

Finance applications using arithmetic sequences and recurrence relations

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to use a recurrence relation to model simple interest.
- To be able to use a recurrence relation to model flat rate depreciation.
- To be able to use a recurrence relation to model unit cost depreciation.
- To be able to use a rule to determine the nth term for linear growth or decay.



Recap

In the previous lesson we looked at how to turn a recurrence relation into a rule. This has build on the knowledge we had about reading and using recurrence relations to create arithmetic sequences.

Whilst it has all been rather theoretical at the moment … what are the real world applications?

Well, finance using recurrence relations and rules. Anything with money and you can guarantee the formula isn't too far behind.

Year 12 General Maths using this concept A LOT! So, get that summary book ready and let's make some decent notes.

$$V_0 = 20, \quad V_{n+1} = V_n + 10$$

$$V_n = V_0 + n.D$$



Recap: The recurrence relation formula

You really do need to commit this to memory. Learn what it means and how to use it and I don't there are too many things which they can thrown at you which you would be able to answer.

$$t_0 = a, \qquad t_{n+1} = t_n + D$$

Add or subtract the value of D

Start number

To get to the next number

Take the current term



Maths is stupid - or Barry is trying to trick us

When we get to real world situations, we seem to have to use different letters! So, please be careful to always read the question to see which letter they have used.

$$V_0 = P, \qquad V_{n+1} = V_n + D$$

The interest or payment being added

Start number or opening balance

To get to the next number

Take the current term



Simple interest

Banks do all they can to make money. They want your money so they can give it to someone else.

To get you to give them your money they “reward” you for saving with them by paying you interest on your money.

They then charge someone a much higher rate of interest to borrow the money and they keep the difference.

This part of the video is going to look at interest.

Simple interest is when the bank pay you the **same amount of interest** (per period) regardless of how much money you have in the bank. They work this out from your **initial investment.**

$1000

10%

10% of 1000

$10/100 \times 1000 = \$100$

$$V_0 = P, \qquad V_{n+1} = V_n + D$$

How much you open the account with

The amount of interest the bank is giving you

To get to the next number

Take the current term



Working out simple interest

100 = 1000 @ 10%

2000 @ 8%

4000 @ 1·2%

To work out the interest (D) you need to know the interest rate the bank are using to reward you.

This is always a **%** and given per year or annum.

To find the interest you build on the percentages work we have already done and get:

$$D = \frac{r}{100} \times V_0$$

We can then use the value of D in the following:

$$V_0 = P, \qquad V_{n+1} = V_n + D$$

How much you open the account with

The amount of interest the bank is giving you

To get to the next number

Take the current term



Example: Finding the amount of simple interest each year

An investment of $3500 pays interest at the rate of 4.2% per annum in the form of simple interest. Find the amount of interest paid each year.

$$D = \frac{r}{100} \times V_0$$

$V_0 = 3500$ $r = 4.2\%$

$$D = \frac{4.2}{100} \times 3500 = \$147$$



Example: Finding the amount of simple interest each year

The following recurrence relation can be used to model a simple interest investment of $2000, paying interest at the rate of 7.5% per annum:

$$V_0 = 2000, V_{n+1} = V_n + 150$$

where Vn is the value of the investment after n years.

- Use the recurrence relation to find the value of the investment after 1, 2 and 3 years.
- When will the investment reach $2750 in value?

$$D = \frac{r}{100} \times V_0$$

$\frac{7.5}{100} \times 2000 = 150$

- $V_0 = 2000$

  $V_1 = 2000 + 150 = \$2150$

  $V_2 = 2150 + 150 = \$2300$

  $V_3 = 2300 + 150 = \$2450$

- 5 years.



Using recurrence relations to model flat rate depreciation

When we buy a car, we start losing money the second you leave for forecourt. When something loses money we say it is **depreciating in value**. When the value of a the depreciation stays the same, we say it has a flat rate of depreciation.

$$D = \frac{r}{100} \times V_0$$

As something is **losing value** we need to change the recurrence relation to have a **negative sign in it.**

$$V_0 = P, \qquad V_{n+1} = V_n - D$$

How much you paid for the item

To get to the next number

Take the current term

The amount you are losing each period



Example: Using recurrence relations to model flat rate depreciation

The following recurrence relation can be used to model the flat rate depreciation of a car purchased for $18500, depreciating at a flat rate of 10% per year.

$$D = \frac{r}{100} \times V_0$$

$$V_0 = 18500, V_{n+1} = V_n - 1850$$

10% of 18 500 = 1850

In the recurrence relation, Vn is the value of the car after n years.

- Use the recurrence relation to find the value of the car after 1, 2 and 3 years.
- How long will it take for the car's value to depreciate to zero?

$V_0 = 18500$

$V_1 = 18500 - 1850 = \$16650$

$V_2 = 16650 - 1850 = \$14800$

$V_3 = 14800 - 1850 = \$12950$

After 10 years



Unit cost depreciation

Again, here is an example of where Maths is stupid!

This is pretty much the same idea as flat rate depreciation. The only difference is that something is losing value per use. An example of this would be a photocopier. Each copy printed will reduce the value of the machine.

The formula stays exactly the same!!!

$$D = \frac{r}{100} \times V_0$$

$$V_0 = P, \qquad V_{n+1} = V_n - D$$

How much you paid for the item

To get to the next number

Take the current term

The amount you are losing each use



Example: Unit cost depreciation

A car with a purchase price of $32000 depreciates at a unit cost of $150 per 1000 kilometres.

$$D = \frac{r}{100} \times V_0$$

- State the recurrence relation for unit cost depreciation of the car, where Vn represents the value of the car after n thousand kilometres.
- Use the recurrence relation to find the value of the car after 1000, 2000 and 3000 kilometres.
- How many kilometres is the car expected to travel before its value depreciates by at least $1000
- ?

$V_0 = 32000$ $150 / 1000 km

$V_0 = 32000$, $V_{n+1} = V_n - 150$

$V_0 = 32000$

$V_1 = 32000 - 150 = \$31850$

$V_2 = 31850 - 150 = \$31700$

$V_3 = 31700 - 150 = \$31550$

7000 km.

32000

31000



Changing the recurrence relation into a rule

If you have remembered the work from the last lesson, we know that we can change a recurrence relation into a rule. As we have changed the letters being used in this chapter, it makes sense to redo the work.

If we start with:

$$V_0 = P, \qquad V_{n+1} = V_n \pm D$$

Then the rule would become:

$$V_n = P \pm nD$$

Note the use of the sign ($\pm$) which means I can write two formulae in one line.



Example: Using a rule for the nth term

The following recurrence relation can be used to model a simple-interest investment of $4000, paying interest at the rate of 6.5% per year.

$$V_0 = 4000, V_{n+1} = V_n + 260$$

- How much interest is added to the investment each year?
- Use a rule to find the value of the investment after 15 years.
- Use a rule to find when the value of the investment first exceeds $10000.

$\frac{6.5}{100} \times 4000 = 260$

- $260
- $V_n = 4000 + n.260$

$V_{15} = 4000 + 15.260$

$= \$7900$

$10000 = 4000 + n.260$

$n = 23.0769$

$n = 24$



Example: Using a rule for the nth term

The following recurrence relation can be used to model a simple-interest investment of $12000, paying interest at the rate of 4.8% per year.

$$V_0 = 12000, V_{n+1} = V_n + 576$$

- How much interest is added to the investment each year?
- Use a rule to find the value of the investment after 10 years.
- Use a rule to find when the value of the investment first exceeds $20000.

$\frac{4.8}{100} \times 12000 = 576$

- $576
- $V_n = V_0 + n.D$

$V_n = 12000 + n.576$

$V_{10} = 12000 + 10.576$

$= \$17760$

$20000 = 12000 + n.576$

$n = 13.888....$

$n = 14$ years



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