# Matrix multiplication

Year 11 General Maths
Units 1 and 2

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## Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- Understand how to perform multiplication of matrices
- Know the rules to be able to perform multiplication of matrices
- Know that, in general, matrix multiplication is not commutative i.e. $AB \neq BA$

$2 \times 3 = 6$

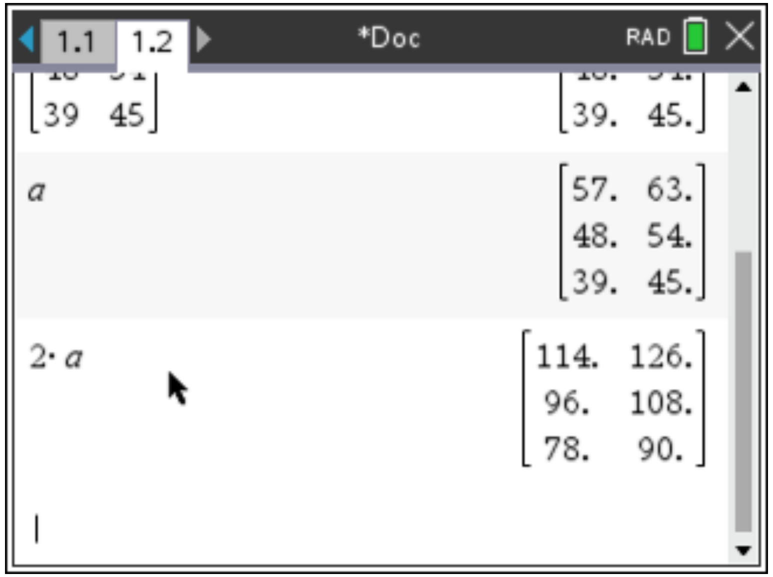
$3 \times 2 = 6$

## Recap of past learning

In a previous video we looked at how to multiply a matrix by a scalar. A scalar is just a number (in this case). It seemed pretty easy!

We are now going to see what happens when we multiply two matrices together. I'm going to start by using the CAS to show you some examples of it working and not working!

So, here is an example of scalar multiplication … this is **not** what we're going to be doing.



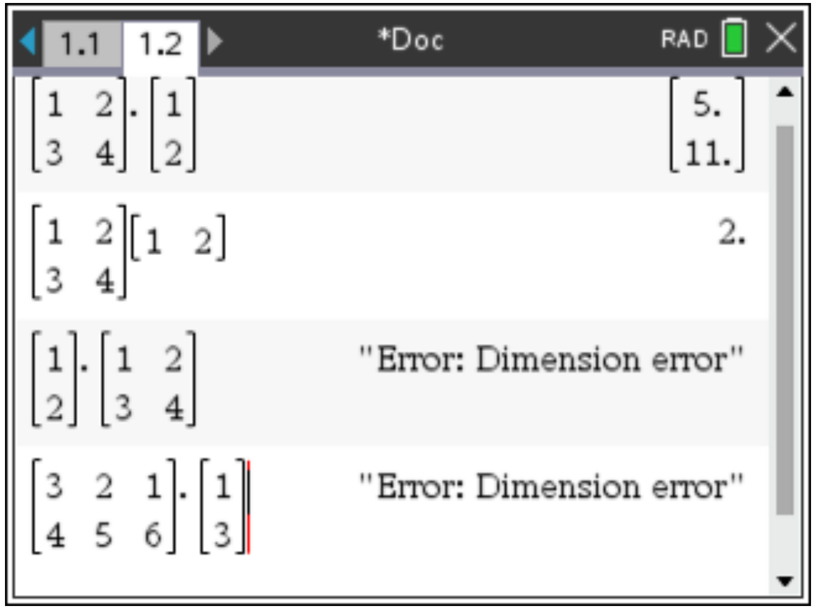

## Odd one out …

Let’s pay a little ”Odd One Out”. Can you find out why some of these are working and some of them are not?

2 by 2    2 by 1

2 by 1    2 by 2

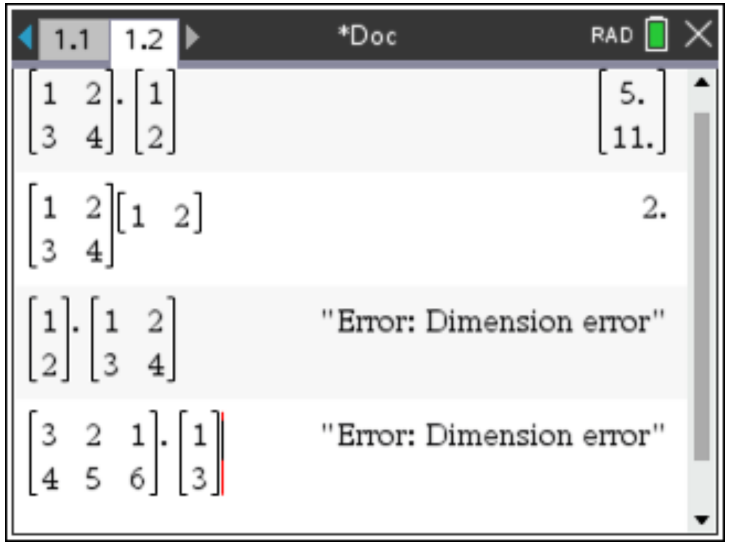
2 by 3    2 by 1




# Recap: Order of a matrix

In a previous video we looked at the order of a matrix. I said that it was going to become really important in later videos! Well, this is the video it becomes important.

Let's look at each of the examples I did and look at the order of each of the matrices written side by side




# The order of the matrices tell us if they can be multiplied.

When the **number of columns** of the first matrix equals the **number of the rows** in the **second** matrix we know they will be able to be multiplied.

But … how do we multiply them?! Hold on tight … this is where things get interesting!

It's important to note that the result of the matrix multiplication is given by the first and last numbers when we write the orders together.

Example:

( Three by two ) and ( two by one )

These are **defined** meaning they can be multiplied.

The result will be a **three by one** matrix.

2 by 2   1 by 2   ✗

2 by 2   2 by 1   ✓

$$\begin{bmatrix} - \\ - \end{bmatrix}$$

## Recap: The address of each element

Remember that a matrix has elements. And each element of the matrix has an address.

I can read the address in another way. It can tell me the row and column I need to multiply to get the answer.

What?

$$C = \begin{array}{ccc} c_{11} & c_{12} & \\ \multicolumn{3}{c}{\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \end{bmatrix}} \end{array}$$

$c_{21}$ tells me to multiply row 2 and column 1 to get this number

$c_{23}$ tells me to multiply row 2 and column 3 to get this number

## Multiplication … let's do it

Lets do the following question from Cambridge:

If $A = \begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$ then find $AB$

**Remember**: Check the order first to see if it's defined and then see what the result matrix will be.

$7\times 20 + 4\times 30$

$140 + 120$

$$\begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 260_{11} \\ 280_{21} \end{bmatrix}$$

$5\times 20 + 6\times 30$

$= 100 + 180$

2 by 2 2 by 1

# Multiplication … let's do it

For the following matrices, decide whether the matrix multiplication in each question below is defined and if it is, give the order of the answer matrix and then do the matrix multiplication

$$A = \begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 8 \\ 9 \end{bmatrix}, C = \begin{bmatrix} 2 & 4 & 7 \end{bmatrix}, D = \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$$

a. $AB$

b. $BA$

c. $CD$

3 by 2 2 by 1

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 58_{11} \\ 86_{21} \\ 35_{31} \end{bmatrix}$$

$5 \times 8 + 2 \times 9$

$40 + 18$

$4 \times 8 + 6 \times 9$

$= 32 + 54$

$1 \times 8 + 3 \times 9$

$8 + 27$

## Multiplication … let’s do it

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2 by 1   3 by 2

x

*a.* $AB$

*b.* $BA$

*c.* $CD$

## Multiplication … let's do it

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$$A = \begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 8 \\ 9 \end{bmatrix}, C = \begin{bmatrix} 2 & 4 & 7 \end{bmatrix}, D = \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$$

a. $AB$

b. $BA$

c. $CD$

1 by 3   3 by 1

✓

$2 \times 8$
$+ 4 \times 6$
$+ 7 \times 5$

$\begin{bmatrix} 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix} = \begin{bmatrix} 75 \end{bmatrix}$

$16 + 24$
$+ 35$

# Matrix powers

We know that, when we raise a number to a power it means to multiply the number by itself power number of times.

e.g. $4^3 = 4 \times 4 \times 4$

The same is true for matrices. Matrices can be raised to a power (which becomes really important in Year 12 General Maths).

Whilst we can, by hand, do matrices which are raised to the power of 2, it becomes really tedious for higher powers. Hence we can use the CAS.

$4^2 = 4 \times 4 = 16$

$4^3 = 4 \times 4 \times 4 = 64$

## Example: Matrix powers

If:

$$A = \begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$$

Determine:

- $A^2$
- $AB^2$

Note: Be very careful of the notation here! Who does the squared belong to?

2 by 2 ✓ 2 by 2

$$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 2 \\ 8 & 5 \end{bmatrix}$$

$$AB^2 = A \times B \times B$$

$$= \begin{bmatrix} -6 & -11 \\ -29 & -52 \end{bmatrix}$$

# The identity matrix

There is a really special matrix which is used (in the next lesson) called the **identity matrix**.

It has the following features:

- It has to be a square matrix
- It has one 1's and zeros
- The ones are on the leading diagonal
- The rest of the elements are zeros.

Examples include …

$$\begin{bmatrix} 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Example: The identity matrix

Something really interesting happens when you multiply a matrix by its corresponding identity matrix.

**Remember: To multiply them together they would need to have the same order as each other.**

Consider the following matrices:

$$A = \begin{vmatrix} 5 & 2 \\ 8 & 3 \end{vmatrix} \; and \; I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

- Find *AI*
- Find *IA*

$$\begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$$

2 by 2 ✓ 2 by 2

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$$

# Using the CAS to multiply two matrices together

We can do the question from before using the CAS.

Consider the following matrices:

$$A = \begin{vmatrix} 5 & 2 \\ 8 & 3 \end{vmatrix} \; and \; B = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

- Find *AI*
- Find *IA*

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