



Collinearity

Year 12 Specialist Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Specialist Mathematics course.

- Understand what it means for three points to be collinear.
- Apply the principle to questions



Recap

We have spent a lot of time looking at the properties of vectors and how we can:

- Find the lengths of vectors
- Draw vectors
- Find the angle between vectors and axes
- Find the angle between two vectors
- Express one vector as a projection onto another.

Now we look at another interesting part of vectors: Collinearity.

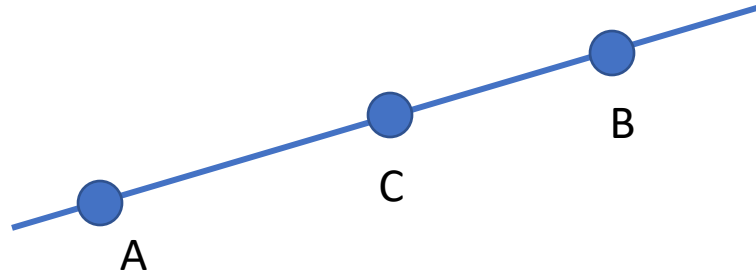


Collinearity

When three or more points all lie on the same line, we say they are **collinear**.

We can use vector mathematics to test if three points all lie on the same line.

For now, it seems to make sense that, when viewing through a vector's lens, the following stands true.



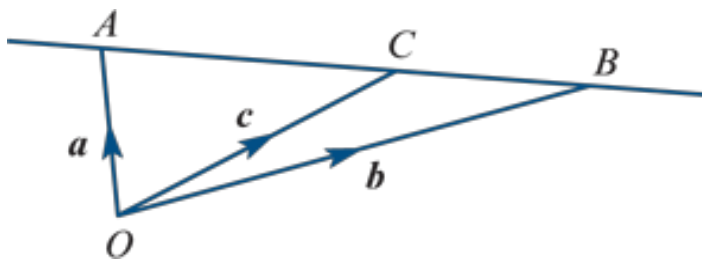
$$\overrightarrow{AC} = m\overrightarrow{AB}$$

This is true if and only if $\overrightarrow{AC}$ and $\overrightarrow{AB}$ are **parallel**



Collinearity: The proof

Consider the following diagram



$$\vec{AC} = m\vec{AB}$$

$$\begin{aligned} c &= \vec{OA} + \vec{AC} \\ &= \underline{a} + m\vec{AB} \\ &= \underline{a} + m(\underline{b} - \underline{a}) \\ &= a + mb - ma \\ &= a(1-m) + mb \\ c &= (1-m)a + mb \end{aligned}$$

This only applies if $c = (1-m)a + mb$

Note that these add to one.



Collinearity: The proof

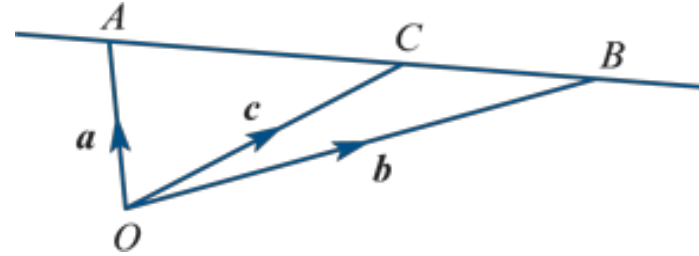
It now makes sense to say that if A, B and C are collinear, then from:

$$c = (1 - m)a + mb$$

$$\overrightarrow{OC} = \lambda \overrightarrow{OA} + \mu \overrightarrow{OB}$$

Where $\lambda + \mu = 1$

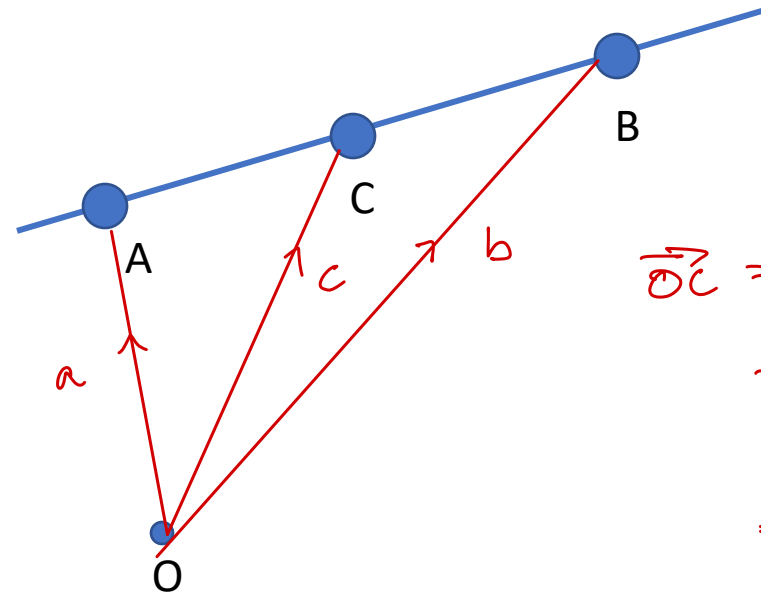
And $0 < \mu < 1$



Example

For distinct points A and B, let $a = \overrightarrow{OA}$ and $b = \overrightarrow{OB}$. Express $\overrightarrow{OC}$ in terms of a and b, where C is:

- the midpoint of AB
- the point of trisection of AB nearer to A
- the point C such that $\overrightarrow{AC} = -2\overrightarrow{AB}$



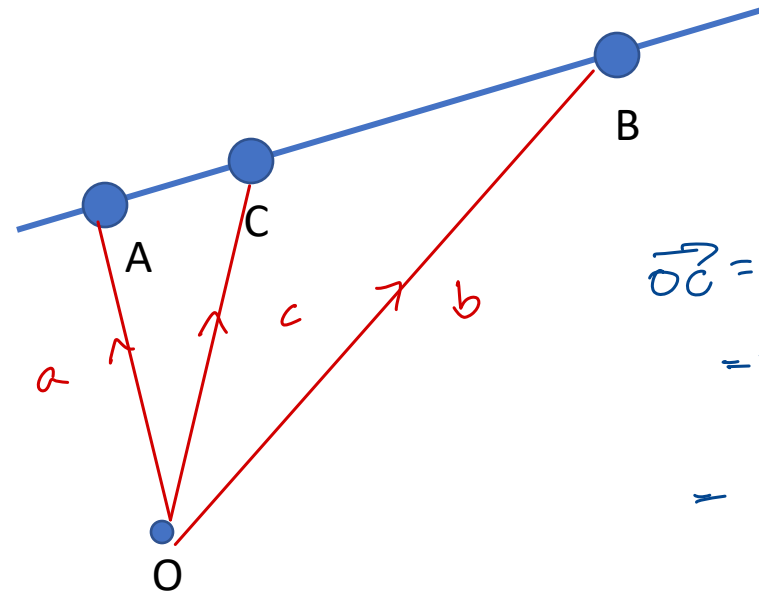
$$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OA} + m\overrightarrow{AB} \\ &= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} \\ &= \underbrace{a} + \frac{1}{2}(b - a) \\ &= \frac{1}{2}a + \frac{1}{2}b \\ &= \end{aligned}$$



Example

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$$\overrightarrow{AC} = \frac{1}{3} \overrightarrow{AB}$$

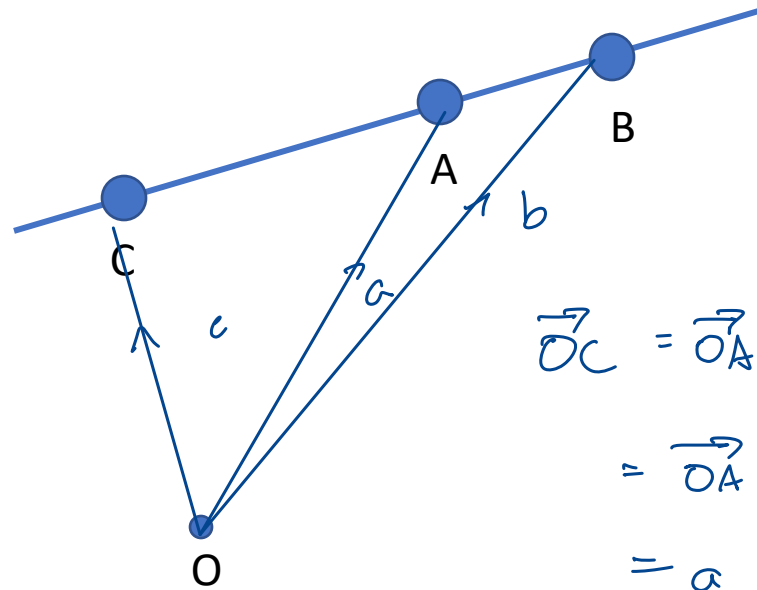
$$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OA} + m \overrightarrow{AB} \\ &= \overrightarrow{OA} + \frac{1}{3} \overrightarrow{AB} \\ &= \underline{a} + \frac{1}{3} (\underline{b} - \underline{a}) \\ &= \underline{\frac{2}{3}a} + \underline{\frac{1}{3}b}\end{aligned}$$



Example

For distinct points A and B, let $a = \overrightarrow{OA}$ and $b = \overrightarrow{OB}$. Express $\overrightarrow{OC}$ in terms of a and b, where C is:

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$$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OA} + \overrightarrow{AC} \\ &= \overrightarrow{OA} - 2\overrightarrow{AB} \\ &= \underline{a} - 2(\underline{b} - \underline{a}) \\ &= \underline{3a} - \underline{2b}\end{aligned}$$

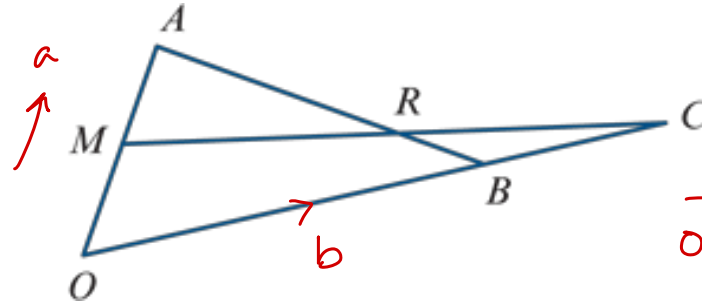


Example

Let $\vec{OA} = a$ and $\vec{OB} = b$, where vectors a and b are linearly independent.

Let M be the midpoint of OA , let C be the point such that $\vec{OC} = \frac{4}{3}\vec{OB}$ and let R be the point of intersection of lines AB and MC .

- Find $\vec{OR}$ in terms of a and b .
- Hence find $AR:RB$



$$\begin{aligned}\vec{OR} &= \vec{OM} + \vec{MR} \\ &= \frac{1}{2}\vec{OA} - \frac{1}{2}ma + \frac{4}{3}mb \\ &= \frac{1}{2}a - \frac{1}{2}ma + \frac{4}{3}mb \\ &= \frac{1}{2}a(1-m) + \frac{4}{3}mb\end{aligned}$$

$$\vec{AR} = n\vec{AB}$$

$$\vec{AR} = n(b-a)$$

$$\begin{aligned}\vec{OR} &= \vec{OA} + \vec{AR} = a + n(b-a) \\ &= (1-n)a + nb\end{aligned}$$

$$\vec{OM} = \frac{1}{2}\vec{OA}$$

$$\vec{OC} = \frac{4}{3}\vec{OB}$$

$$\vec{MR} = m\vec{MC}$$

$$= m(\vec{MO} + \vec{OC})$$

$$= m\left(-\frac{1}{2}\vec{OA} + \frac{4}{3}\vec{OB}\right)$$

$$= m\left(-\frac{1}{2}a + \frac{4}{3}b\right)$$

$$= -\frac{1}{2}ma + \frac{4}{3}mb$$

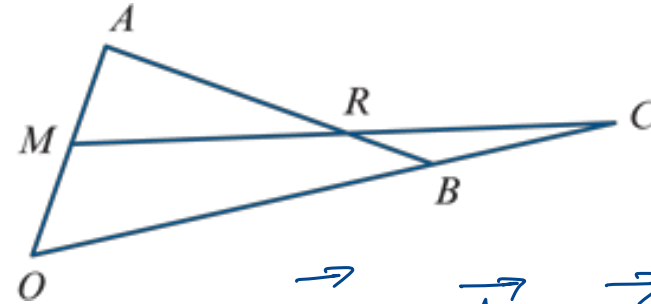


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$$\frac{1}{2}(1-m) = 1-n \quad \frac{4m}{3} = n$$

$$m = \frac{3}{5} \quad n = \frac{4}{5}$$

$$\therefore \overrightarrow{OR} = \frac{1}{5}a + \frac{4}{5}b$$

$$\begin{aligned}\overrightarrow{AR} &= \overrightarrow{AO} + \overrightarrow{OR} \\ &= -a + \frac{1}{5}a + \frac{4}{5}b \\ &= -\frac{4}{5}a + \frac{4}{5}b \\ &= \frac{4}{5}(b-a)\end{aligned}$$

$$AR:RB$$

$$\frac{4}{5} : \frac{1}{5} \quad \therefore \underline{\underline{4:1}}$$



Learning Objectives: Reviewed

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Work to be completed

The following represents the minimum work which should be completed.

The more questions you answer from each exercise, chapter review and Checkpoints the better your chance of gaining an excellent study score in November.

Specialist Mathematics Units 3 and 4 Textbook

Chapter 2

Exercise 2E: Collinearity

Questions: 1, 3, 5 and 7

