# Gradient and direct proportion

Year 9
Mathematics

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# Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- Understand what it means to be proportional
- To know the form of the equation that links two variables that are proportional
- Know what rate of change means
- Know what constant rate of change means

# RECAP

The following table in the textbook for Question 8 was really thought provoking. It tested your **understanding** and not your ability to regurgitate. Given two pieces of information, can we find the third missing piece of the puzzle?

Complete this table showing the gradient, $x$-intercept and $y$-intercept for straight lines.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| **Gradient** | 3 | $-1$ | $\frac{1}{2}$ | $-\frac{2}{3}$ | 0.4 | $-1.25$ |
| ***x*-intercept** | $-3$ | | | 6 | 1 | |
| ***y*-intercept** | | $-4$ | $\frac{1}{2}$ | | | 3 |

# RECAP

In previous lessons we have looked at straight lines and more recently, what the gradient means.

Gradient is a measure of slope and, for straight lines, will be constant for the whole line regardless which two points I choose to measure it.

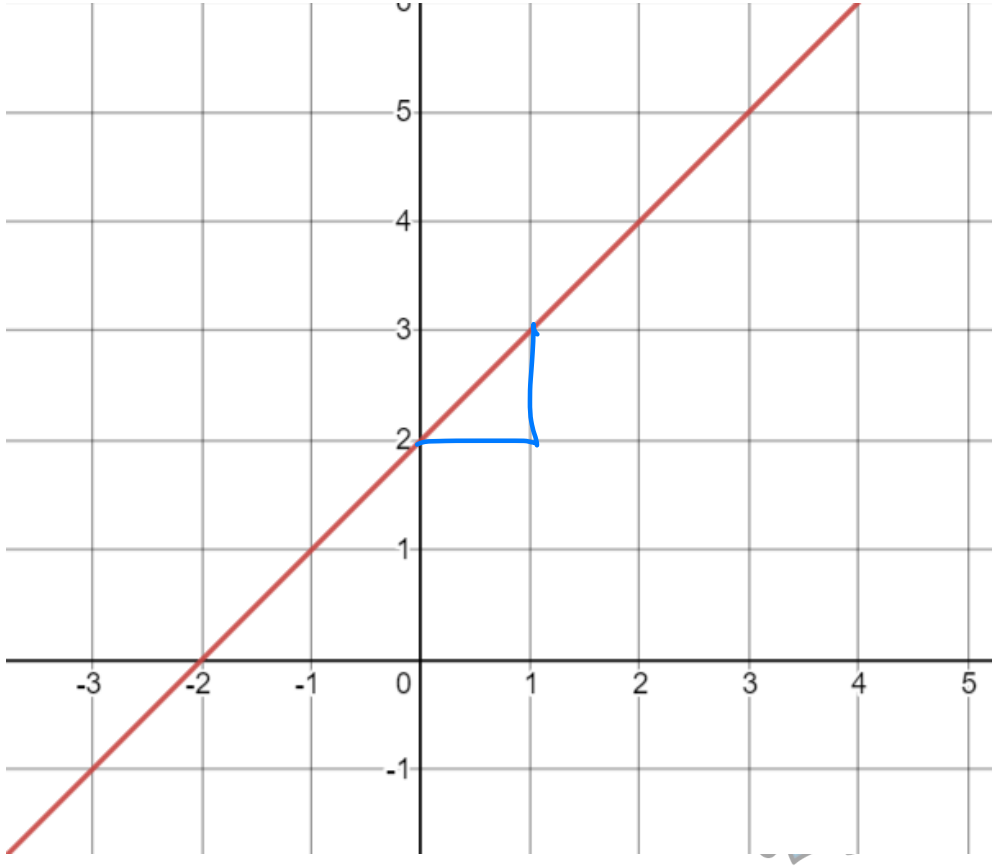
Straight line graphs show **direct proportionality**.

The **rate of change** of the y-value is proportional to the rate of change of the x-value.

So, for things to be proportional they must increase at the same rate.

This is what we're going to look at today.

$$m = \frac{\text{rise}}{\text{run}}$$



# Rate of Change

It is important to note the following:

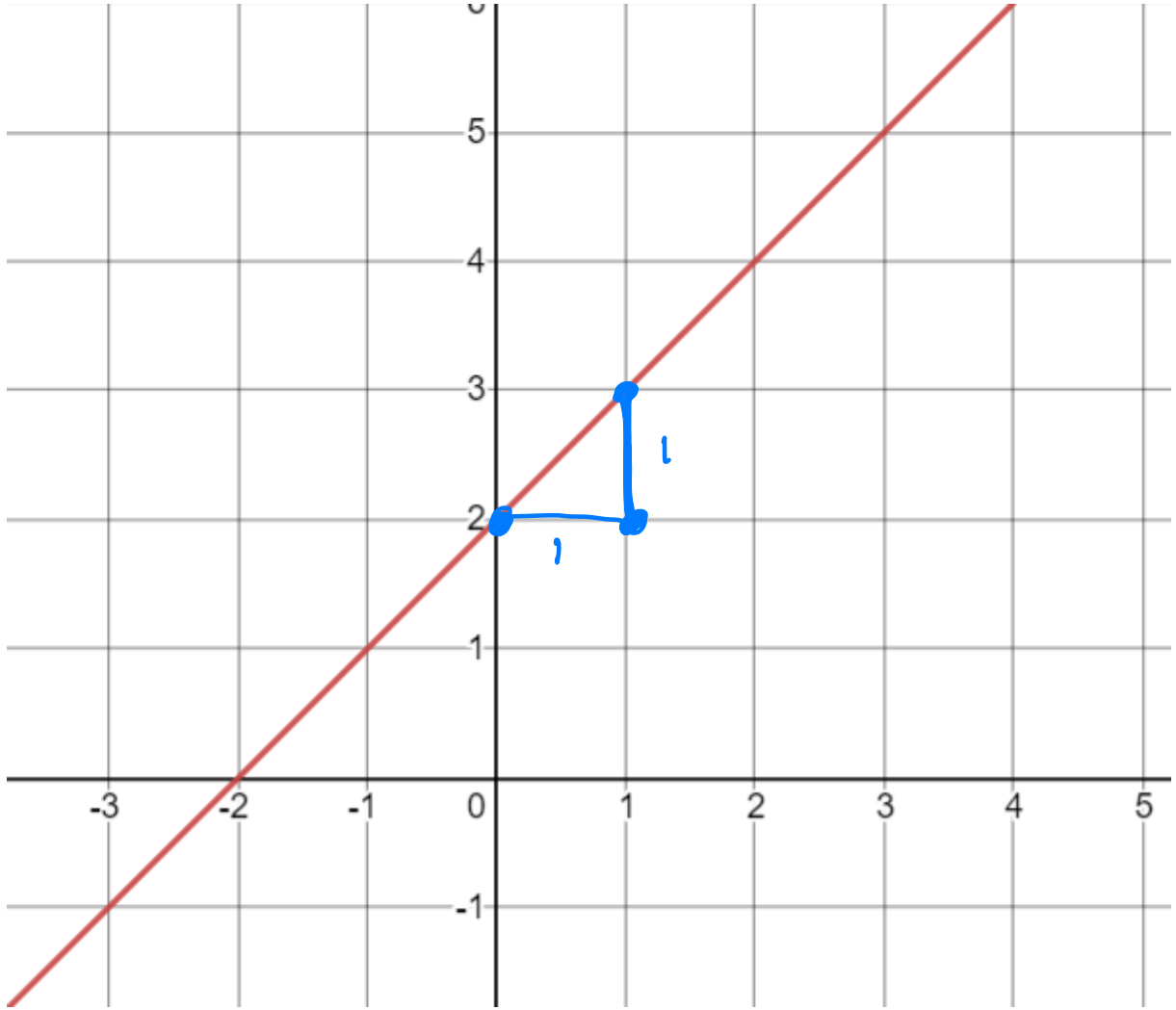
$$Rate\ of\ change = \frac{change\ in\ y - values}{change\ in\ x - values}$$

Hold on … this seems to be the same as:

$$Rate\ of\ change = \frac{y_2 - y_1}{x_2 - x_1}$$

Which is the same as the gradient.

And … Barry strikes again! **Gradient** and **rate of change** are the **same thing**. Sigh.



# Average Speed: Direct Proportion

Average speed is a great one as this is a measure of direct proportion.

If I travel for 5 hours and cover 420 km I can make an assumption about my trip.

The speed ai travel is constant.

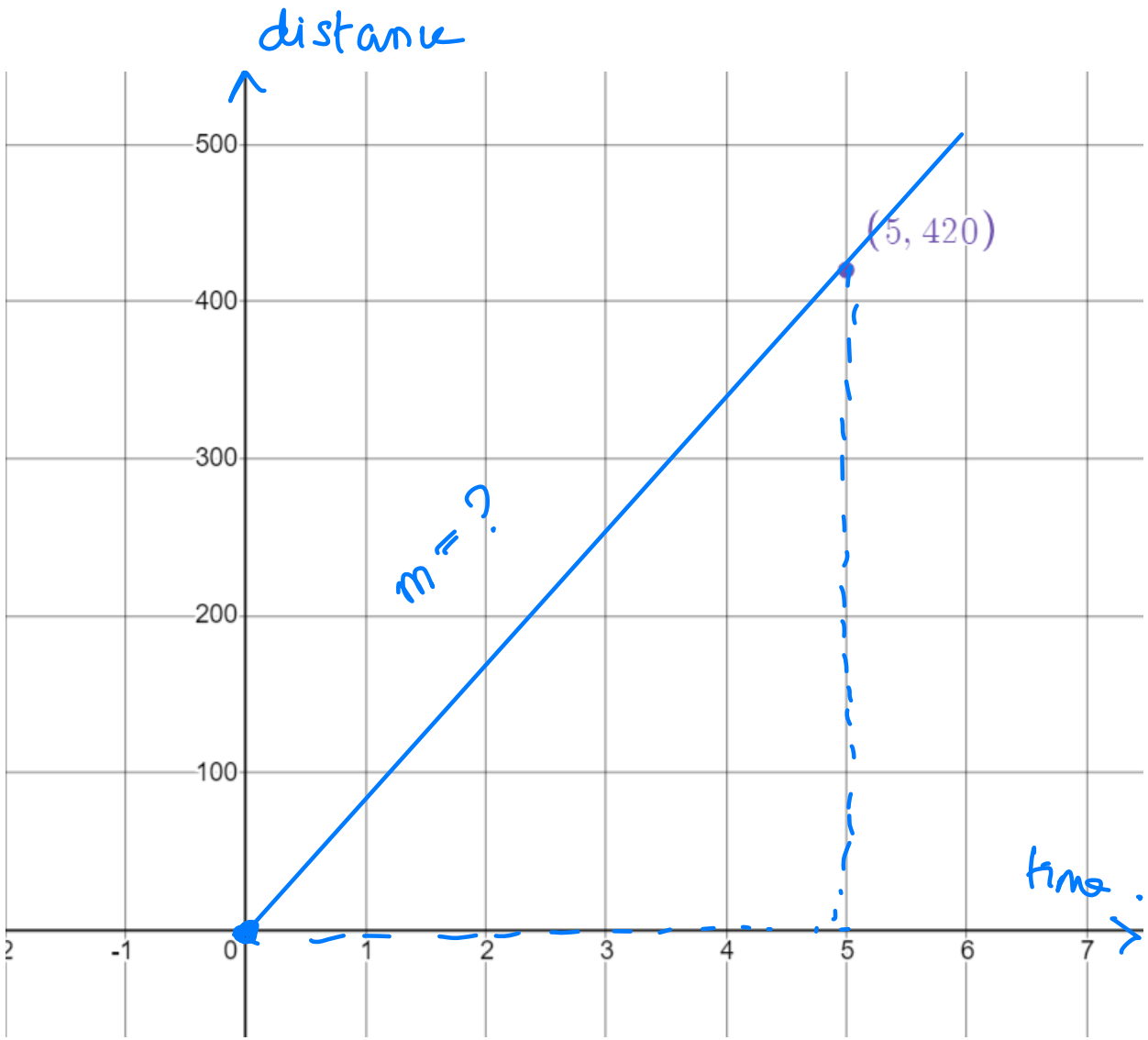
I also notice that the information given is **a point on a graph** which allows me to draw straight lines.

It makes sense that, in 2.5 hours I will have travelled 210km. In 1 hour I will have travelled 84 km.

$m = \frac{420}{5} = 84$

$1\text{hr} = 84\text{ km}$

$\frac{1}{2}\text{hr} = 42\text{ km}$



# Using information from a question to help plot points

The questions you will be looking at will all be direct proportion. The rate of change will be the same. Hence, the question **must** give you one (or two) points to enable you to answer questions (or to draw a graph).

**Example:**
Water is poured into an empty tank at a constant rate. It takes 3 hours to fill the tank with
600 litres.

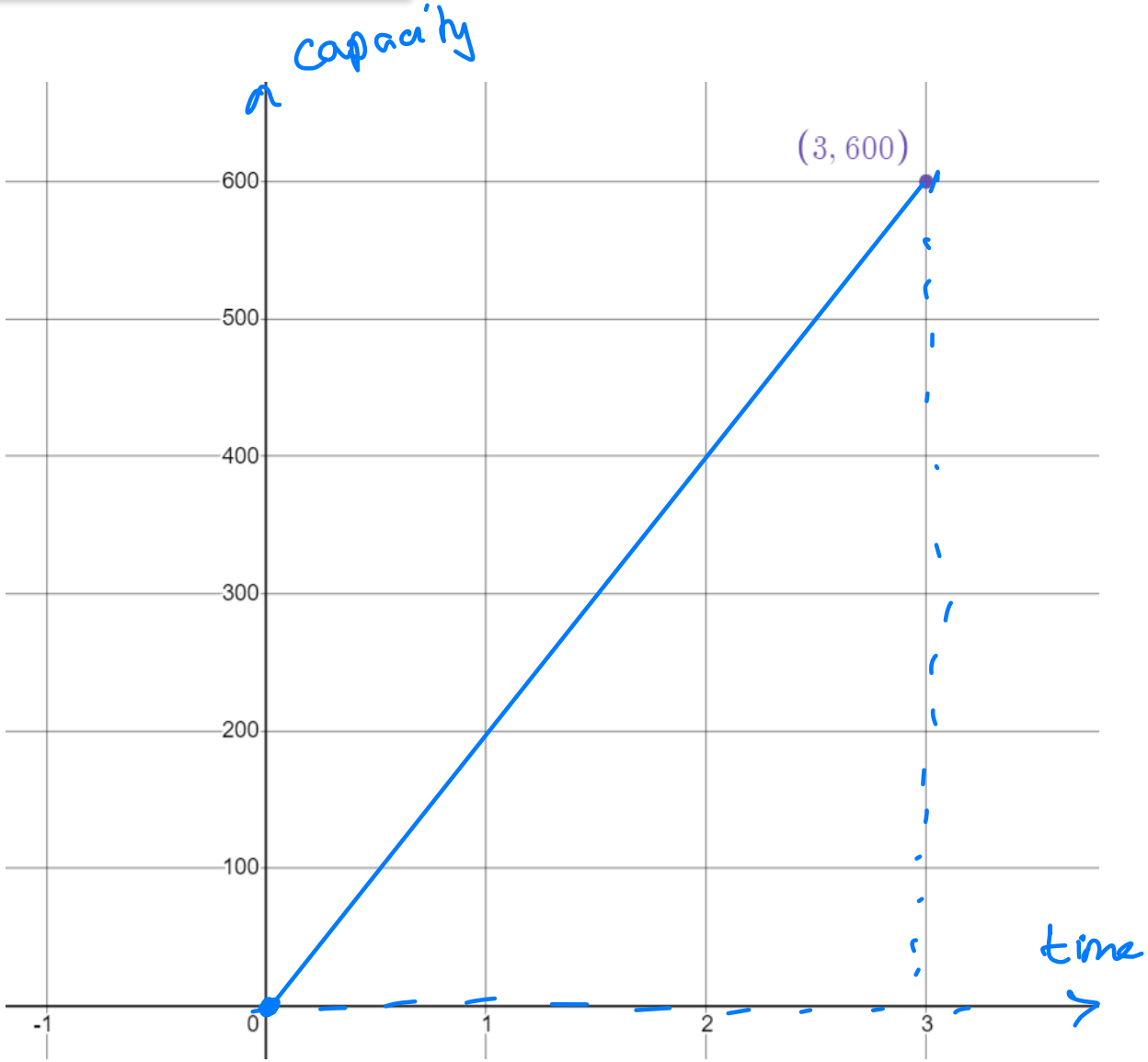
(3, 600)

The highlighted parts are a coordinate so I can draw a straight-line graph.

This will allow me to answer other questions.

$m = \frac{600}{3} = 200$

1 hr = 200 litres

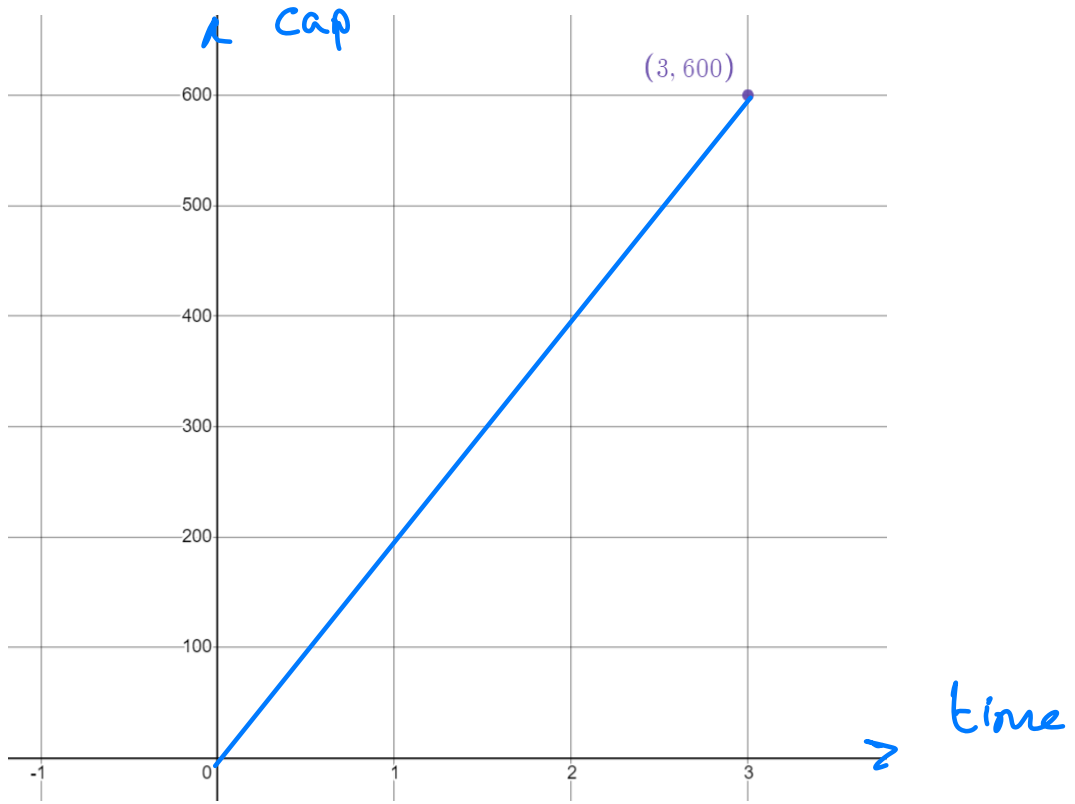


# Finding rules from lines

Hopefully it is become clear that the rule for an equation can be gained from the following scaffold:

$$y\ variable = gradient \times x - variable \pm constant$$

Capacity = 200 x time



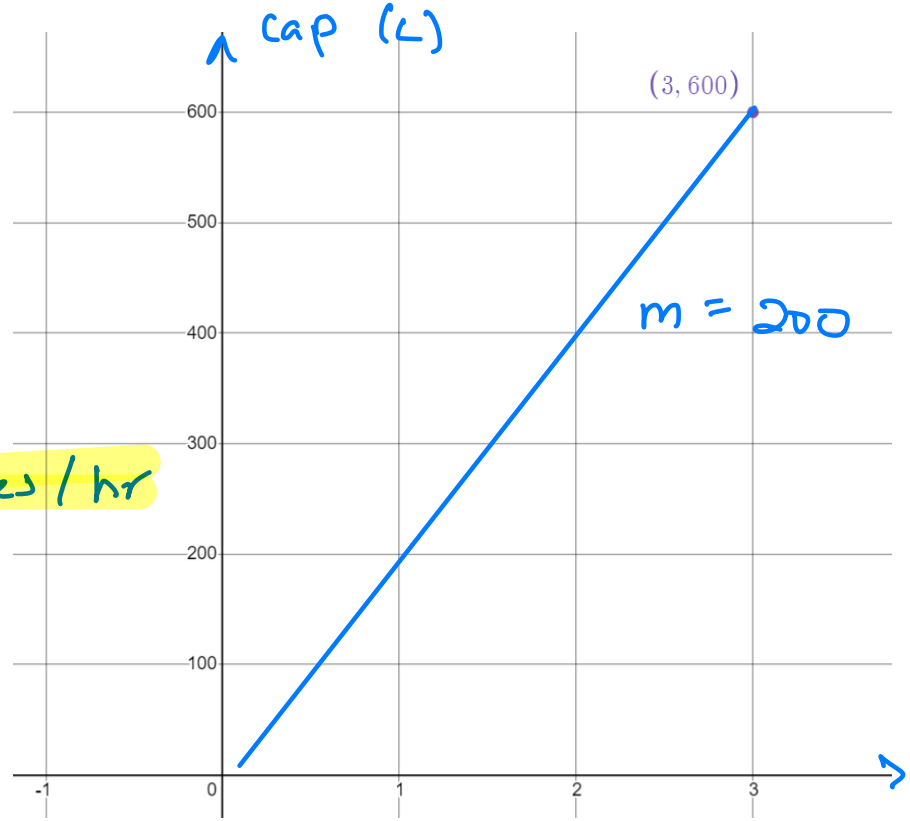

# Using information from a question to help plot points

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) Using $0 \leq t \leq 3$.

c Find:
  i the gradient of your graph
  ii the rule for V.

d Use your rule to find:
  i the volume after 1.5 hours
  ii the time to fill 5000 litres.




rate = $\frac{600}{3}$ = 200 litres / hr

m = 200

$0 \leq t \leq 3$

Cap = 200 x time

Cap = 200 × 1·5

= 300 Litres

5000 = 200 × t

$t = \frac{5000}{200}$

= 25 hrs

# Using information from a question to help plot points

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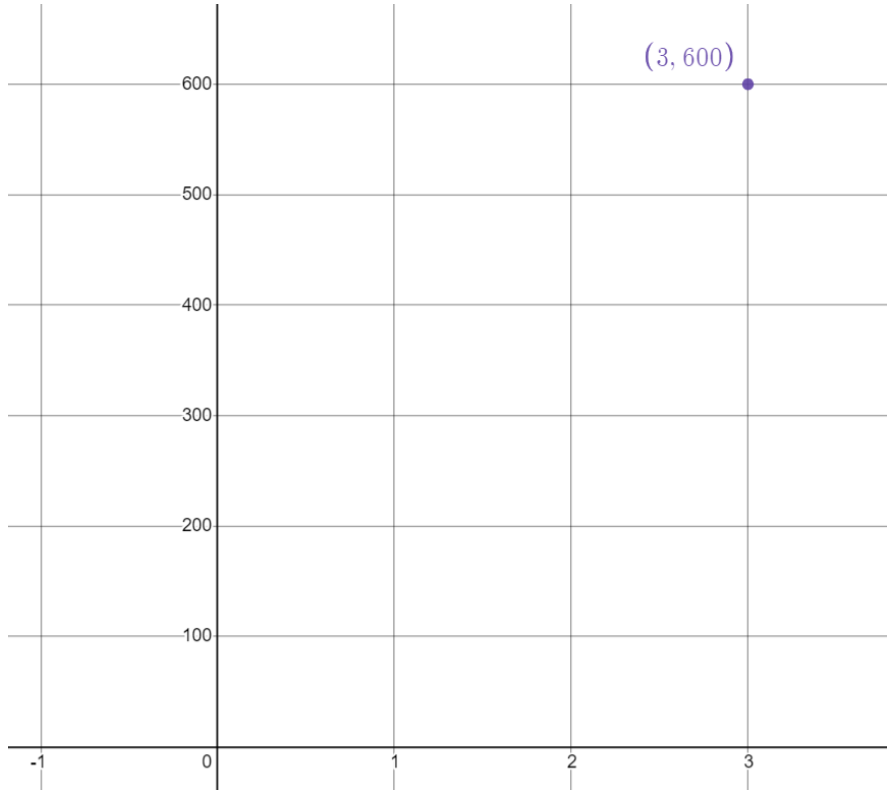



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