

Finance applications using geometric sequences and recurrence relations

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to use a recurrence relation to model compound interest for investments or loans.
- To be able to write a recurrence relation to model a loan that compounds with a different compounding period.
- To be able to use a recurrence relation to model reducing-balance depreciation.

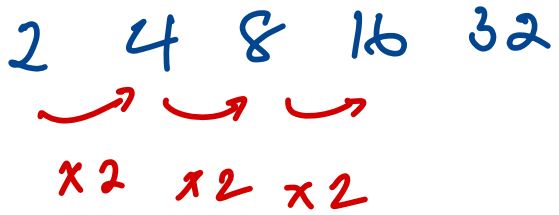


Recap

In the last lesson we looked at how we can go from a recurrence relation to a rule for a Geometric Sequence. This is going to be good knowledge to have now that we once again return to applying all the learning to real world situations.

In a previous lesson we looked at the idea of Simple Interest. We decided that this didn't seem particularly fair as we wouldn't be getting any interest on the interest. We wanted our money to **compound** and grow bigger but not in a linear way.

This lesson will look at a real world situation where money grows in a non linear way.





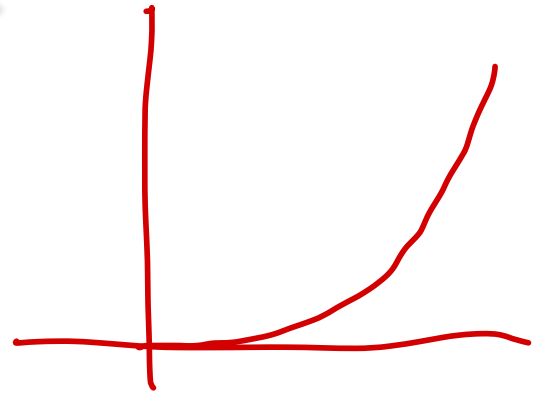
Recap: Geometric Growth and Decay

When something grows (or shrinks) in a non-linear way we say it is decaying or growing geometrically (so long as we use the same formula to go from one period to the next).

The balance of a bank account would grow geometrically (as long as we don't take money out of the account and interest rates stay the same).

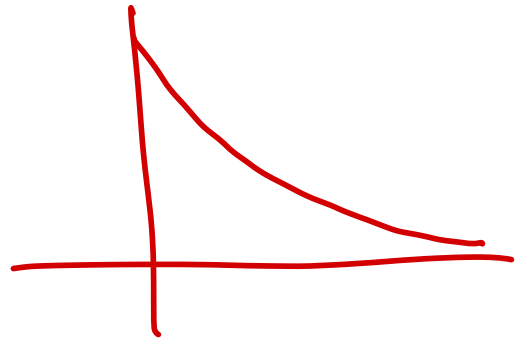
A new car actually depreciates geometrically as it loses a percentage of its value each year.

Knowing this means we can use the formulae we have to rule the world!



grow $R > 1$

decay $0 < R < 1$





Recap: Recurrence relations and rules

As happened with Arithmetic sequences, as soon as we start talking about and using money, the pronumerals used in the equations all seem to change.

So, please note that we will now be using:

$$V_0 = P, \qquad V_{n+1} = R \times V_n$$

And

$$V_n = R^n \times V_0$$

Note: It is always important to use the letters they give in the question. I have known SACs to use different letters and remove marks when students didn't notice.



Recap: Recurrence relations and rules

The value of R in the following equation is really important.

When it is greater than one, the investment or sequence will **grow**.

$$V_0 = P, \qquad V_{n+1} = R \times V_n$$

When the value is between 0 and 1 it will decay (or get smaller)

You cannot have a negative value of R.



Compound interest

Interest is always **added to your principal.**

This means something will always grow assuming you leave it alone to do so.

Banks pay compound interest. They pay interest on what you have in your account on any one day. The more you have, the more interest you earn. The interest is always added on.

You can work out the value of R from the rate of interest being paid:

$$R = \frac{r}{100} + 1$$

Where 'r' is the rate of interest being paid.

$1000 @ 10%

10% of 1000 = 100

∴ $1100 @ 10%

10% of 1100 = $110

$1100 + 110 = $1210

$V_n = R^n \times V_0$



Example: Compound interest

The following recurrence relation can be used to model a compound interest investment of $1000, paying interest at the rate of 8% per annum.

$$V_0 = 1000, V_{n+1} = 1.08Vn$$

In the recurrence relation, Vn is the value of the investment after n years.

- Use the recurrence relation to find the value of the investment after 1, 2 and 3 years, **to the nearest cent**.
- Determine when the value of the investment will first exceed $1500.

$R = 1 + \frac{r}{100}$

$= 1 + \frac{8}{100}$

$= 1 + 0.08 = 1.08$

$V_0 = 1000$

$V_1 = 1.08 \times 1000 = \$1080$

$V_2 = 1.08 \times 1080 = \$1166.40$

$V_3 = 1.08 \times 1166.40 = \$1259.71$

$\therefore$ 6 years.



Example: Compound interest

Sally borrows \$6000 from a bank. Interest will accrue at the rate of 4.2% per annum.

Let Vn be the value of the loan after n compounding periods.

Write down a **recurrence relation** to model the value of Sally's loan if interest is compounded:

- **yearly**
- quarterly
- monthly

yearly

$V_0 = 6000, \quad V_{n+1} = 1.042 \times V_n$

$V_0 = 6000$

$r = 4.2\%$

$R = 1 + \frac{4.2}{100}$

$= 1.042$



Example: Compound interest

Sally borrows $6000 from a bank. Interest will accrue at the rate of 4.2% per annum.

Let Vn be the value of the loan after n compounding periods.

Write down a **recurrence relation** to model the value of Sally's loan if interest is compounded:

- yearly
- **quarterly**
- monthly

$V_0 = 6000, \quad V_{n+1} = 1.0105 \times V_n$

Quarters

$4.2 \div 4$

$= 1.05\%$

$R = 1 + \frac{r}{100}$

$= 1 + \frac{1.05}{100}$

$= 1.0105$



Example: Compound interest

Sally borrows \$6000 from a bank. Interest will accrue at the rate of 4.2% per annum.

Let Vn be the value of the loan after n compounding periods.

Write down a **recurrence relation** to model the value of Sally's loan if interest is compounded:

- yearly
- quarterly
- **monthly**

Yearly = 4.2%

Monthly = $\frac{4.2}{12}$ = 0.35%

$V_0 = 6000, \quad V_{n+1} = 1.0035 \times V_n$

$R = 1 + \frac{r}{100}$

$= 1 + \frac{0.35}{100}$

$= 1.0035$



Reducing balance depreciation

Whilst interest is always added on, there are lots of cases where things lose value.

We keep talking about cars losing value, but there are lots of things which can also be subject to reducing balance depreciation.

Because something is losing money, the value of R must fall between 0 and 1.

The formula doesn't change and the value of R does not become negative! So we still use:

$$V_0 = P, \qquad V_{n+1} = R \times V_n$$

But the value of R will be worked out as:

$$R = 1 - \frac{r}{100}$$



Example: Reducing balance depreciation

A car is purchased for $18500. The following recurrence relation can be used to model the car's value as it depreciates by 10% of its value each year.

$$V_0 = 18500, V_{n+1} = 0.9 \times Vn$$

In the recurrence relation, Vn is the value of the car after n years.

- Use the recurrence relation to find the value of the car after 1, 2 and 3 years.
- When will the value of the car first be worth less than $10000?

$r = 10\%$

$R = 1 - \frac{10}{100}$

$= 1 - 0.1$

$= 0.9$

$V_0 = 18500$

$V_1 = 0.9 \times 18500 = \$16650$

$V_2 = 0.9 \times 16650 = \$14985$

$V_3 = 0.9 \times 14985 = \$13486.50$

$\therefore$ 6 years



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