

Introduction to transition matrices

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to set up a transition matrix.
- To be able to interpret a transition matrix.



Recap

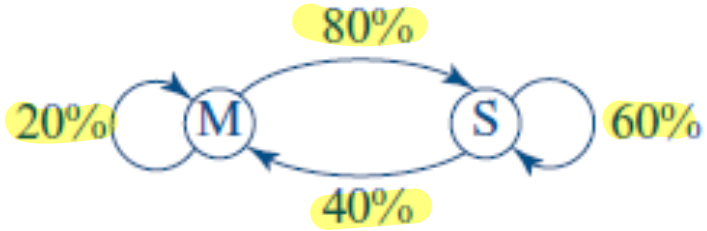


We have, in the past lessons, been looking at how matrices can be added, subtracted, raised to a power, multiplied by each other and a scalar. We have also looked at how they can be used to find the solutions to simultaneous equations. We have also looked at how we can apply Matrices in real world situations.

Everything changes in life. Nothing seems to be constant.

For some things in life, we can model them using Matrices. We can also model the changes which might take place. For example, let's look at the following diagram.

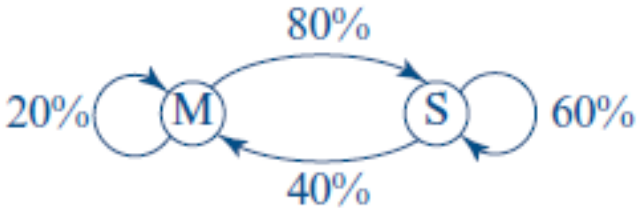
What does it show us?





A transition diagram

Here is the "worded" version of the diagram:

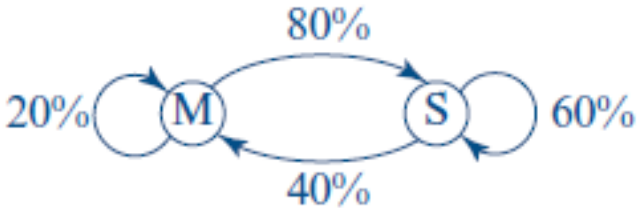


Suppose that in a certain town you have two choices of activity each Saturday night: going to the movies (M) or staying at home (H). Suppose further that it has been established that:

- 20% of people who go to the movies this Saturday will go again next Saturday (and hence, 80% of people who go to the movies this Saturday will stay home next Saturday).
- 40% of people who stay home this Saturday will go to the movies next Saturday (and hence, 60% of people who stay home this Saturday will stay home again next Saturday.)



A transition matrix



We can turn the transition diagram into a transition matrix.

The following are really important things to note about transition matrices:

1. The columns must always add to 1 (one).
2. The columns relate to the "start state". For example, "today"
3. The rows relate to the "end state". For example. "tomorrow"
4. The numbers are always decimals less than one and stand for percentages
5. The transition matrix is always square.

$$T = \begin{matrix} & \text{from} \\ & \begin{matrix} M & S \end{matrix} \\ \begin{matrix} \\ \text{to} \\ \\ \end{matrix} & \begin{matrix} M \\ S \end{matrix} \end{matrix} \begin{bmatrix} 0.2 & 0.4 \\ 0.8 & 0.6 \end{bmatrix}$$



A transition matrix

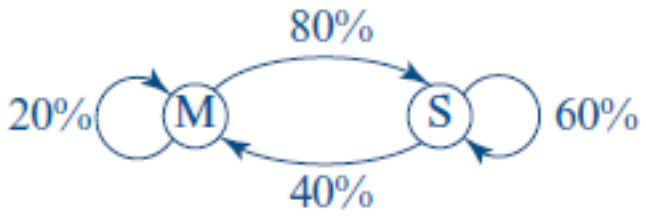
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this Saturday

	M	S	
	0.2	0.4	M
	0.8	0.6	S

next Saturday



Example: setting up a transition matrix

An amusement park has two rides for pre-school children. These are the Ferris wheel (F) and the Merry-go-round (M). After each ride, children can choose which ride they want to go on next. The park observes the following:

- 70% of children on the Ferris wheel will go on the Ferris wheel again.
- 40% of children on the Merry-go-round will go on the Merry-go-round again.

a Find the percentage of children on the Ferris wheel who will go on the Merry-go-round next time.

b Find the percentage of children on the Merry-go-round who will go on the Ferris wheel next time.

c Construct a transition matrix that describes the transition from one step to the next for this situation.

30%.

60%.

from

F M

$$T = \begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix} \begin{matrix} F \\ M \end{matrix}$$

to



Example: setting up a transition matrix

An observant sales assistant has noted whether customers buy hot food (H) or cold food (C) in the store for lunch. They notice that:

- 80% of customers who buy hot food for lunch will buy hot food again the next day.
- 60% of customers who buy cold food for lunch will buy cold food again the next day.

Construct a transition matrix that describes the percentages of customers choosing hot or cold food for lunch from one day to the next.

M → T → W → Th

....

today

H C

$$T = \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} \begin{matrix} H \\ C \end{matrix}$$ tom



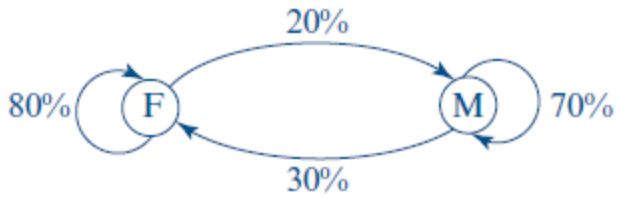
Example: Using a transition matrix

Creating a transition matrix is important, but the numbers in each element mean something. They can be used to extract real world information.

One of the most important things you can do this year is fully understand what a transition matrix does.

Let's look at the following transition matrix and diagram. What do the numbers mean?

$$T = \begin{array}{c} \\ F \\ M \end{array} \begin{array}{c} \text{this ride} \\ \begin{array}{cc} F & M \end{array} \\ \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \end{array} \begin{array}{l} F \\ M \end{array} \text{ next ride}$$





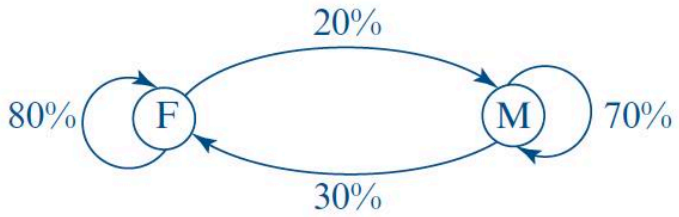
Example: Using a transition matrix

The following transition matrix, T, and its transition diagram can be used to describe the movement of children between a Ferris wheel and a Merry-go-round.

80% of 140

80 ÷ 100 × 140

$$T = \begin{array}{c} \\ F \\ M \end{array} \begin{array}{c} \text{this ride} \\ \begin{array}{cc} F & M \end{array} \\ \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \end{array} \begin{array}{l} \\ F \\ M \end{array} \text{ next ride}$$



a What percentage of children on the Ferris wheel are predicted to go on the:

i Ferris wheel next time? 80%

ii Merry-go-round next time? 20%

b 140 children went on the Ferris wheel. How many of these children do you expect to go on the:

i Ferris wheel next time? 112

ii Merry-go-round next time? 28

c If 140 children went on the Ferris wheel and 80 children went on the Merry-go-round, how many children do you predict will go on the Ferris wheel next time?

80% of 140 + 30% of 80 = 136.

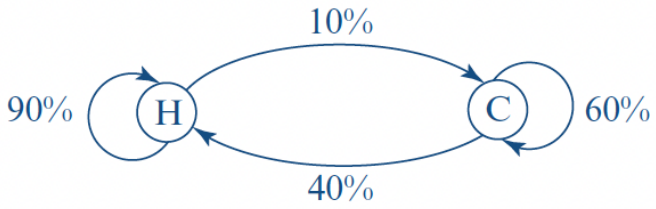


Example: Using a transition matrix

The following transition matrix, T, and its transition diagram can be used to describe customers choosing between a hot and a cold meal, given their previous choice.

90% of 400

$$T = \begin{array}{c} \\ H \\ C \end{array} \begin{array}{c} \begin{array}{cc} \text{today} & \\ H & C \end{array} \\ \begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix} \end{array} \begin{array}{l} H \\ C \end{array} \text{ tomorrow}$$



90% } 100%
10% }

a Of the customers who purchased hot food today, what percentage of people do you expect to buy hot meals and cold meals tomorrow?

b If the store sold 400 hot meals on one day, how many of those who purchased hot meals do you expect to purchase hot meals the next day?

360

c If the store sold 400 hot meals and 500 cold meals on one day, how many hot meals in total do you expect them to sell the next day?

90% of 400 + 40% of 500 = 360 + 200 = 560



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