Gradient



Year 9
Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- Understand that gradient is a measure of slope
- Understand that gradient is a ratio between the vertical change and horizontal change of a graph
- Know that gradient is constant across the whole of a straight line graph.
- Find the gradient of a line using a graph or two points.

$$\frac{\text{grad}}{\text{int.}}$$

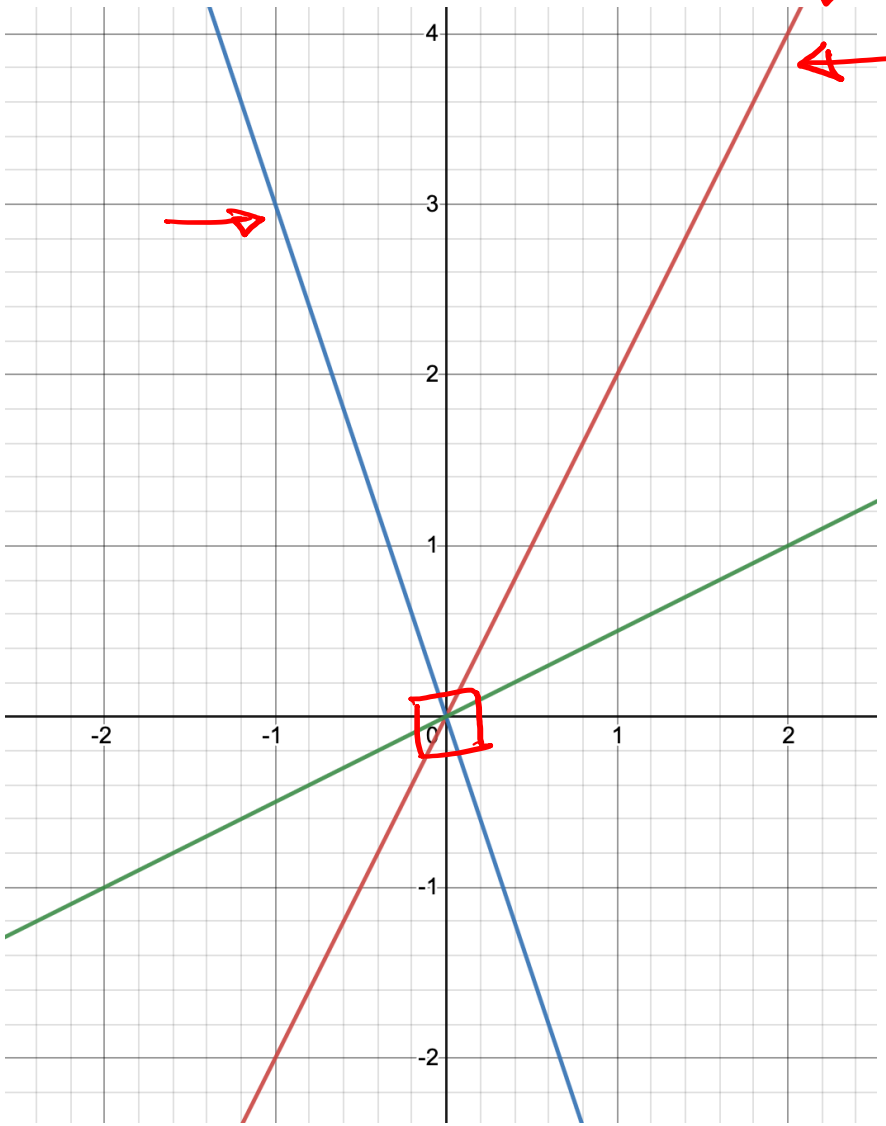
RECAP

We have been looking, in previous lessons, how to take a relationship and draw it as a graph. We can plot individual points and then join them together to make a straight line which extends to infinity and beyond.

We can draw lots and lots of different lines. We met some of them in the last lesson.

The lines shown seem to all go through the same point, but have a different slope. Two of them seem to slop up and one seems to slope down (if we travel from left to right on the axes).

This slope is called **gradient** and can have a numerical measure associated with it. This is the content of the lesson today.



origin
(0,0)

What is gradient?

Gradient is a measure of slope.

It is a measure of steepness.

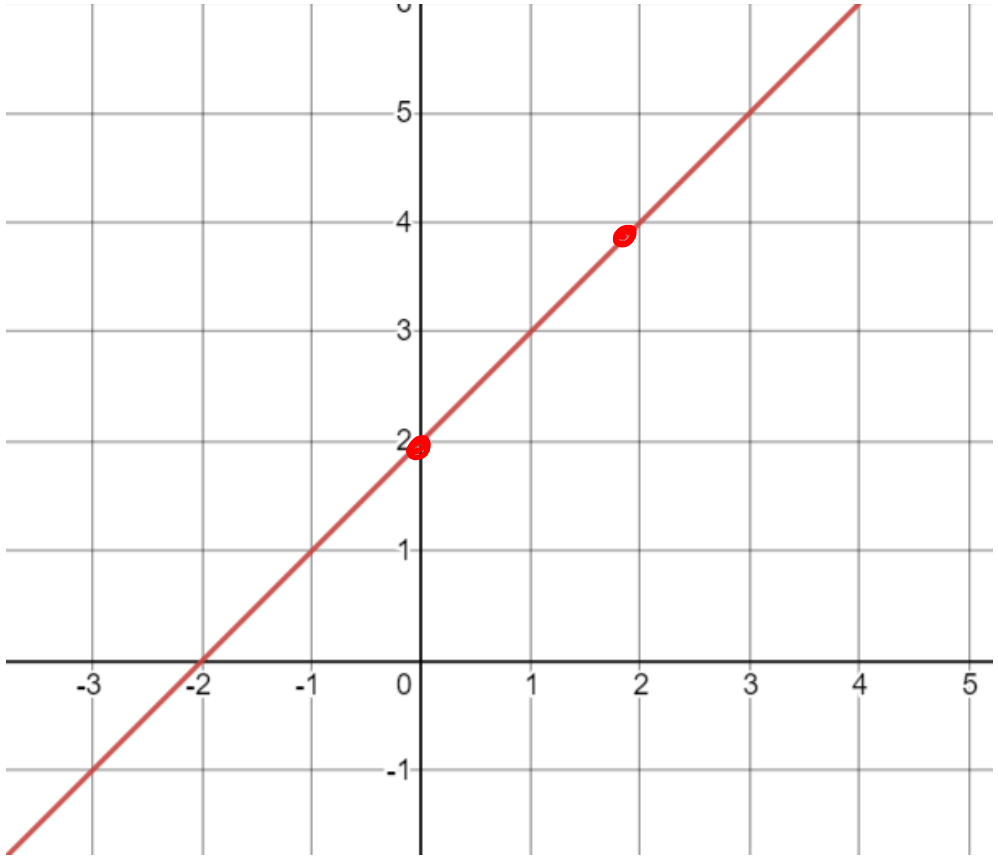
It is a measure of how many units a graph changes vertically given a change in units horizontally.

Basically, it's given as:

$$gradient = \frac{rise}{run}$$

Where **rise** is a change in height, and **run** is a change in horizontal distance.

The gradient of a straight line is constant.



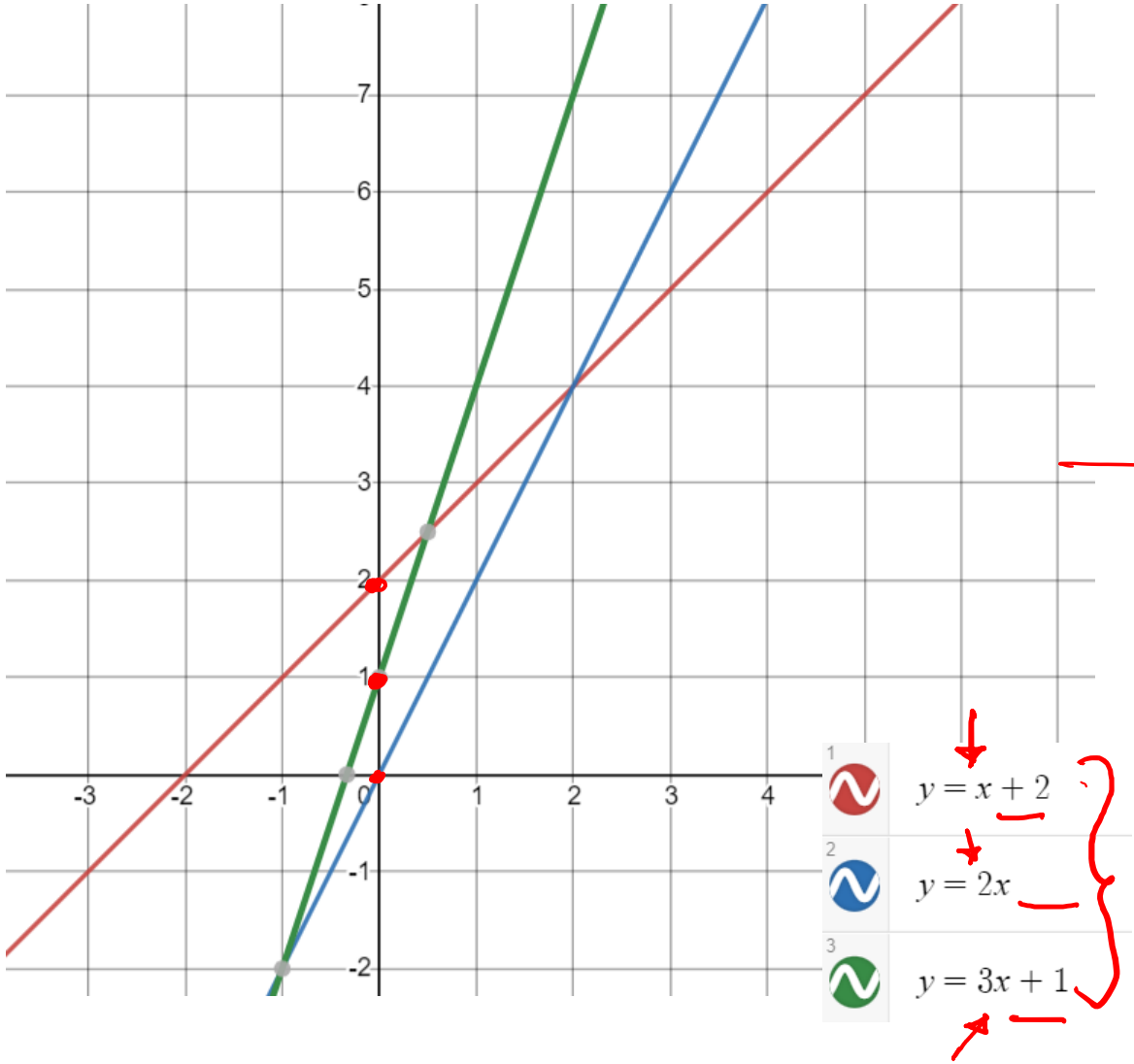
m

Positive, negative, zero and undefined gradients

Gradients can take lots of values. It is a number. So it can be positive, negative, zero and undefined. Positive gradients always slope up (when reading from left to right).

All the gradients in the graphs slope up. So, they are **positive**.

We can measure the gradients for each one.



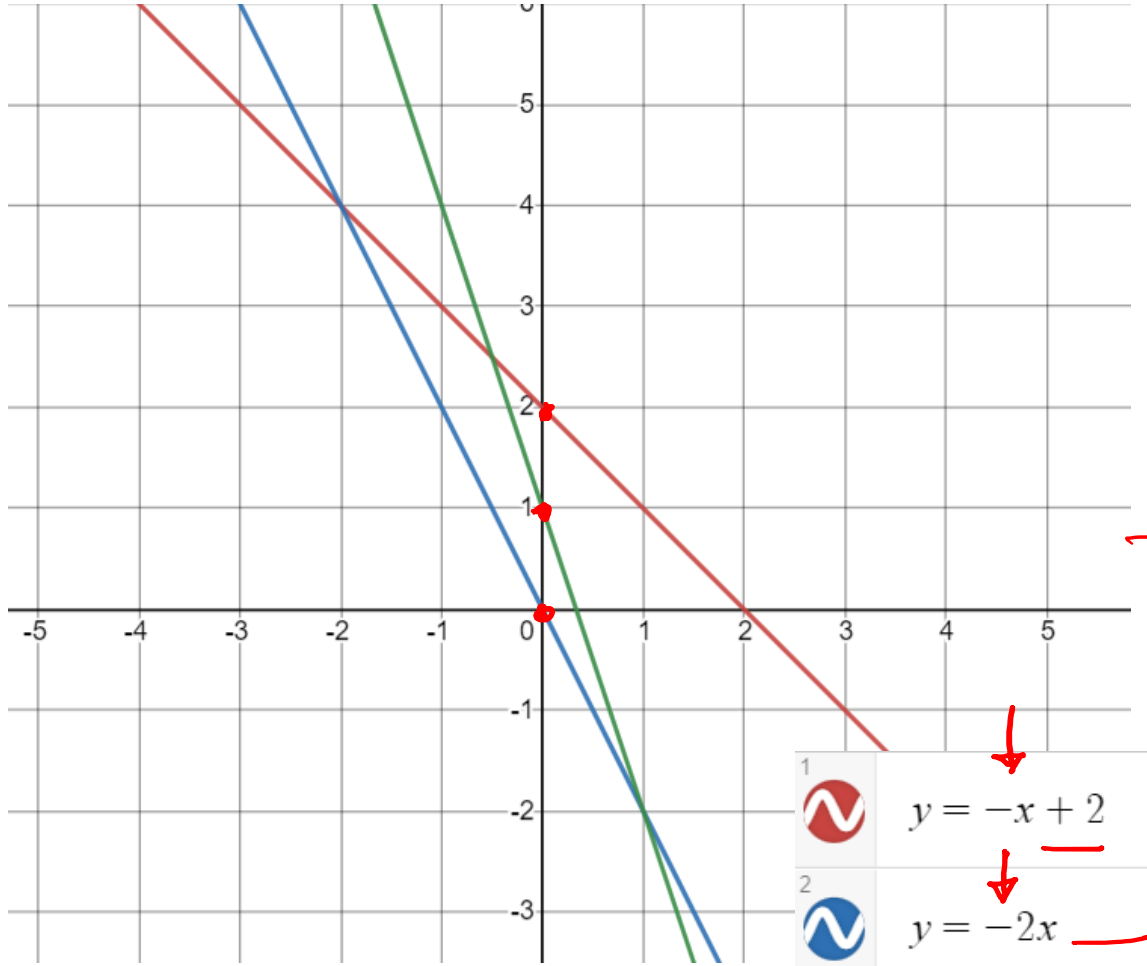


Positive, negative, zero and undefined gradients

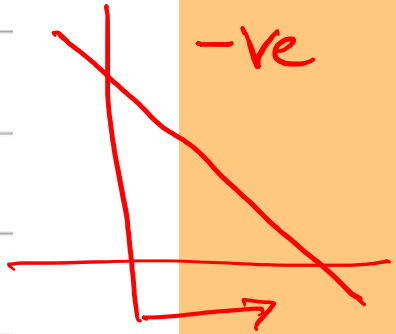
The gradients in the graphs shown on the right are **negative**. They all slope down when reading from left to right.

Again, the gradients can be measured.

$y = -1x + 2$



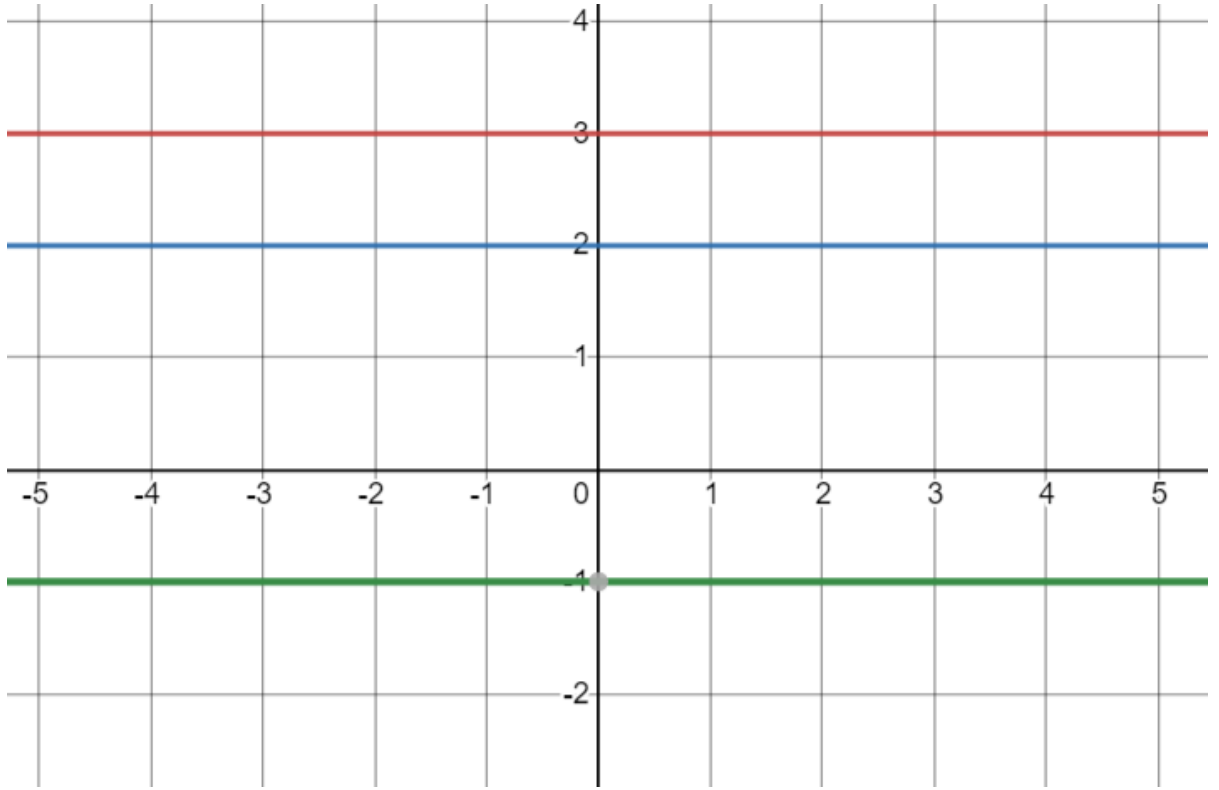
1. $y = -x + 2$
2. $y = -2x$
3. $y = -3x + 1$



Positive, negative, zero and undefined gradients

Horizontal lines have a zero gradient.

We can see this as there is no change in height.



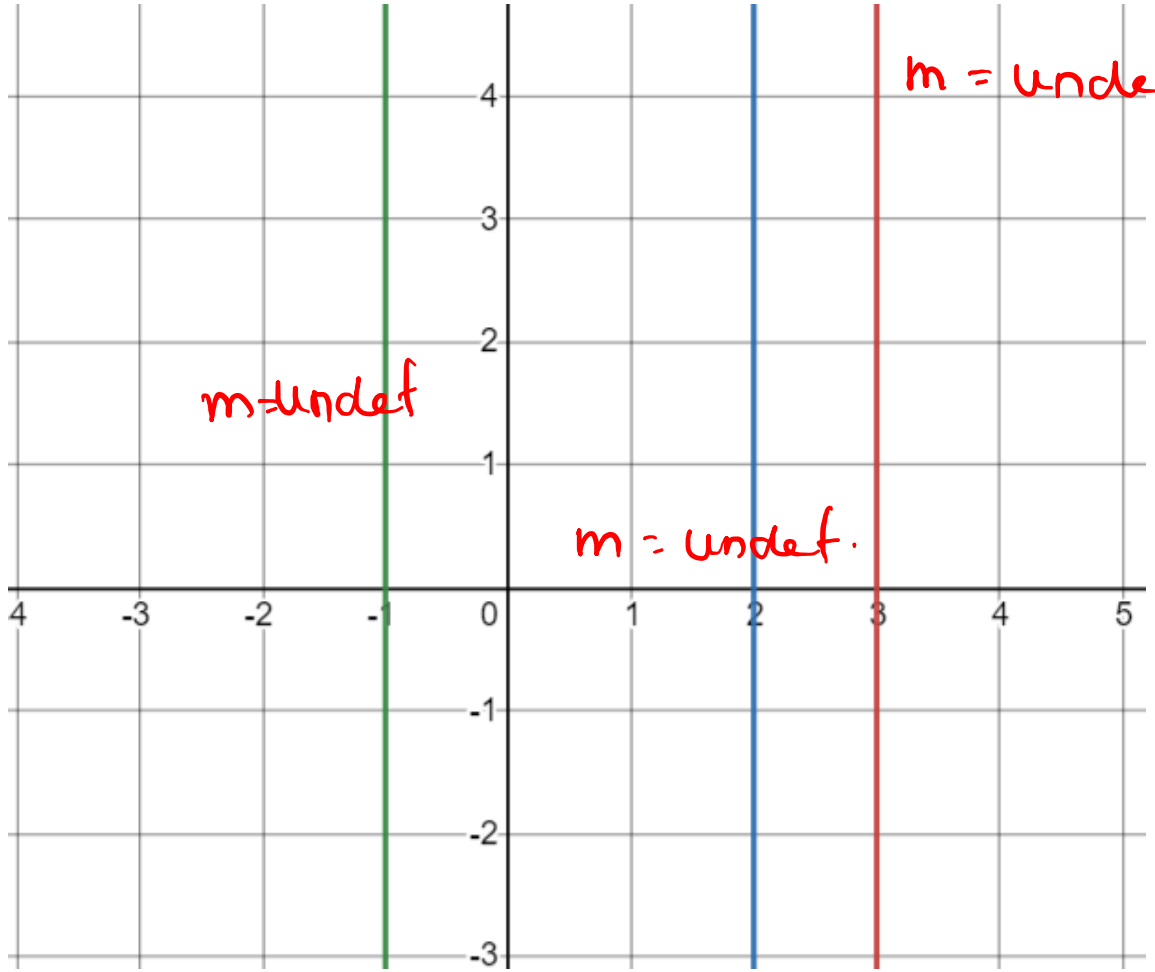
Positive, negative, zero and undefined gradients

Vertical lines have an **undefined** gradient as we cannot divide by zero. Hence, the gradient cannot be measured.

We can also say that the gradient is infinite.

m = "undefined"

m = ∞



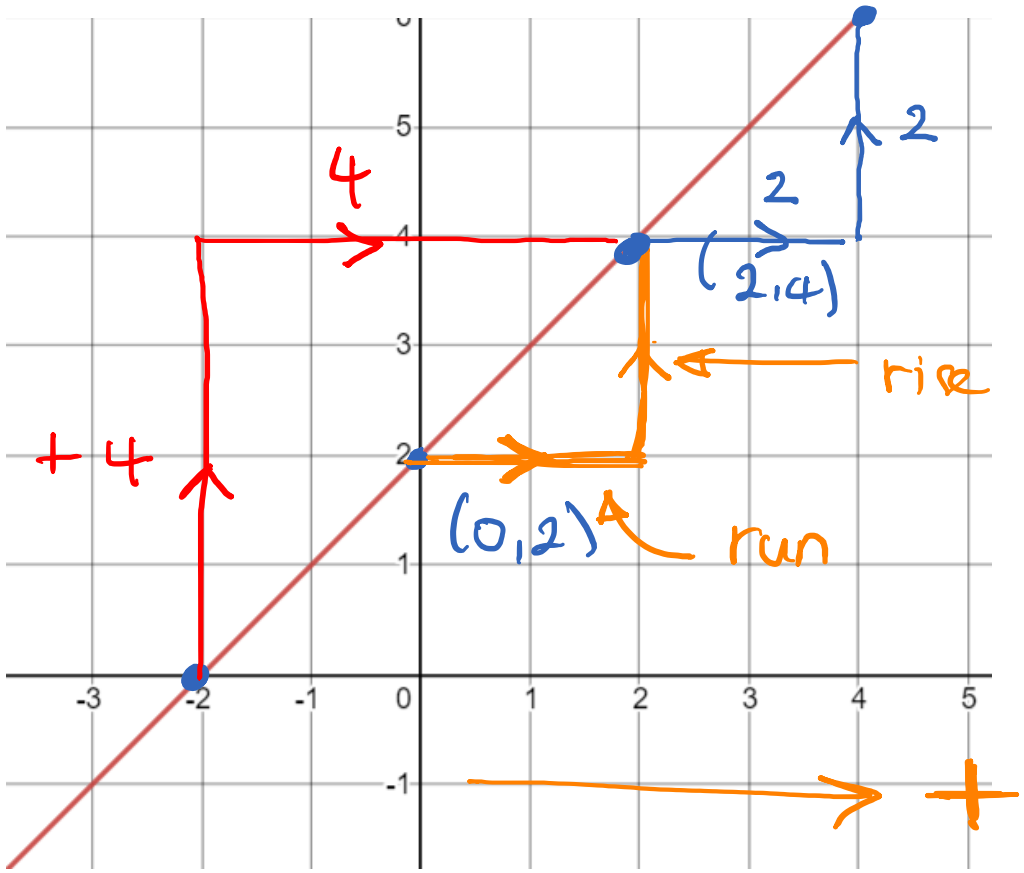
Finding gradients from graphs

We find gradients by choosing **any two points** on the graph and counting squares! Once we know how many squares up (or down!) a graph travels and how many across, we simply put it into the following formula:

$$gradient = \frac{rise}{run}$$

Remember, the rise can be **negative**.

Run is always positive as it's read from left to right.



You can use any two points on a straight line graph.

$m = \frac{\#2}{\#2} = 1$

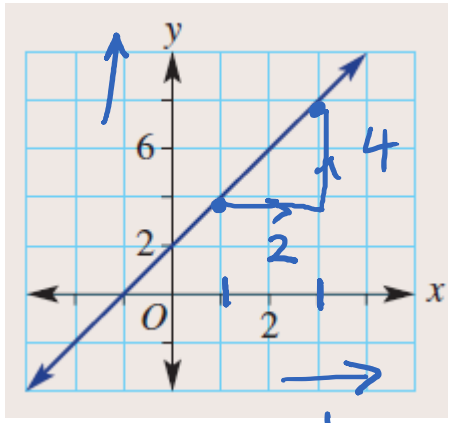
$\frac{2}{2} = 1$

+ve

$m = \frac{4}{4} = 1$

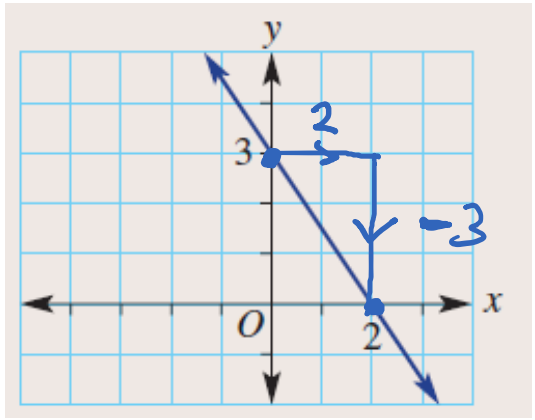
Examples

For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.



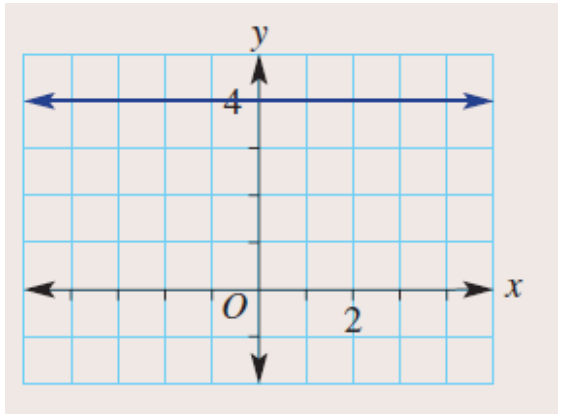
+ve

$m = \frac{4}{2} = 2$



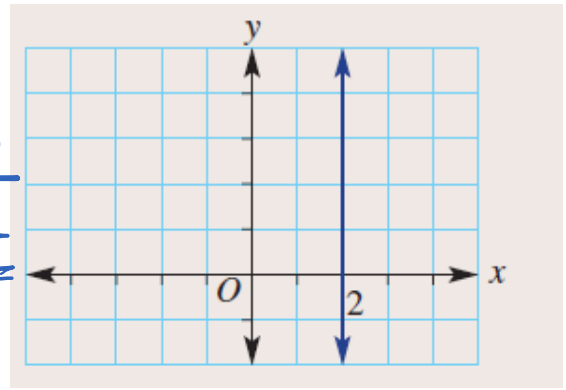
-ve

$m = \frac{-3}{2}$



zero

$m = 0$



undefined

Finding the gradient using two points

Sometimes, it's actually easier to find the gradient from two points than to draw a graph! No. Really. Let's look at an example of finding the gradient from two points:

$$A(3,4) \text{ and } B(5,6)$$

Remember: A coordinate is nothing more than two numbers telling you a position on a set of axes.

If you subtract the x-values you find the change in horizontal distance. If you subtract the values you get the change in vertical height.

You just need to be consistent. To help us, the formula for gradient can also be written:

$$gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6-4}{5-3}$$

$$= \frac{2}{2} = 1$$

$x_1\ y_1$ $x_2\ y_2$

$A(3,4)$ $B(5,6)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ ← rise, ← run

Finding the gradient using two points

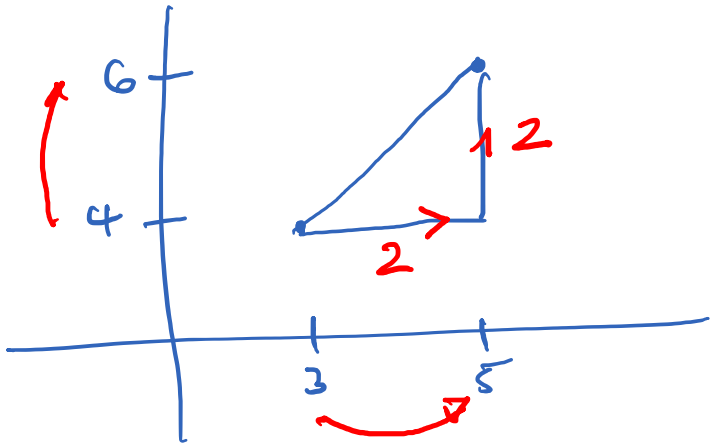
Sometimes, it's actually easier to find the gradient from two points than to draw a graph! No. Really. Let's look at an example of finding the gradient from two points:

$$A(3,4) \text{ and } B(5,6)$$

So we can use the formula to find the gradient.

$$gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: There are many times I will still draw a little pencil sketch of a triangle to make sure I'm doing this correctly.



$m = \frac{2}{2} = 1$

Finding a missing value of a coordinate

This tests **real** understanding of what you're doing!

What if I told you I had two coordinates $(0,2)$ and $(1,?)$. I know the gradient is 4. Can I find the value of the question mark?

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \begin{matrix}(\text{rise}) \\ (\text{run})\end{matrix}$$

x_1, y_1
$(0,2)$

x_2, y_2
$(1, ?)$

$$4 = \frac{? - 2}{1 - 0}$$

$$4 = \frac{? - 2}{1}$$

$$4 = ? - 2$$

$$? = 6$$