Trigonometric Ratios

- Year 9 Mathematics

Year 9 Mathematics Mainstream

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- Know what the three trigonometric ratios are
- Know how to find trigonometric ratios from right angled triangles
- Know how to evaluate trigonometric ratios

RECAP

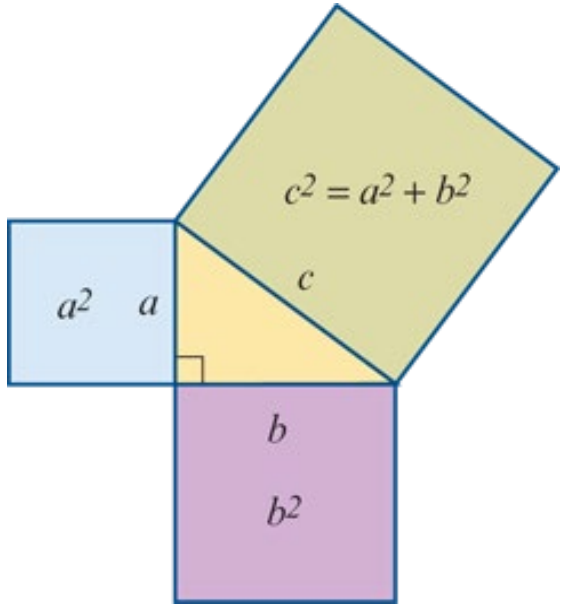
In a previous lesson we looked at how, for right angled triangles, there is a relationship between the sum of the areas of the two shortest sides of a right angled triangle and the area of the hypotenuse.

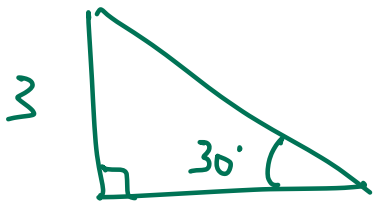
This is **only true** for right angled triangles.

The relationship is more formally known as:

$$c^2 = a^2 + b^2$$

This allowed us to find missing side lengths when given two other side lengths. But, what happens when we are given an angle and a side length? Is there a way to find the lengths of the other sides of a **right angled triangle**?





SOHCAHTOA

I love this topic as you can pretty much smash it using the word SOHCAHTOA.

Or, as my Maths teacher at school used to make us recite:

Silly
Old
Harry
Caught
A
Herring
Trawling
Off
America

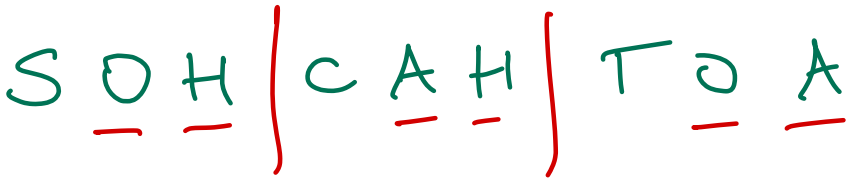
This is a Herring – it's a type of fish!

This is a Trawler – it's a fishing boat!

I’m sure he hate us

I really don’t understand why he made us learn the following:

Silly
Old
Harry
Caught
A
Herring
Trawling
Off
America



I really think he hated us … sigh … but on with the Maths!

Labelling a right-angled triangle

The first really important thing to note it … **THIS ONLY WORKS WITH RIGHT-ANGLED TRIANGLES.**

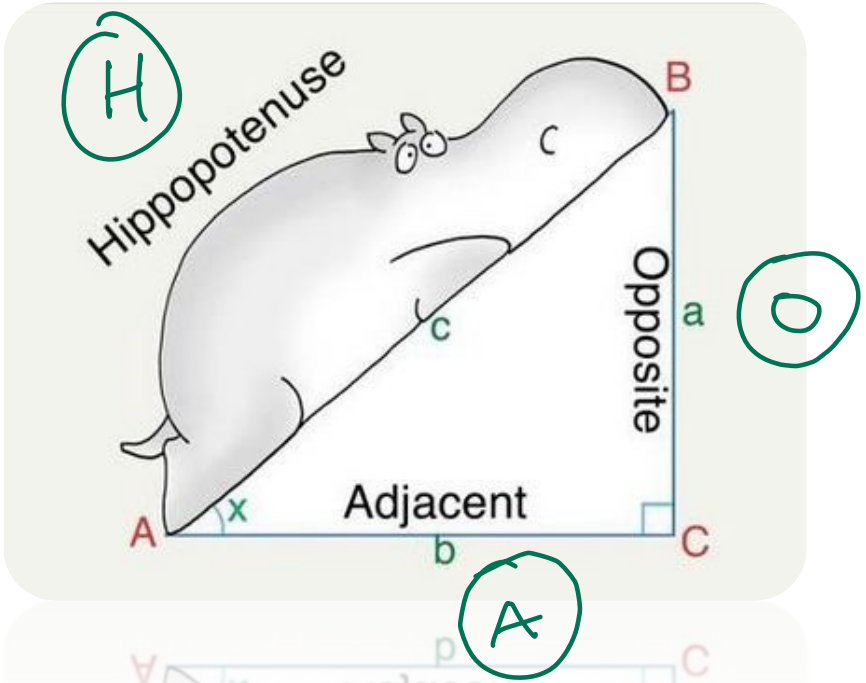
The longest side of a right-angled triangle is called the **Hypotenuse**.

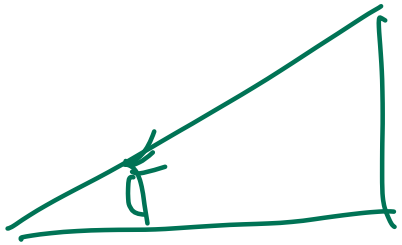
I used to think people were just struggling to say Hippopotamus.

The rest of the sides can only be labelled with reference to an **known** (or unknown by given) **angle**.

The diagram uses ‘x’ to be the angle.

But we can also use any one of the Greek letters. Generally we use θ.

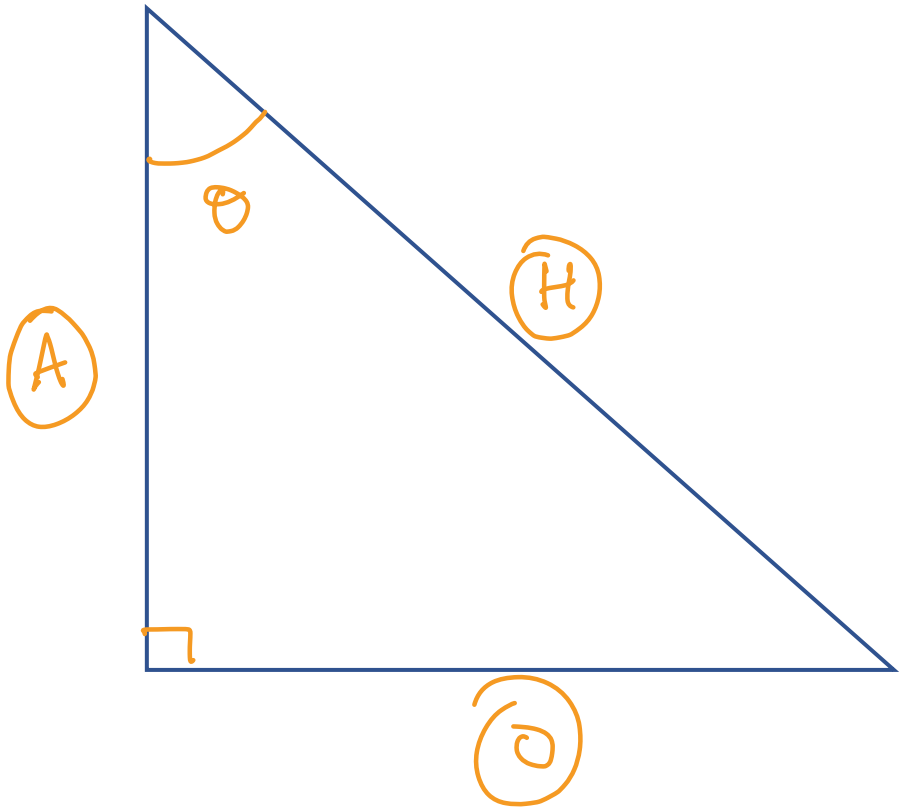




SOH | CAH | TOA

Labelling a right-angled triangle

When the angle is in the other corner, then the adjacent and opposite sides will naturally switch.



Adjacent

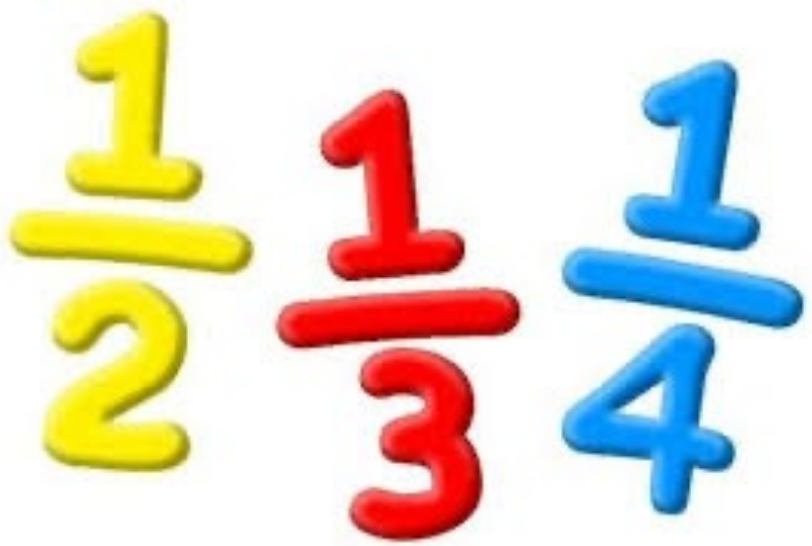
Opposite

What is a ratio?

A ratio is a **fraction**.

When you write something as a ratio you leave it as a fraction!

Trigonometric ratios are always written as fractions.



Using SOHCAHTOA

I love using SOHCAHTOA. It is an "easy" way to remember the following …

S O H C A H T O A

sin θ
cos θ
tan θ

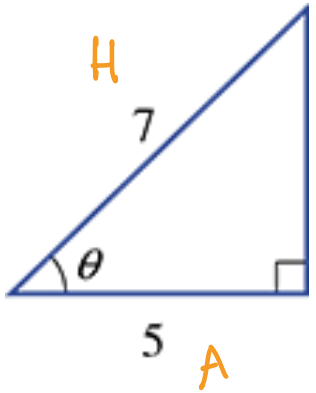
Or …

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad cos\,\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

Examples

Using the given sides, write the trigonometric ratio (in fraction form) for each of the following triangles:

O A O
S H C H T A

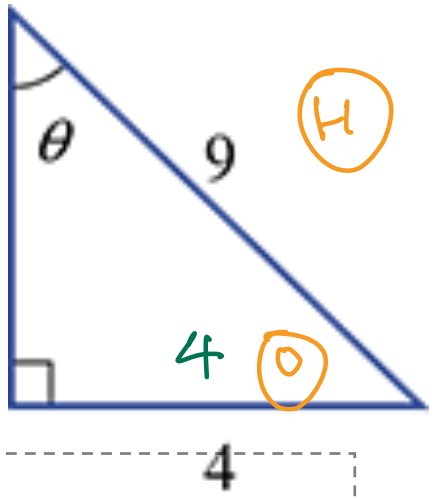


F. $\cos\theta = \frac{A}{H}$

S. $\cos\theta = \frac{5}{7}$

Examples

Using the given sides, write the trigonometric ratio (in fraction form) for each of the following triangles:



H

O

SOH CAH TOA

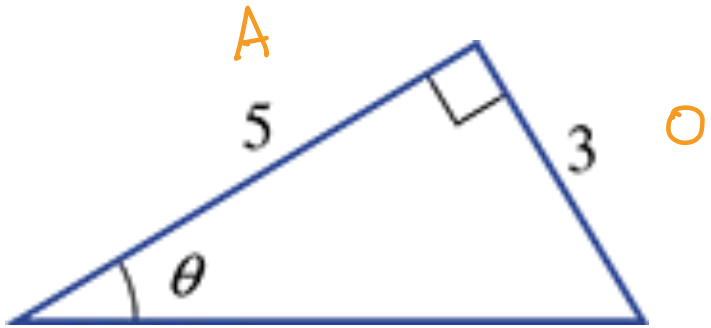
F. $\sin\theta = \frac{O}{H}$

S. $\sin\theta = \frac{4}{9}$

Examples

Using the given sides, write the trigonometric ratio (in fraction form) for each of the following triangles:

O A O
S H C H T A



F. $\tan\theta = \frac{O}{A}$

S. $\tan\theta = \frac{3}{5}$

Drawing a triangle from a ratio

We can draw a triangle from a ratio by simply using SOHCAHTOA.

Remember: Any whole number is really a fraction.

E.g. given that $\cos\theta = \frac{1}{2}$, find $\sin\theta$ and $\tan\theta$

Note: **This is also going to use the Pythag stuff we did.**

O A O
S H C H T A

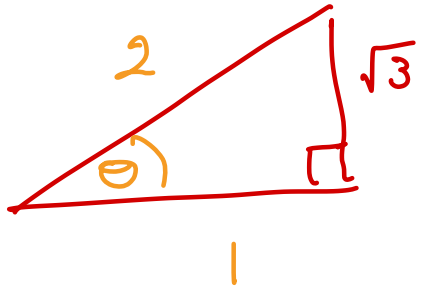
$\cos\theta = \frac{1}{2} = \frac{A}{H}$

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{O}{A}$

$\tan\theta = \sqrt{3}$



$c^2 = a^2 + b^2$

$2^2 = 1^2 + b^2$

$4 = 1 + b^2$

$b^2 = 3$

$b = \sqrt{3}$

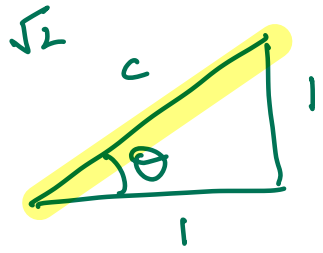
Drawing a triangle from a ratio

We can draw a triangle from a ratio by simply using SOHCAHTOA.

Remember: Any whole number is really a fraction.

E.g. given that $\tan\theta = 1$, find $\sin\theta$ and $\tan\theta$

O A O
S H C H T A



$\tan\theta = \frac{1}{1} = \frac{O}{A}$

$c^2 = a^2 + b^2$

$c^2 = 1^2 + 1^2$

$c^2 = 1 + 1$

$c^2 = 2$

$c = \sqrt{2}$

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{1}{\sqrt{2}}$

Questions to complete:

The questions I would like you to complete for this lesson are:

Exercise 3E: Trigonometric ratios
Questions: 2, 4, 5, 7 10, 12

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