Introduction to linear relations



Year 9
Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- Understand what a cartesian plane is
- Understand what a coordinate is and how it relates an x- and y-value
- Know what it means to be a relation and,
- Know what it means to be a linear relation
- Understand what the x-intercept and the y-intercepts are

RECAP

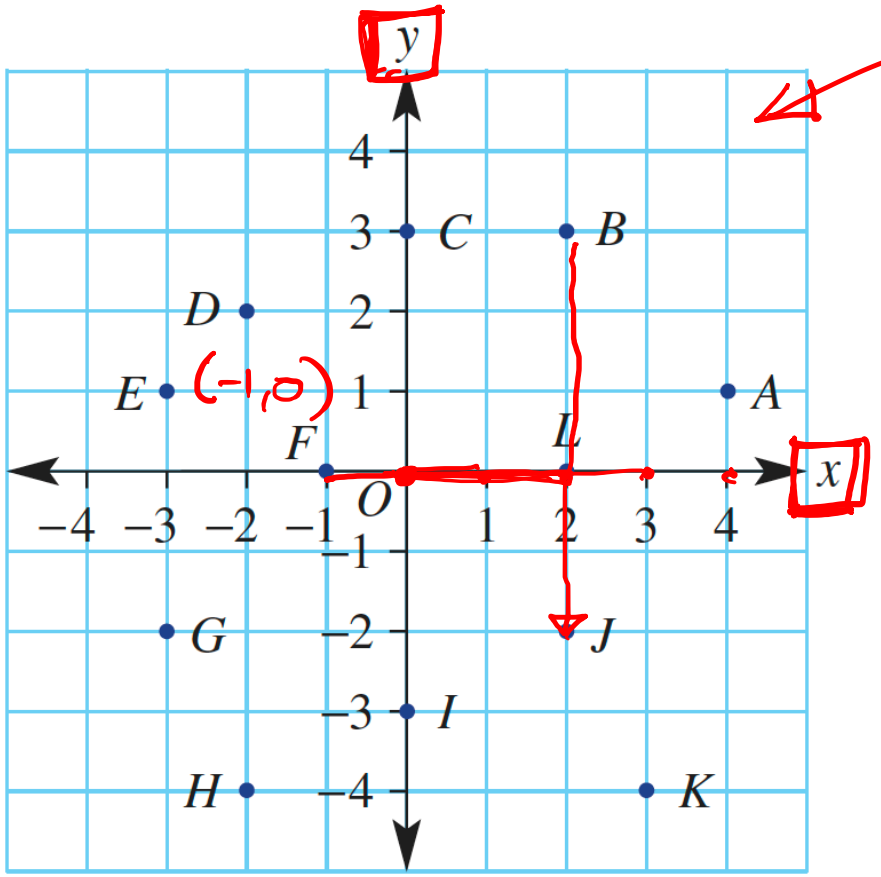
There is not much is a recap in this section of the course as it's a new topic. However, much of what we will cover should build on stuff we have done before. One of the most important things we will need to master is what a coordinate is.

A coordinate is a point in space. Normally, we use a cartesian plan which has an x- and y-axis. These axes are numbered using equal divisions between **tick marks.**

A coordinate has the following form:

$$(x, value\ y\ value)$$

Note that the **brackets** are curved and required. A comma must separate the values.



Let's play a game

You give me a number and I give one in return.

You simply have to tell me how I have gone from your number to mine.

This might not be as easy as it seem.



x 2 +1

Your guess	0	1	~~8~~ 2	~~5~~ 3	7
My answer			7	11	15

The use of a table just makes things a little easier for me to keep track of the numbers we have already used. **Or is there an another motive?**

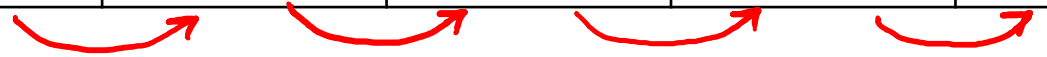
There is always a pattern to these things!

If you have played the game(s) properly, then you should start to see that there is a pattern to the way we can play. If you go up using a sensible sequence, then we might be able to glean the **pattern** or **rule** more easily!

It makes sense to start at **zero** and then go up in **ones**.

Doing this will make life much easier in later sections of this course.

Your guess	0	1	2	3	4
My answer	1	3	5	7	9



$\times 2 + 1$



Writing a rule mathematically

Rules and relationships are really the same thing.

A relationship takes place between two things.

Example:

$$My\ answer = Your\ guess \times 2 + 3$$

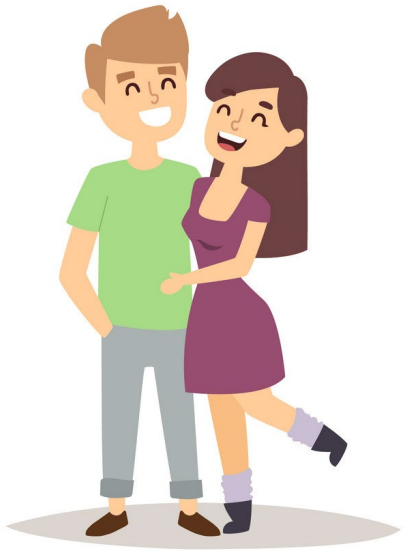
This doesn't seem very Mathematical.

So, we could use letters instead:

$$y = x \times 2 + 3$$

Which we know we can write in a "better way":

$$y = 2x + 3$$



Why would we choose the letters x and y?

Because, the types of relationship we're going to look at today will be plotted on a cartesian plane.

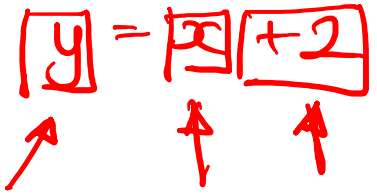
Relationships and coordinates

Let's look at $y = x + 2$

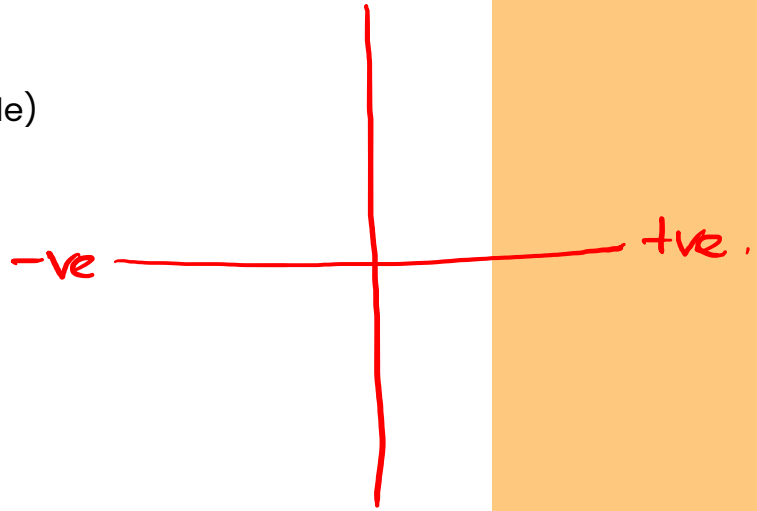
This is a relationship. It says, to get any 'y' value, take an 'x' value and add two.

This can be placed into a table:

x	–3	–2	–1	0	1	2	3
y	–1	0	1	2	3	4	5



It is normal for us to "choose" the x-values and work out the y-values using the equation (or rule) given. The question asked us to use values of x from -3 to 3.

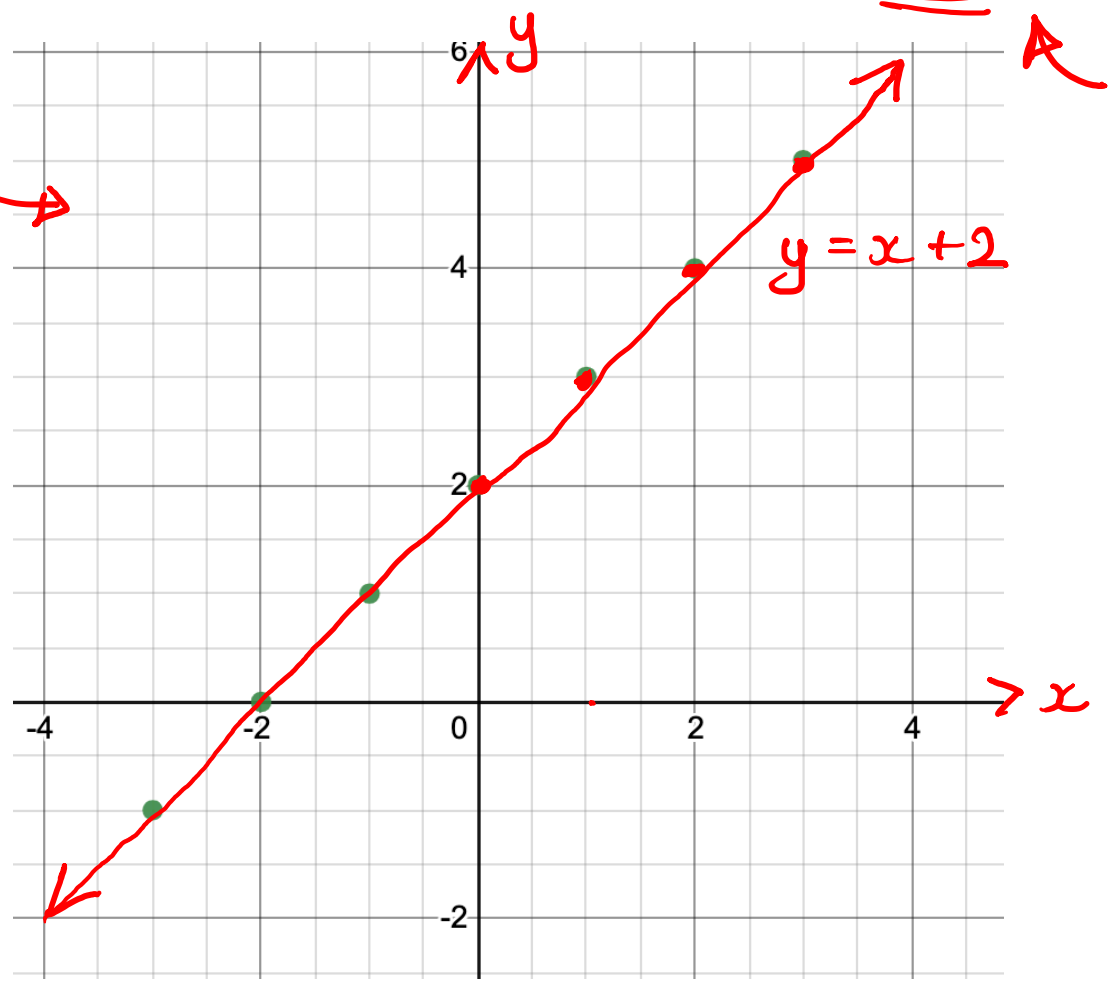


x	-2	-1	0	1	2
y	0	1	2	3	4

Relationships can be plotted on a cartesian plane

If we take the values we have, and plot them on a cartesian plane, we notice something pretty interesting!

x	–3	–2	–1	0	1	2	3
y	–1	0	1	2	3	4	5



The points (coordinates) all sit on a **straight line**.

When this happens, we say the rule given is **linear**.

This line was created using $y = x + 2$

A different rule and line

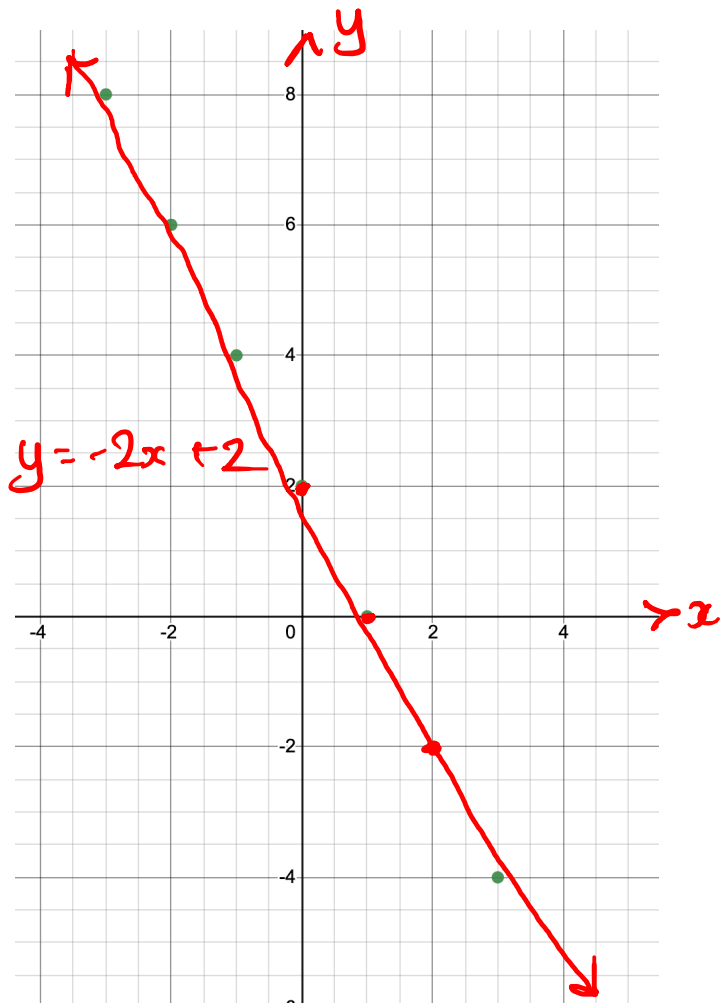
Let's try again with another rule: $y = -2x + 2$

We create the table to results shown below using values provided to us.

x	–3	–2	–1	0	1	2	3
y	8	6	4	2	0	–2	–4

Which, when plotted on a cartesian plane, gives us another **linear relationship**.

This set of dots seems to be getting less high as we move from left to right. This is going to be important later.



$y = -2x + 2$

$y = -2 \times 0 + 2$

$= 2$

$y = -2 \times 1 + 2$

$= 0$

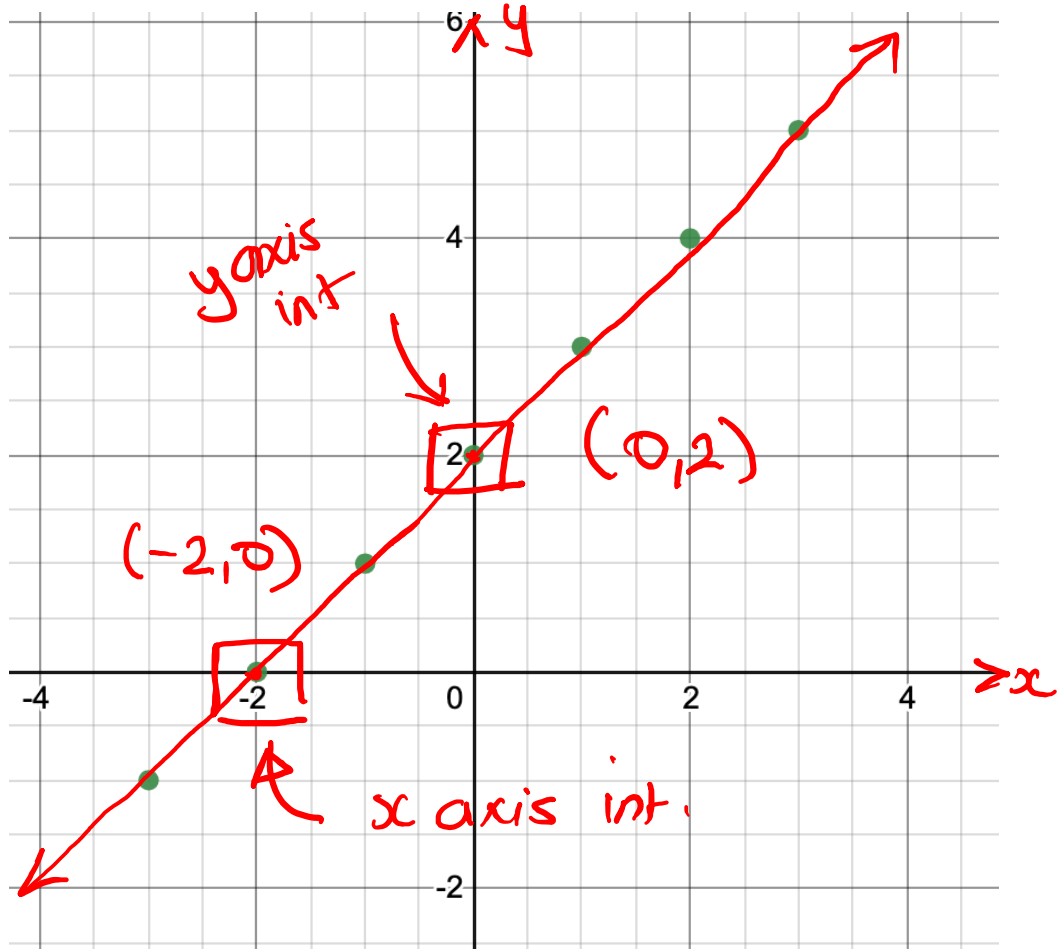
Intercepts – where lines cross axes.

One of the most important parts of this lesson is understanding what an intercept is and how we can find them.

Look back at the first graph.

There are two intercepts; an x-axis and y-axis intercept.

This is where points lie on the x-axis and the y-axis.

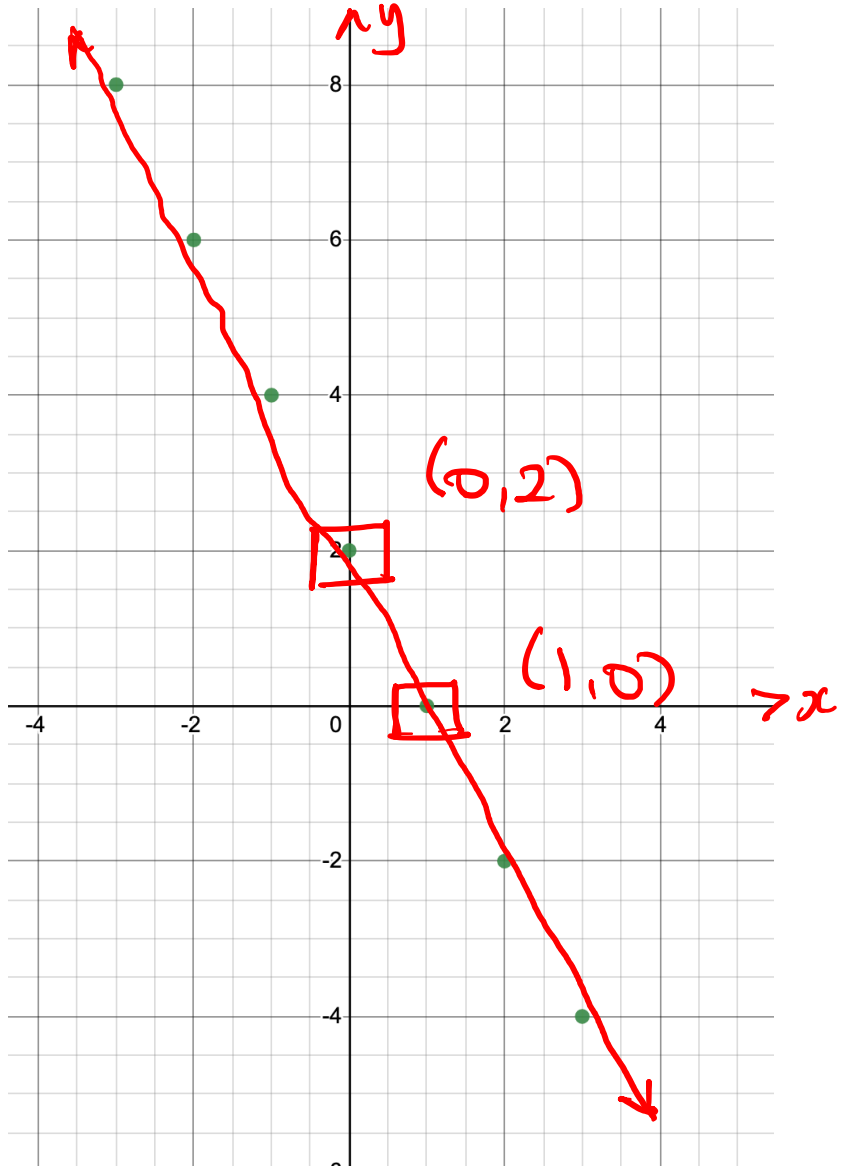


Intercepts – where lines cross axes.

It turns out these two points will form much of the Mathematics we will do later in the course and in years 10, 11 and 12. Two points – three years to study them!

We can see we can also find these intercepts from the table of values we created:

x	–3	–2	–1	0	1	2	3
y	8	6	4	2	0	–2	–4



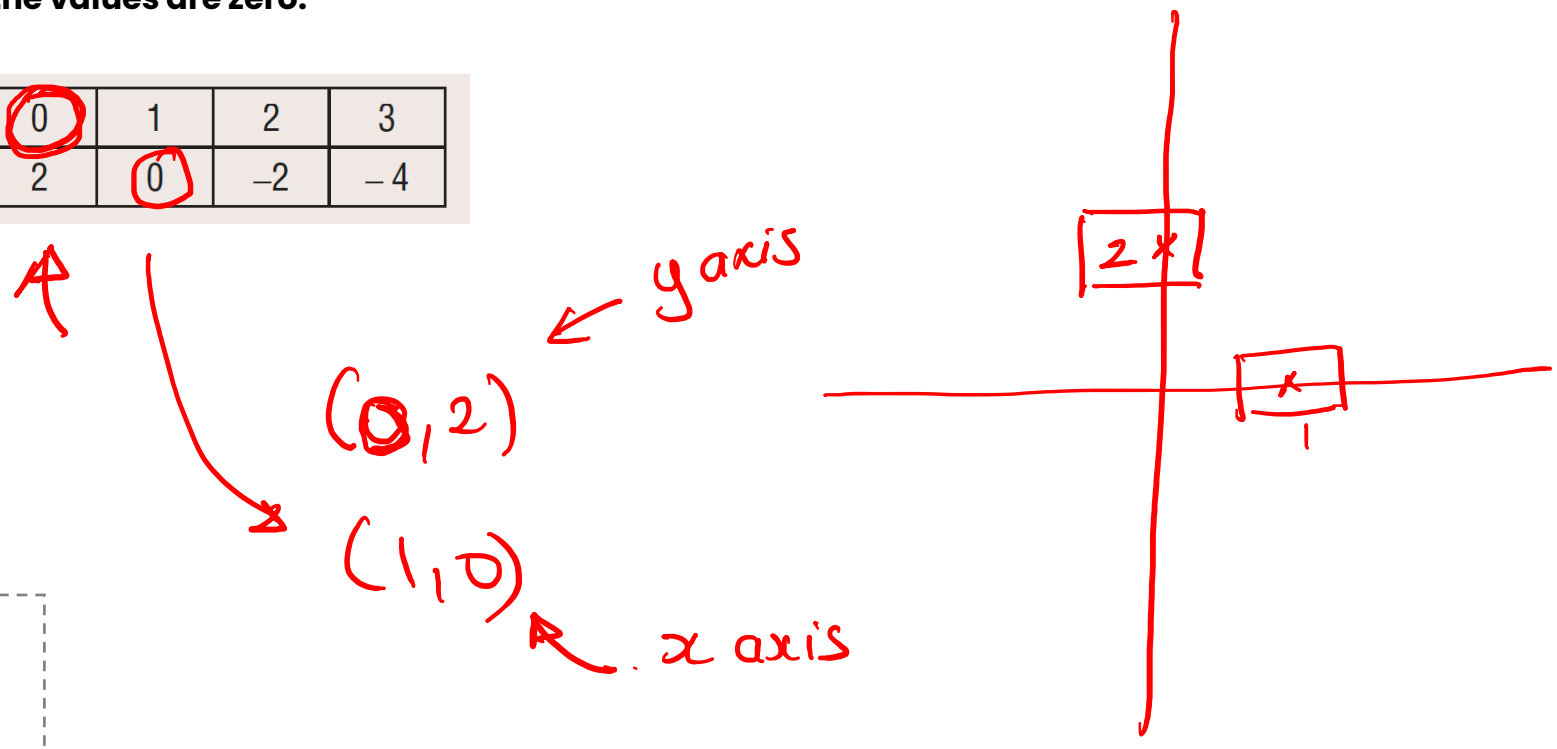
It's really important to know the difference between the x-axis and y-axis intercepts.

Finding an intercept from a table of values

Here is a table of values. Can you now find the x- and y-axis intercepts?

Hint: Look for where the values are zero.

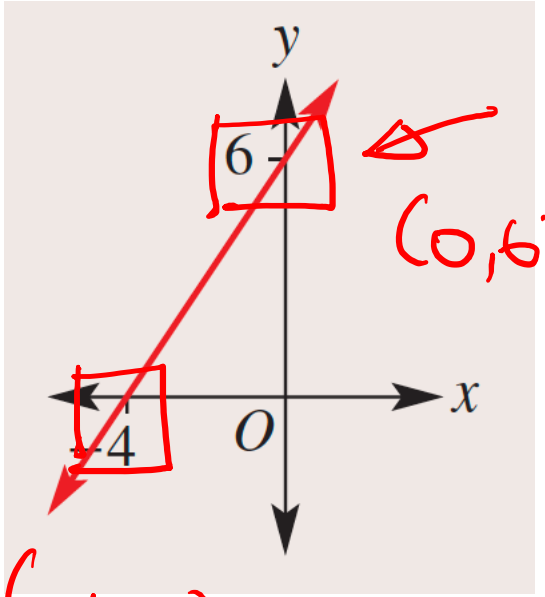
x	–2	–1	0	1	2	3
y	6	4	2	0	–2	– 4



Finding an intercept from a table of values

Here is a graph. Can you now find the x- and y-axis intercepts?

Hint: Look for where the values are zero.



The rule is the key to understanding!

If we have the rule (or linear relationship) we can learn a lot.

Later in the course, we will know that the rule provides us the gradient of the line, the x- and y-intercepts. It can also tell us if a point given lies on the line (fulfils the relationship).

Question:
Decide if the point (-2, 4) is on the line with the given rules.

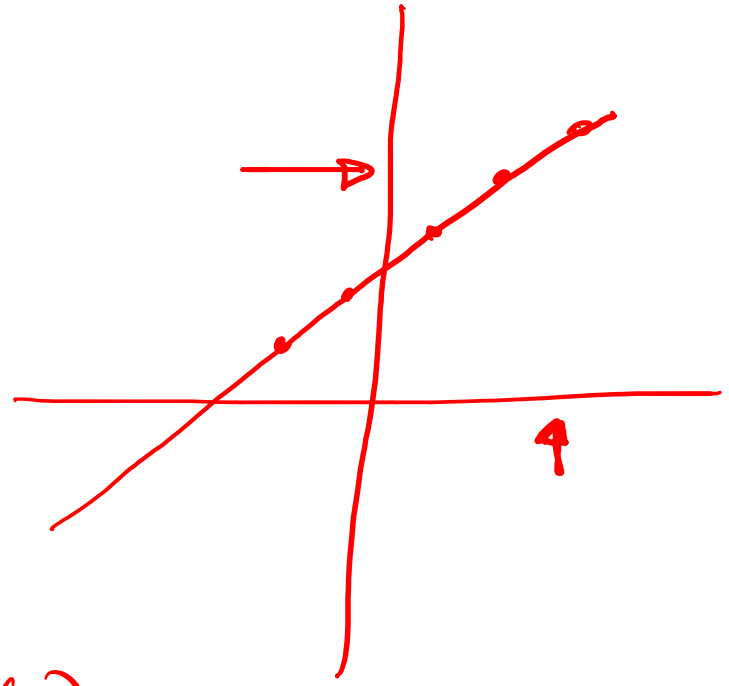
$$y = 2x + 10$$

x, y

$y = 2x - 2 + 10$

$y = -4 + 10$

$y = 6$



$(-2, 6)$

The rule is the key to understanding!

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Question:
Decide if the point (-2, 4) is on the line with the given rules.

$$y = -x + 2$$

$(-2, 4)$

$y = -x + 2$

$y = -(-2) + 2$

$y = 2 + 2$

$y = 4$

Finding a rule from a table of values

This is a taste of what it to come, but we can find the linear relationship from a table of values.

The form of the relationship is:

$$y = mx + c$$

Where the letters m and c are just numbers.

To do this, we simply need to look at what change takes place in the y-value, given a chance in the x-value.

x	0	1	2	3	4
y	2	3	4	5	6

$y = 1x + 2$

$y = 1x + 2$

$= x + 2$

Another example

Let's look at this table and see if we can find the rule which links the x- and y-values.

x	−1	0	1	2	3
y	−2	0	2	4	6

$y = 2x + 0$

$y = 2x$.

Another example

Let's look at this table and see if we can find the rule which links the x- and y-values.

x	1	2	3	4	5	6	7	8	9
y	1	1½	2	2½	3		4		5

1 2 3 4

1 1½ 2 2½

x	1	3
y	1	2

$y = \frac{1}{2}x + \frac{1}{2}$

$y = \frac{1}{2}x + \frac{1}{2}$

$\frac{1}{2}$