

Using matrices to model road and communication networks

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to summarise relationships in a network in a matrix.
- To be able to represent a social network in a communication matrix.



Recap

We have, in the past lessons, been looking at how matrices can be added, subtracted, raised to a power, multiplied by each other and a scalar. We have also looked at how they can be used to find the solutions to simultaneous equations.

But how are they used in real world situations?

Well, they can be used to model and describe communications networks and road networks.

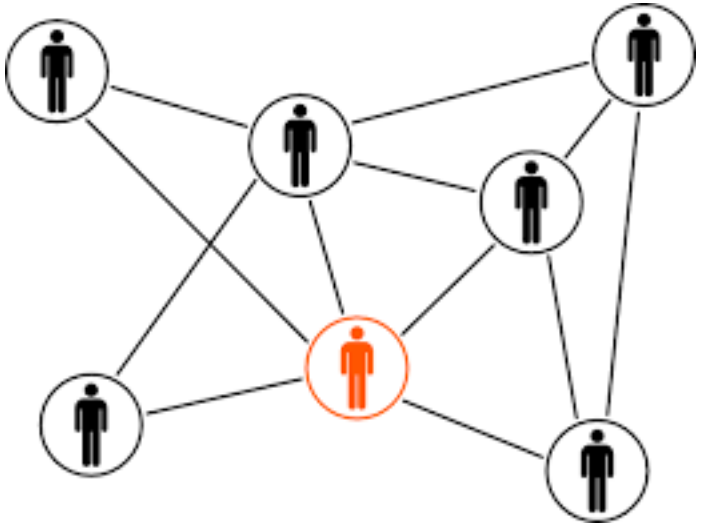
I know … exciting stuff right?





What is a network?

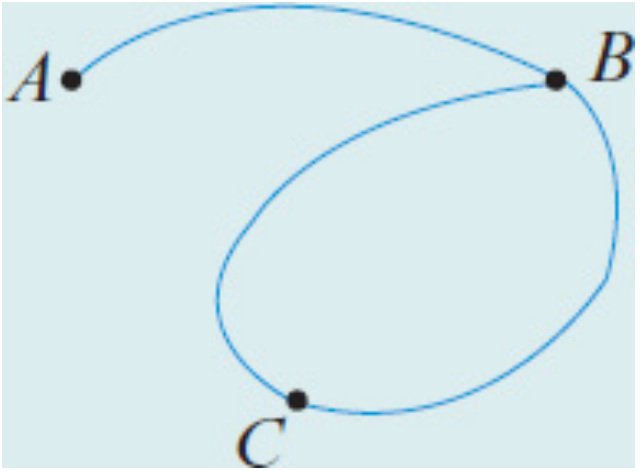
By watching this video (if you are!) you are using a network. A network connects things together using cables, roads, wireless signals, to name but a few. Those connections can be shown using a matrix.





Example: Using a matrix to represent connections

The network diagram on the right shows road connections between three towns, A,B and C



- Use a matrix to represent the road connections. Each element should describe the number of ways to travel directly from one town to another.
- What information is given by the sum of the elements in column B?

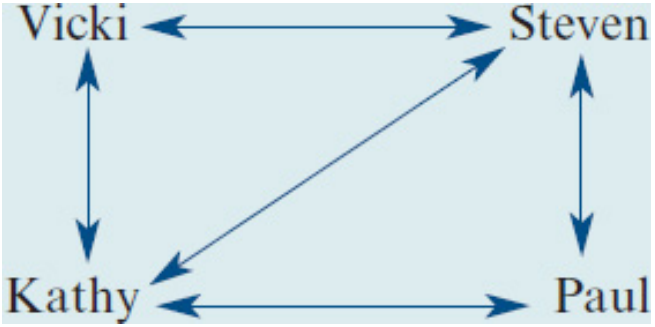
$$\begin{matrix} A & B & C & \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} & & & \begin{matrix} A \\ B \\ C \end{matrix} \end{matrix}$$



Communication matrices

Social media, for its good and bad, connects people. It allows us to speak with lots and lots of people. The idea of speaking and communicating can also be expressed using a matrix.

The diagram on the right shows the number of ways that four people can communicate with each other.



We can see that Vicki can talk directly to Steven and Kathy. She can't speak to Paul. Perhaps Paul speaks a different language.

Steven on the other hand can speak with three people.

$$N = \begin{matrix} V \\ S \\ P \\ K \end{matrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} V & S & P & K \end{matrix}$$



One-step and two-step communications

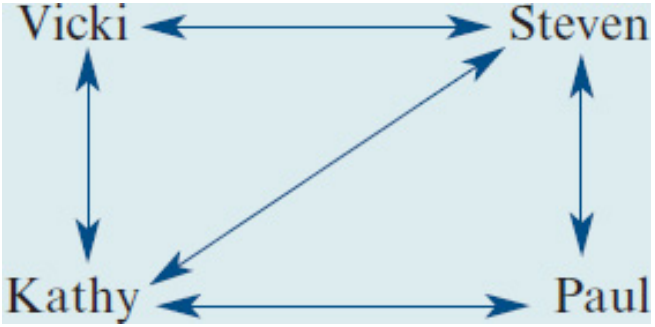
Whilst Vickie can't speak directly with Paul, she can send a message through two other people.

She can pass a message through Steven or Kathy.

When this takes place, we call it a two-step communication.

Vicki -> Steven -> Paul
Vicki -> Kathy -> Paul

We can express two-step communications in a matrix too!





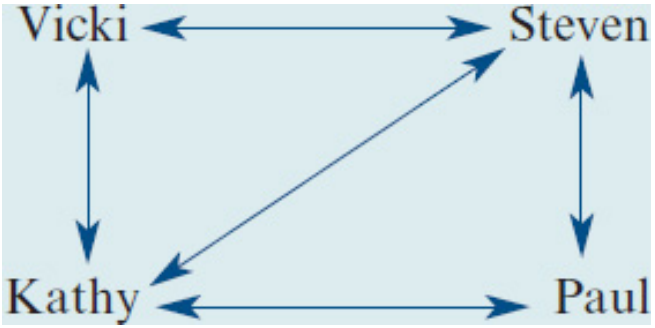
One-step and two-step communications

Was there a quicker way to do this?

Actually yes! Using the CAS and a little trial and error.

Let's assume our one-step communication matrix is given as shown. What happens when we find N^2 on the CAS?

What would N^3 represent?



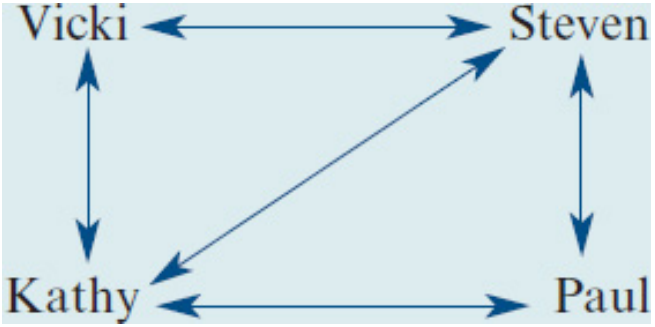
$$N = \begin{array}{c} \begin{matrix} V & S & K & P \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} V \\ S \\ K \\ P \end{matrix}$$

$$N^2 = \begin{array}{c} \begin{matrix} V & S & K & P \end{matrix} \\ \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \end{array} \begin{matrix} V \\ S \\ K \\ P \end{matrix}$$



Example: Communication matrices

The diagram shows the communications within a group of friends. In this diagram:



- Use a matrix, N to record the presence or absence of direct communication links between the people in the network. Use the first letter of each name to label the columns and rows. Explain how the matrix can be read, using the labels.
- Why is the matrix symmetric?
- What information is given by the sum of column K?
- N^2 gives the number of two-step communications between people. Namely, how many ways one person can communicate with someone via another person. Find the matrix N^2, the square of matrix N.
- Use the matrix N^2 to find the number of two-step ways Kathy can communicate with Steven, and write the connections.
- In the N^2 matrix, there is a 3 where the S column meets the S row. This indicates that there are three two-step communications which Steven can have with himself. Explain how this can be given a sensible interpretation.



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