

Perpendicular and Parallel lines

Year 9
Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- Know what it means by lines being perpendicular and parallel
- Know the how the gradients of parallel and perpendicular lines are related
- Be able to find the gradient of a perpendicular line
- Be able to find the equation of a line given a point and gradient of a parallel or perpendicular line.



RECAP

There have been so many videos and lessons before which have looked at the equation of a straight line. We know that straight lines have the following relationship:

$$y = mx + c$$

$y = mx + c$

All lines have a gradient (except vertical lines) which can be found using rise over run.

There are certain lines which have a very interesting relationship with each other – or their gradients do!

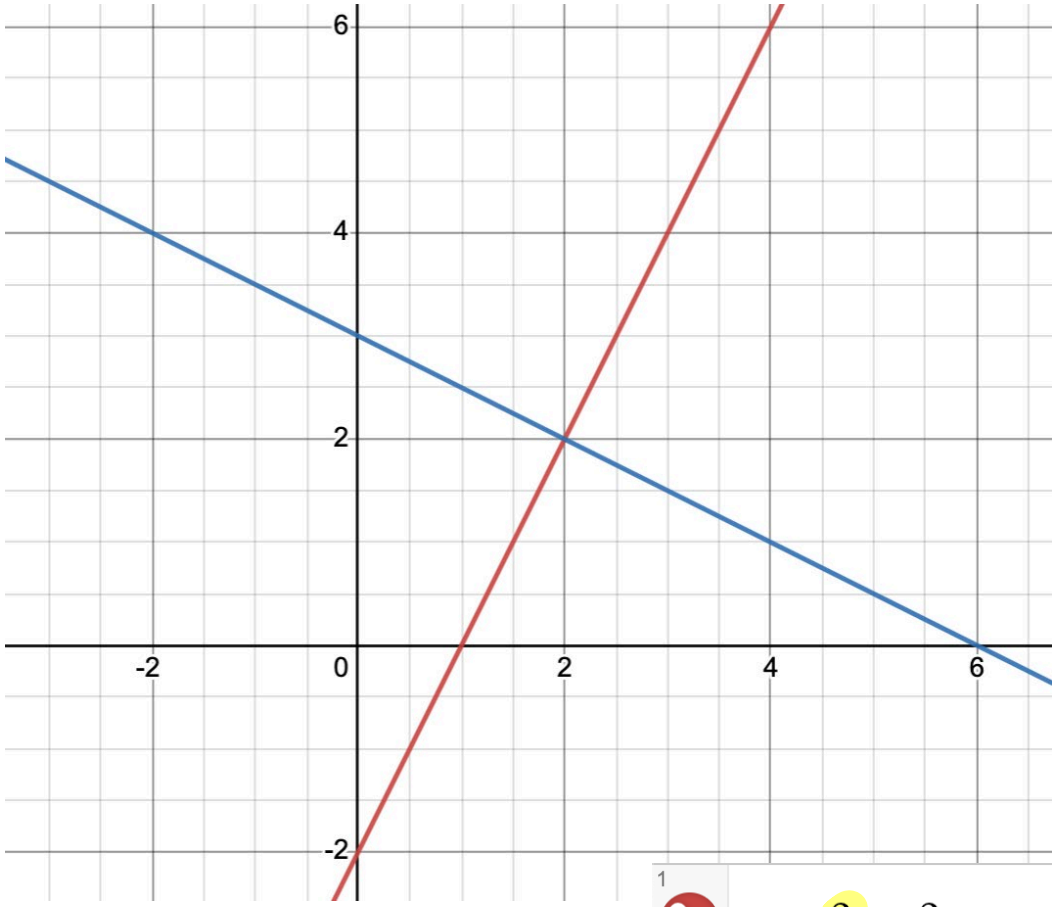


What do you notice about the lines and their equations?

Look at the two lines and see if you can see a link between their lines and equations,

2

$-\frac{1}{2}$



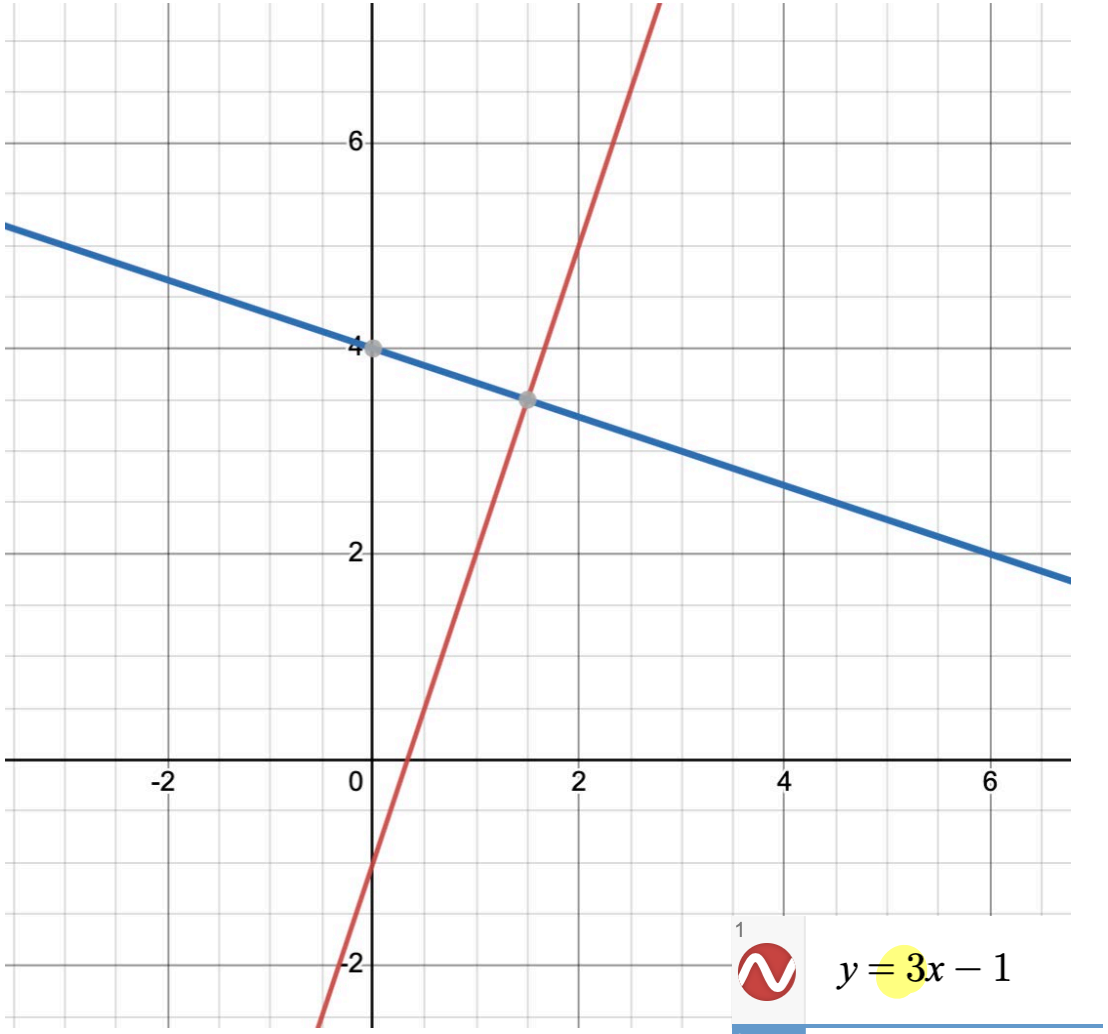
1 $y = 2x - 2$

2 $y = -\frac{1}{2}x + 3$



What do you notice about the lines and their equations?

What about these two lines?



$m = 3$

$m = -\frac{1}{3}$

1 $y = 3x - 1$

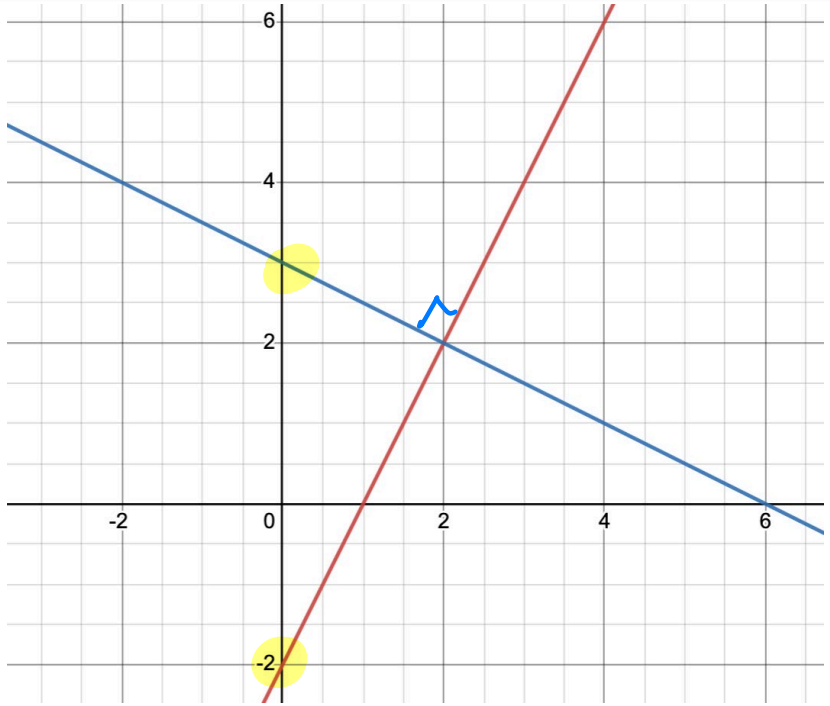
2 $y = -\frac{1}{3}x + 4$



They show two lines which are perpendicular to each other …

When two lines are perpendicular to each other, it means they intersect at 90 degrees (or right angles).

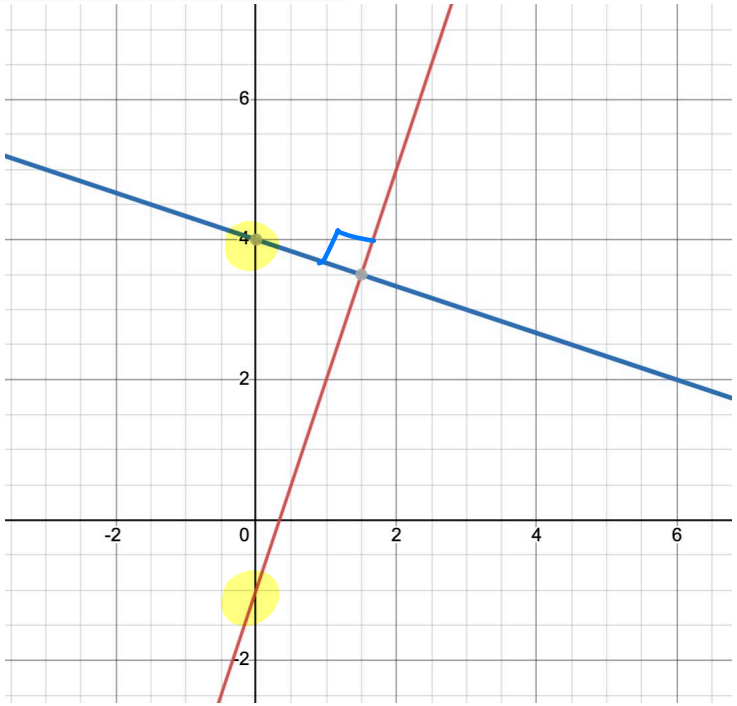
Look at their respect equations. What do you notice?



1. $y = 2x - 2$
2. $y = -\frac{1}{2}x + 3$

$m = \frac{2}{1}$

$m = -\frac{1}{2}$



1. $y = 3x - 1$
2. $y = -\frac{1}{3}x + 4$

$m = \frac{3}{1}$

$m = -\frac{1}{3}$



They show two lines which are perpendicular to each other …

If you've looked closely enough, we are interested only in the **gradients** of each pair.

Do you notice that they are the reciprocal of each other???

That is, in fact, the whole point of this lesson.

Lines which are **perpendicular to each other will have gradients which are the reciprocal and opposite sign to each other.**

1	$y = 2x - 2$
2	$y = -\frac{1}{2}x + 3$

1	$y = 3x - 1$
2	$y = -\frac{1}{3}x + 4$

$m = \frac{4}{1}$ $\quad m_{\perp} = -\frac{1}{4}$

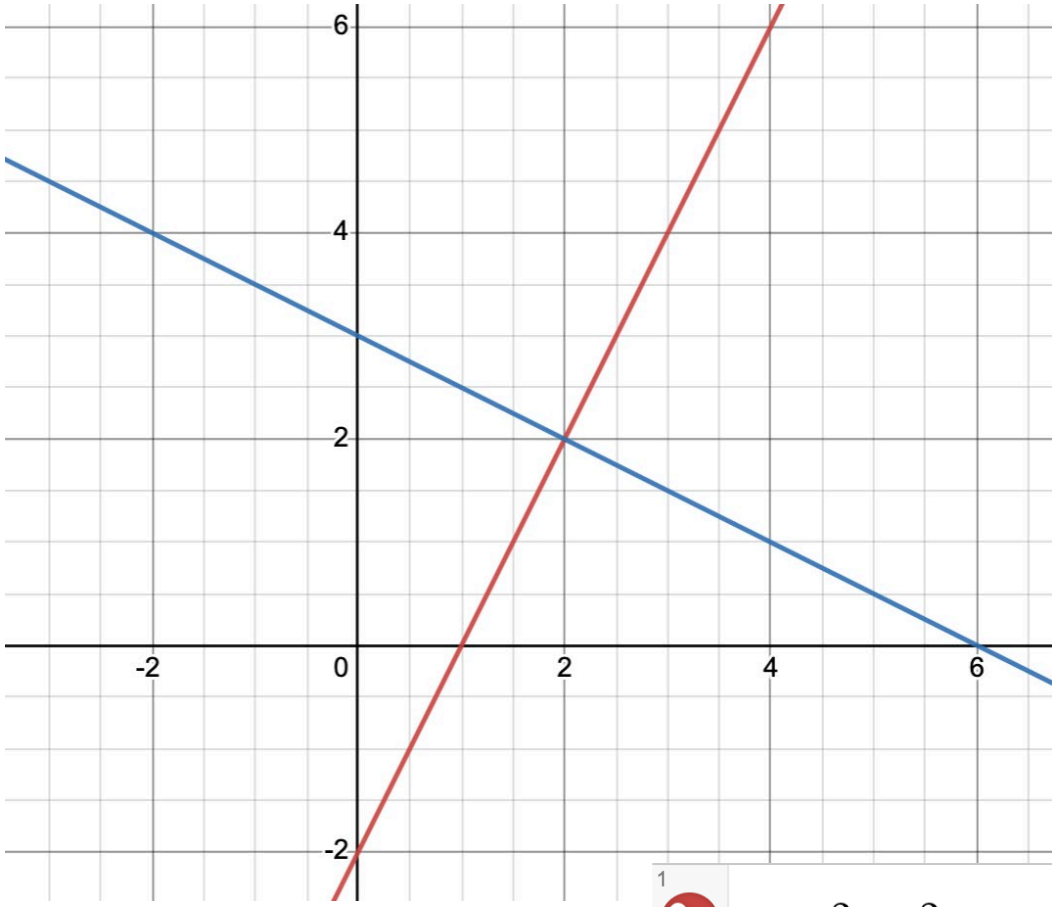
$m = -\frac{2}{3}$ $\quad m_{\perp} = \frac{3}{2}$



Intercepts are NOT the same!!!

One of the most important things to notice is that perpendicular lines will never share the same intercept.

This will become really important when we need to find equations of perpendicular lines.



1 $y = 2x - 2$

2 $y = -\frac{1}{2}x + 3$



Parallel lines

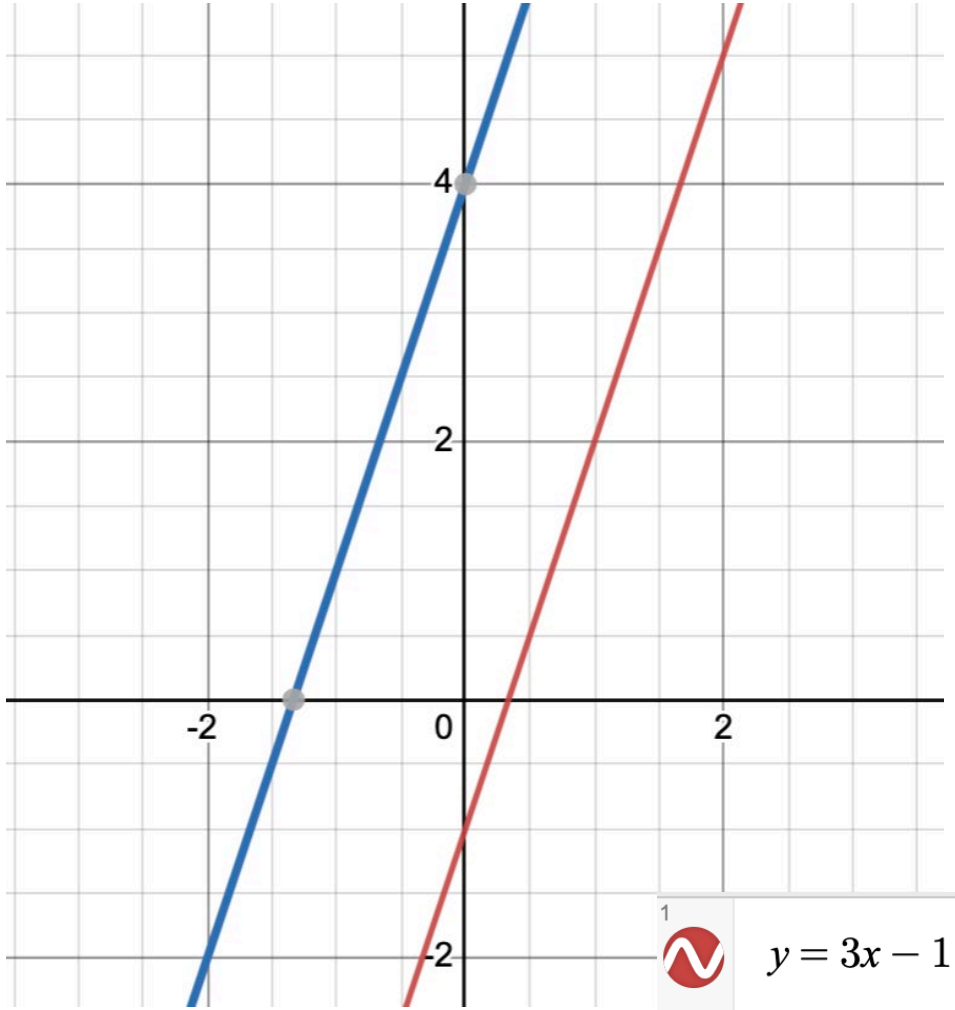
We have met these before.

Parallel lines will have the same gradient and (**generally**) different intercepts too.

So … we now have ways to identify if lines are parallel or perpendicular to each other!

If the gradients are the same then they will be **parallel.**

If the gradients are the opposite sign and reciprocal of each other, they are **perpendicular**.



1 $y = 3x - 1$

2 $y = 3x + 4$



An easier way to find perpendicular gradients

There has GOT to be an easier way to find the gradients of perpendicular lines!!!

Yup! We might see the following:

$$m_2 = -\frac{1}{m_1}$$

$m_1 . m_2 = -1$

Or it can even be written as:

$$m_1 \times m_2 = -1$$

$\frac{1}{2} \cdot \frac{-2}{1} = -1$

These are the same …

I tend to (1) swap the sign and (2) take the reciprocal …



Examples

Find the gradients which are perpendicular to the following:

- $m = 3$
- $m = -2$
- $m = \frac{1}{4}$
- $m = -\frac{1}{5}$

$m = \frac{3}{1}$ $\quad m_{\perp} = -\frac{1}{3}$

$m = -\frac{2}{1}$ $\quad m_{\perp} = \frac{1}{2}$

$m = \frac{1}{4}$ $\quad m_{\perp} = -\frac{4}{1} = -4$

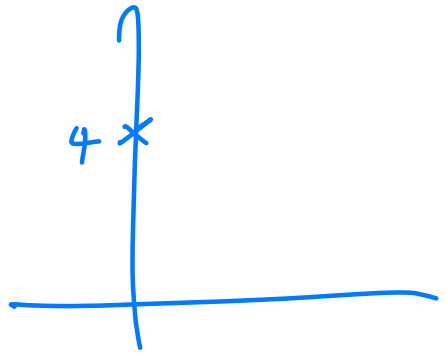
$m = -\frac{1}{5}$ $\quad m = \frac{5}{1} = 5$



Find equations of parallel lines

We have already shown that lines which are parallel to each other share the same gradient but have **different intercepts**.
This is important when given questions such as the following:

Example:
Find the equation of a line which is parallel to $y = 3x - 1$ and passes through $(0, 4)$.



$y = 3x - 1$

$y = mx + c$

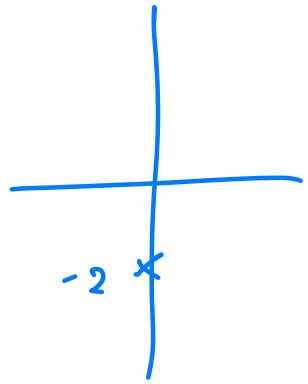
$y = 3x + 4$



Find equations of parallel lines

We have already shown that lines which are parallel to each other share the same gradient but have **different intercepts**.
This is important when given questions such as the following:

Example:
Find the equation of a line which is parallel to $y = 4x + 3$ and passes through $(0, -2)$.



$$y = mx + c$$

$$y = 4x - 2$$



Find equations of perpendicular lines

So, it's going to be important for each question to work out the new value of 'c' each time. We can do this like we have been doing in other lessons by using $y = mx + c$ and one coordinate.

We can now do complicated examples like:

Find the equation of a line which is perpendicular to the line $y = 2x - 3$ and passes through $(0, -1)$.

$m = \frac{2}{1}$

$m_{\perp} = -\frac{1}{2}$

$y = mx + c$

$y = -\frac{1}{2} \cdot x - 1$



Find equations of perpendicular lines

What about this example?

Find the equation of a line which is perpendicular to the line $y = -3x + 1$ and passes through $(0, 4)$

$m = -\frac{3}{1}$

$m_{\perp} = \frac{1}{3}$

$y = mx + c$

$y = \frac{1}{3} \cdot x + 4$



Extension: Perpendicular bisectors

When a line is cut in half by another line, we call this the **perpendicular bisector.**

We can find the equation of this line by using some of the maths in this lesson and the last one.

We know we can find the midpoint of a line. This is a coordinate!

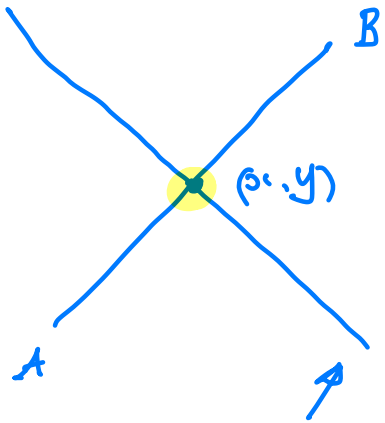
We can also find the gradient of any line.

So, with the gradient and the midpoint, we can find the equation of a **perpendicular bisector.**

Example: Find the midpoint and then the equation of the perpendicular bisector between the two points $A(1,1)$ *and* $B(3,5)$

$m_{AB} = 2$

$m = \frac{4}{2} = 2$



$m = -\frac{1}{2}$

$mid_{AB} = (2,3)$

$y = mx + c$

$3 = -\frac{1}{2} \cdot 2 + c$

$(2,3)$

$3 = -1 + c$

$c = 4$

$\therefore y = -\frac{1}{2}x + 4$



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