1, 4, 9, 16, 25\}$$

So, we can use the $\cap$ sign to show an intersection:

$$A \cap B = \{1, 9\}$$

But what if they don’t have anything is common? Well, we say that A and B are disjoint.

$$A \cap B = \emptyset = \{\}$$

What if we want to list all the elements in every set? We can use the $\cup$ sign.

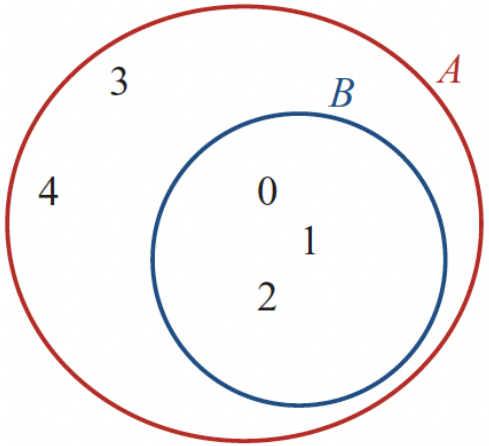
$$A \cup B = \{1, 3, 4, 5, 7, 9, 16, 25\}$$

What about looking at those elements in A which are not in B?

$$A \backslash \mathrm{B} = \{3, 5, 7\}$$

Venn Diagrams

We have met these before and they are a pictorial representation of sets of objects.

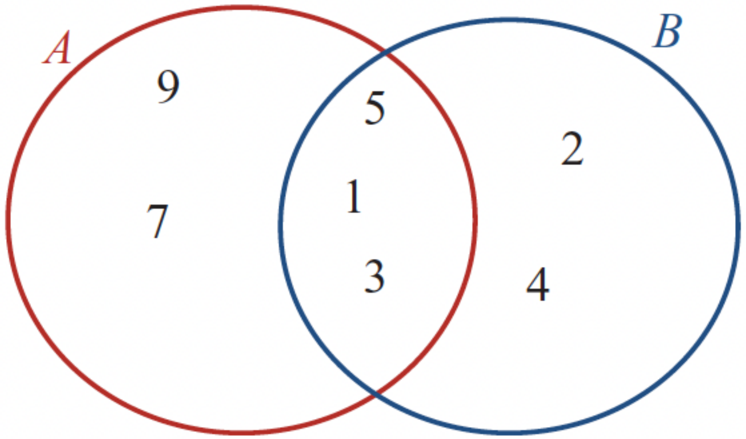


This is a prime example of B being contained completely within A

$B \subset A$

The numbers in the centre are the intersection of A and B.

All the numbers would be A union B.



Examples

For $A = \{1, 2, 3, 7\}$ and $B = \{3, 4, 5, 6, 7\}$, find:

a $A \cap B$ **b** $A \cup B$

$A \cap B = \{3, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

Examples

For $A = \{1, 2, 3, 7\}$ and $B = \{3, 4, 5, 6, 7\}$, find:

a $A \setminus B$ **b** $B \setminus A$

$A \setminus B = \{1, 2\}$

$B \setminus A = \{4, 5, 6\}$

Numbers everywhere!

We can classify numbers in a range of different ways and it's really important to not only know these ways but also the notational short hand for each one. Many of the questions you will face later in this course will use the notation and the ideas in this part of the lesson.

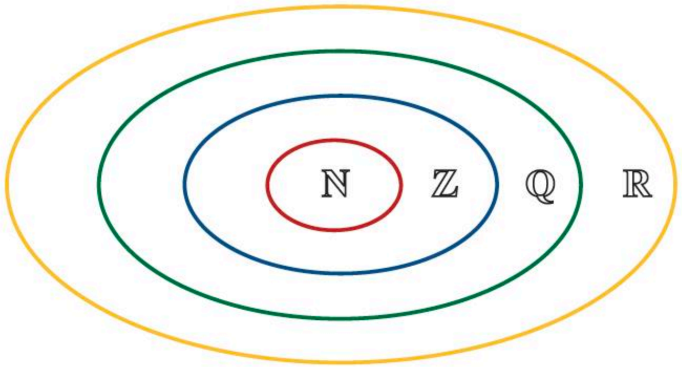
Real Numbers $\mathbb{R}$: Every number under the sun.

Natural Numbers $\mathbb{N}$: Positive whole numbers starting from 1.

Integers $\mathbb{Z}$: Whole numbers which can be positive or negative (including 0)

Rational Numbers $\mathbb{Q}$: Numbers which can be expressed as a fraction.

Irrational Numbers : Those numbers which cannot be expressed as a fraction.



Ever decreasing ovals!

Positive Real Numbers: $\mathbb{R}^+$

Negative Real Numbers: $\mathbb{R}^-$

Real number excluding zero: $\mathbb{R}\setminus\{0\}$

R^- R^+

0

$y = \frac{1}{x}$

Describing a set of numbers

Often times we will need to limit the numbers which we will use in a question and need to find a way to show that limit (or subset).

For example, it is only possible to put real numbers from 0 to infinity under a square root to gain a real answer in return. How would we express this?

One way is the following:

$$\{x: 0 \leq x \leq \infty\}$$

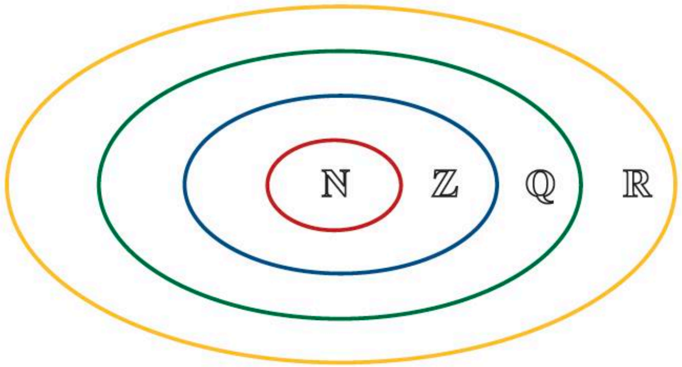
This reads the set of all x from 0 to infinity inclusive

$$\{x: x \neq 0\}$$

This means x can take all real values except zero

$$\{2n: n = \mathbb{N}\}$$

This is the set of all even natural numbers



Ever decreasing ovals!

Interval notation

Whilst the previous slide is perfectly acceptable … we tend to use something a little more succinct; interval notation.

It is simplified by using round and square brackets and a comma.

Round brackets mean the number beside it is **NOT** included.
Square brackets meant he number beside it **IS** included.

$$(0, 10]$$

$(0, 10]$

The above would mean that the set contains all real numbers between 0 and 10. It includes 10 but NOT zero.

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Converting between notation styles:

$$(a, b) = \{x : a < x < b\} \qquad [a, b] = \{x : a \le x \le b\}$$

$$(a, b] = \{x : a < x \le b\} \qquad [a, b) = \{x : a \le x < b\}$$

$$(a, \infty) = \{x : a < x\} \qquad [a, \infty) = \{x : a \le x\}$$

$$(-\infty, b) = \{x : x < b\} \qquad (-\infty, b] = \{x : x \le b\}$$

Interval notation and number lines

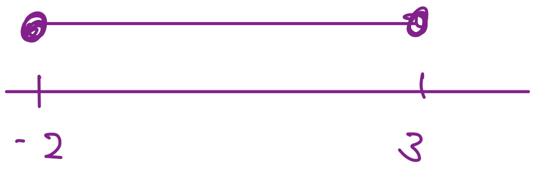
A picture saves 1000 words apparently.

Whilst I can't believe this is always true, it's nice to know we can draw number lines for interval notation!

Illustrate each of the following intervals of real numbers:

a $[-2, 3]$ **b** $(-3, 4]$ **c** $(-2, 4)$ **d** $(-3, \infty)$ **e** $(-\infty, 5]$

a)



b)

c)

d)

e)

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