

Inverse matrices and solving simultaneous equations using matrices

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to find the inverse of a matrix.
- To be able to solve simultaneous equations using matrices.



Recap

In the previous lessons we have been looking at the “basics” of matrices. How to:

- Add and subtract them
- Multiply by a scalar
- Multiply two matrices together (by hand and using the CAS)

The one thing I haven’t said is how to **divide** matrices.

That is because you cannot. However, we can use a nice trick to help us. This trick uses the ideas covered in the previous lessons.



Recap: The identity matrix

In the previous lesson we came across a nice little matrix called the **Identity matrix.**

Seemed a little pointless at the time as, when we multiply something by the identity matrix, we are really just multiplying by one.

However, it comes in handy when we are told the following:

$$A \times A^{-1} = I$$
$$A^{-1} \times A = I$$

Where *I* is the identity matrix.

OK. But what is this A^{-1} matrix?

$$A \times A^{-1} = I$$

$$A^{-1} \times A = I$$



The inverse matrix

Let me just go straight into showing you how to find the inverse of a matrix.

Given a matrix, A, below:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \times \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The inverse if found to be:

$$A^{-1} = \frac{1}{ad - bc} \times \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Notice how the values of a and d have swapped place.
The values in the place of c and d have their signs swapped.

The bottom part of the fraction is called the determinant



The inverse matrix: Using the CAS

$16 - 12 = 4$

OK!

That was a bit nuts, but the good thing is, we can use the CAS to find the inverse of a matrix.

$A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -4 \\ -3 & 8 \end{bmatrix}$

Find the inverse of matrix, A, below:

$$A = \begin{bmatrix} 8 & 4 \\ 3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{8 \times 2 - 3 \times 4}\begin{bmatrix} 2 & -4 \\ -3 & 8 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2 & -4 \\ -3 & 8 \end{bmatrix}$$



The determinant of a matrix

If we look back at what we had before:

$$A^{-1} = \frac{1}{ad - bc} \times \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The bit at the bottom of the fraction is really important and called the **determinant**.

This number, if it isn't zero means that a matrix will have an inverse.
If this number is a zero, then it means a matrix will not have an inverse. This is really important when solving simultaneous equations.

So, $\det(A) = ad - bc$.

$$\det(\text{Matrix}) = ad - bc$$



Example: The determinant of a matrix

Find the determinant of each of the following matrices:

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 7 \\ 2 & -2 \end{bmatrix}$$

$\det(A) = 5 \times 6 - 3 \times 2$

$= 30 - 6$

$= \underline{\underline{24}}$

$\det(B) = (-3)(-2) - 2 \times 7$

$= 6 - 14$

$= \underline{\underline{-8}}$



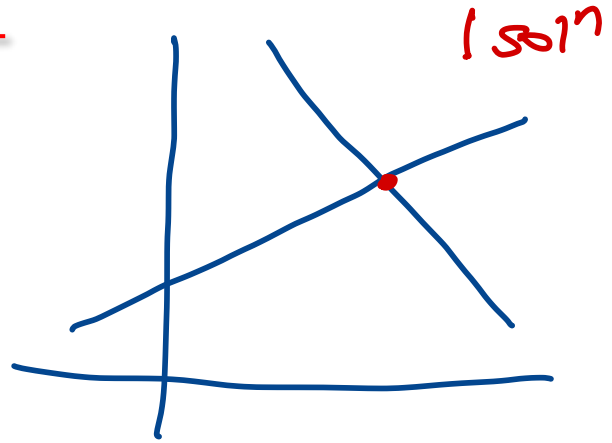
Simultaneous equations

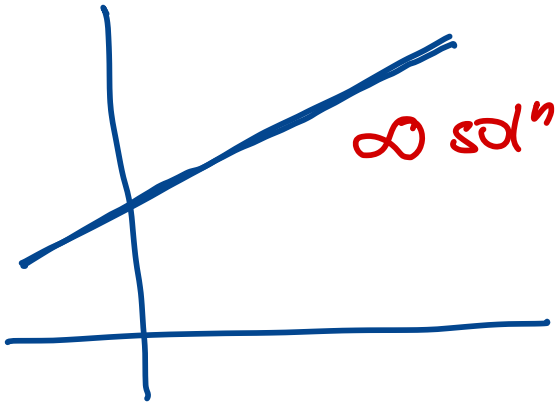
This is a massive subject in itself and looks at finding the point where two lines cross.

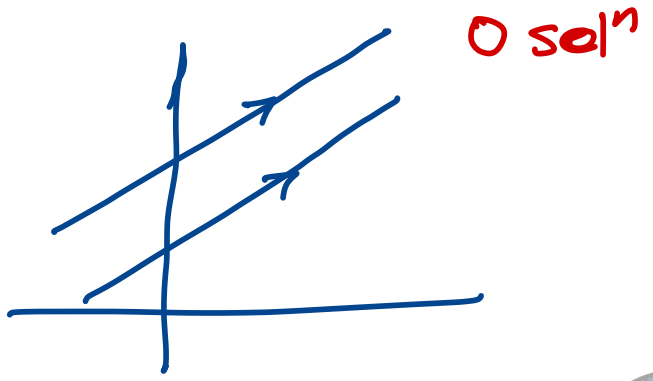
You may have done this in Year 10 using elimination, substitution or graphing. We can actually solve them using matrices.

Two lines can cross in one of three ways; one point, no points, and infinite points.

When the determinant is **anything other than zero**, we know there will only ever be one crossing point. This is why it's good to find the determinant for all matrices.









Solving simultaneous equations using matrices

Let's do some examples to show how to convert simultaneous equations and write them in **matrix form**.

Express the following simultaneous equations as matrices:

$$8x + 2y = 46$$
$$5x - 3y = 19$$

$$\begin{bmatrix} 8 & 2 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 46 \\ 19 \end{bmatrix}$$

$$8x + 2y = 46$$
$$5x - 3y = 19$$



Solving simultaneous equations using matrices

Let's do some examples to show how to convert simultaneous equations and write them in **matrix form**.

Express the following simultaneous equations as matrices:

$$6x - y = 15$$
$$-2x + 5y = -19$$

$$\begin{bmatrix} 6 & -1 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -19 \end{bmatrix}$$



Matrix algebra

When we have written the simultaneous equations in matrix form we have a bit of a problem; we cannot divide matrices.

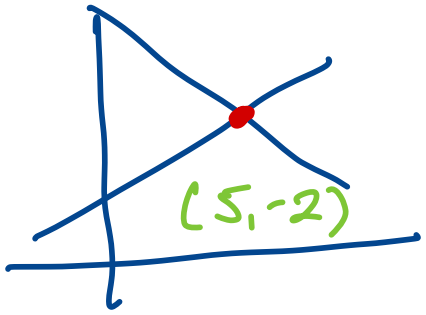
However, we can use some interesting algebra and the identity matrix to make things a little easier for us.

Solve to find x and y:

$$5x + 2y = 21$$
$$7x + 3y = 29$$

$$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21 \\ 29 \end{bmatrix}$$

$$A \cdot X = C$$



$$A.X = C$$

$$A^{-1}.A.X = A^{-1}.C$$

$$I.X = A^{-1}.C$$

$$X = A^{-1}.C$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 21 \\ 29 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$x = 5$

$y = -2$



Short cut and using the CAS

To make things easier for you, we can short cut the learning and just know that we can move the matrix from the start on the left to the right and raise it to the power of negative one.

Solve to find x and y:

$$5x + 2y = 21$$
$$7x + 3y = 29$$

$$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21 \\ 29 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 21 \\ 29 \end{bmatrix}$$

$$=$$

$$\ldots$$



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