

Pythagoras in three dimensions

Year 9 Mathematics
Mainstream

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- Know how to apply the understanding we have to finding lengths of sides of right angled triangles in three dimensional solids



RECAP

In a previous lesson we looked at how, for right angled triangles, there is a relationship between the sum of the areas of the two shortest sides of a right angled triangle and the area of the hypotenuse.

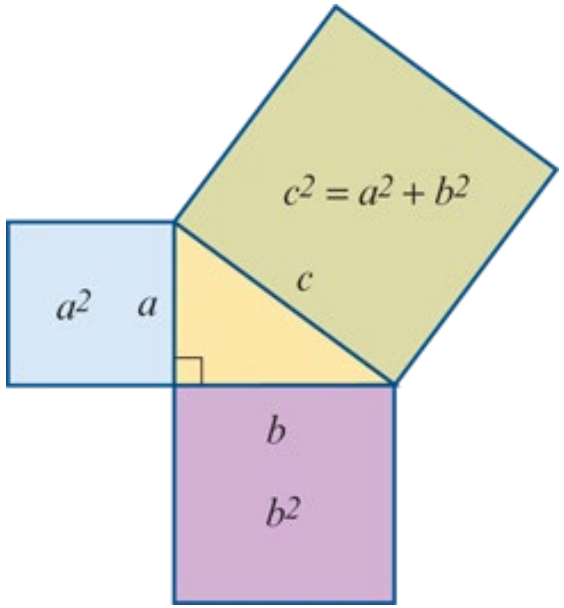
This is **only true** for right angled triangles.

The relationship is more formally known as:

$$c^2 = a^2 + b^2$$

In the previous lessons we looked at how to find any missing side length of a right angled triangle **so long as we have two other sides given to me**.

Let's see how we can apply them to more complex problems.

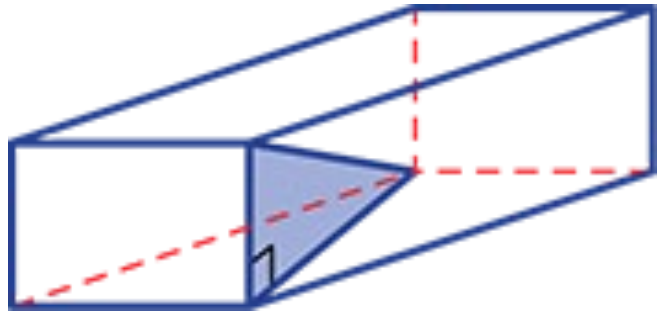


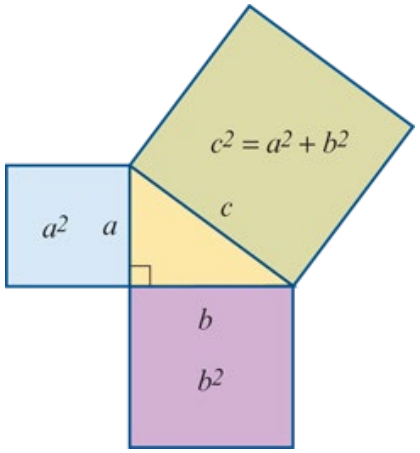


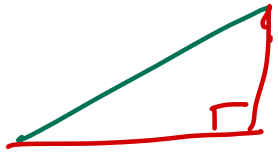
Triangles are everywhere

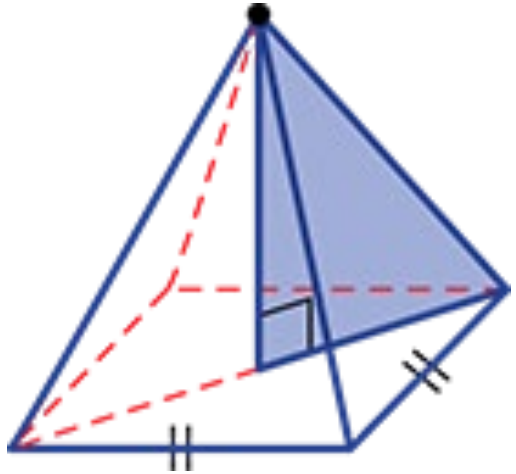
We can see right angled triangles everywhere. All the shapes we have looked at so far are all two-dimensional. Let's open things up to three dimensional shapes.

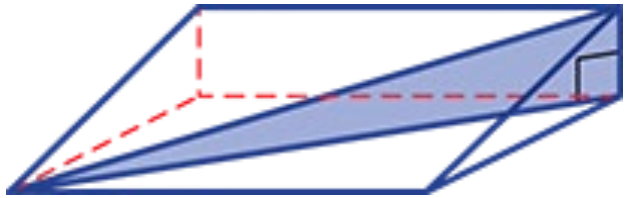
Where ever there is a slanted line, there is a right angled triangle.







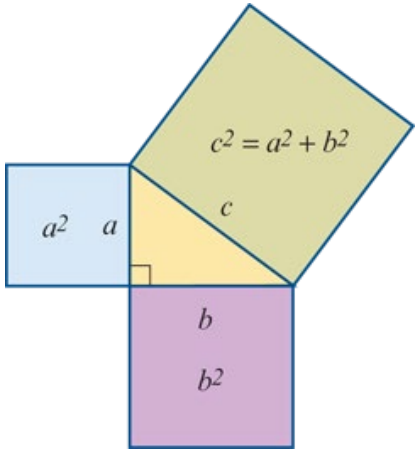


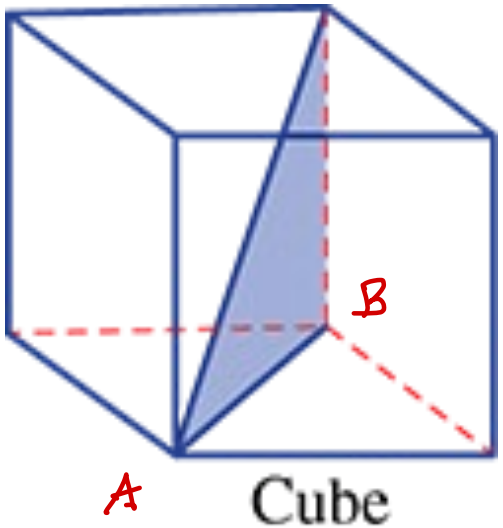




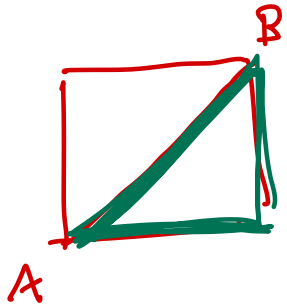
Finding the length of sides may require two calculations to take place

In order to find the length between the two corners shown, we would need to find the diagonal along the bottom of the shape first.





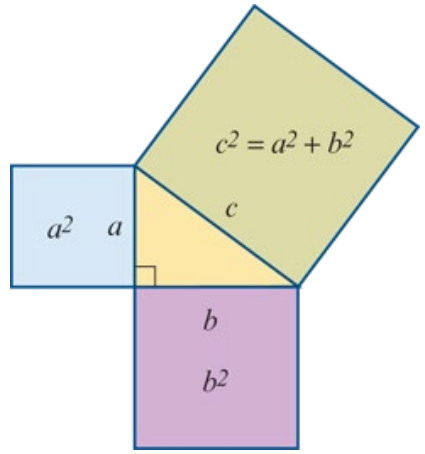
Cube

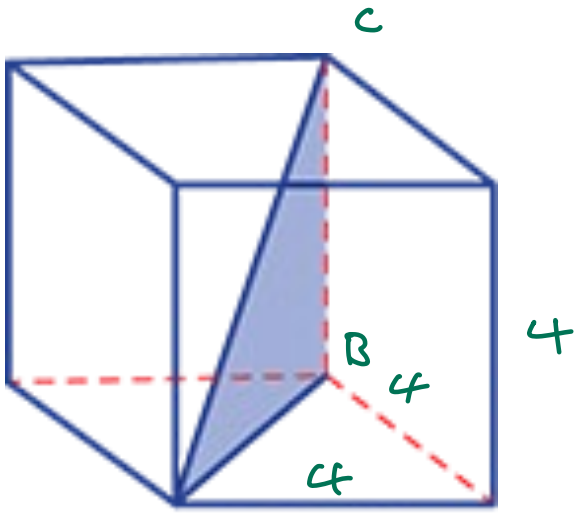




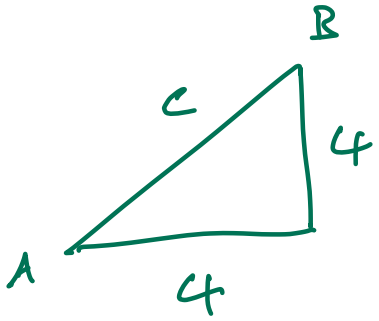
Using surds to make our life easier

When finding the length along the bottom of the shape, it actually makes our life easier to keep the answer as a surd. This makes finding the diagonal through the shape (from the bottom left corner to the back right corner) easier (and more accurate!).





Cube

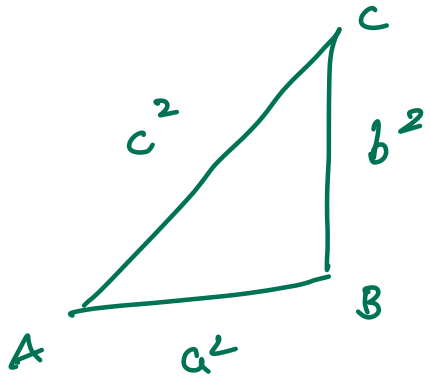


$c^2 = 4^2 + 4^2$

$= 16 + 16$

$c^2 = 32$

$c = \sqrt{32}$

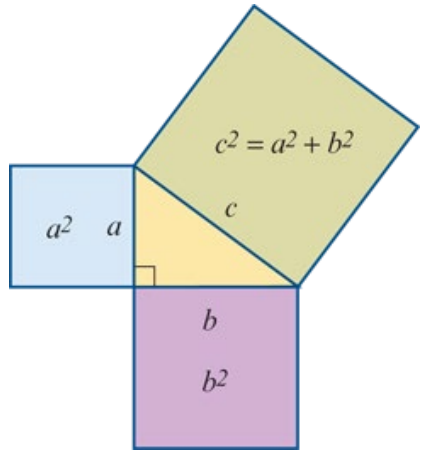


$4^2 = 16$

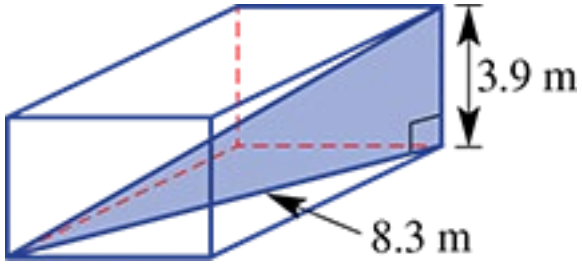
32

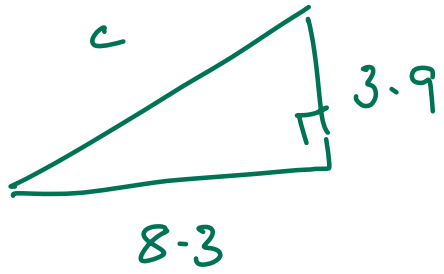


Example 1



The length of the diagonal on the base of a rectangular prism is 8.3 m and the height of the rectangular prism is 3.9 m. Find the distance from one corner of the rectangular prism to the opposite corner. Give your answer correct to two decimal places.





$c^2 = 8.3^2 + 3.9^2$

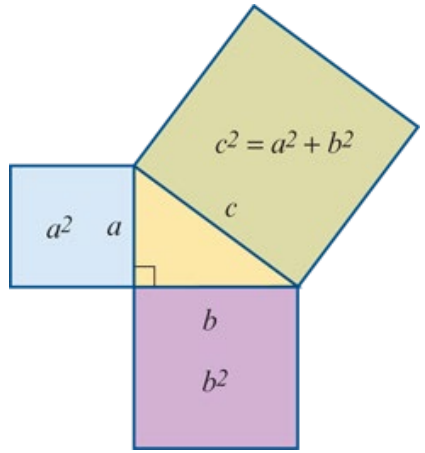
$c^2 = 84.1$

$c = \sqrt{84.1}$

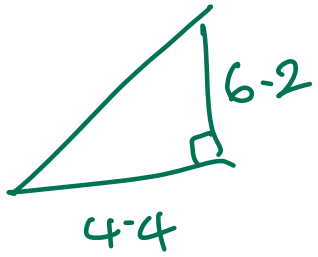
$c = 9.17\text{ m}$



Example 2



A cylinder has diameter 4.4 cm and height 6.2 cm. Find the distance from a point on the circumference of the base to a point on the opposite side at the top. Give your answer correct to two decimal places.

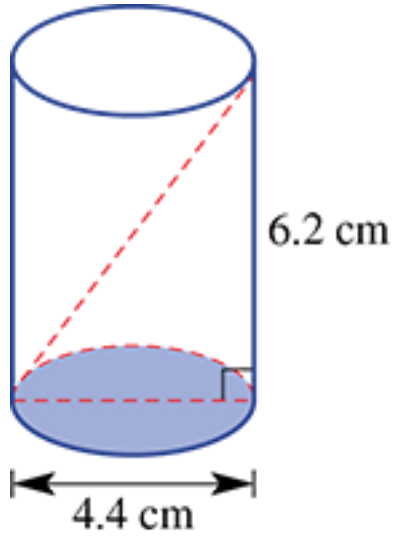


$c^2 = a^2 + b^2$

$c^2 = 4.4^2 + 6.2^2$

$c^2 = 57.8$

$c = \sqrt{57.8} = 7.60$ cm

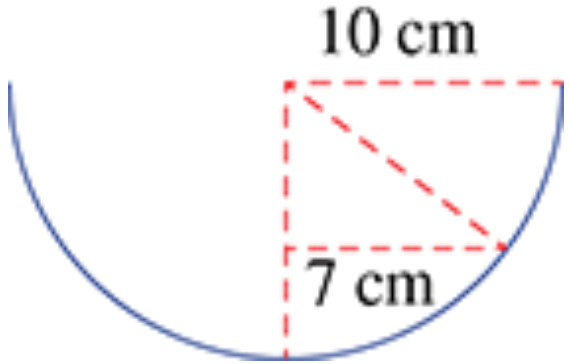


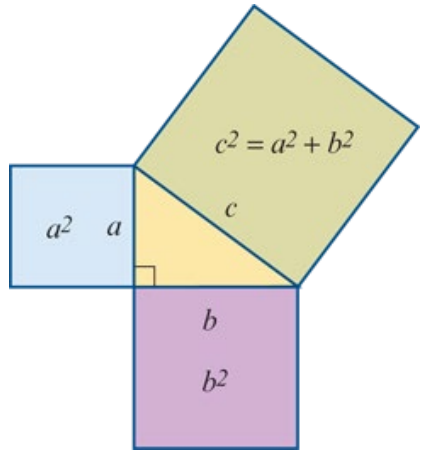


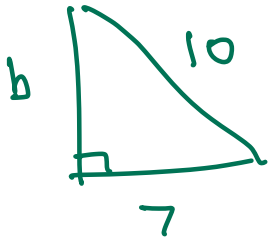
A challenging question

I have seen the following question in many year 12 exams!

A bowl is in the shape of a hemisphere (half sphere) with radius 10 cm. The surface of the water in the container has a radius of 7 cm. How deep is the water? Give your answer to two decimal places.







$c^2 = a^2 + b^2$

$10^2 = 7^2 + b^2$

$100 = 49 + b^2$ $\quad -49$ $\quad -49$

$51 = b^2$

$b = \sqrt{51}$

$= 7.14$ cm

$\therefore$ height $= 10 - 7.14$

$= 2.86$ cm



Questions to complete:

The questions I would like you to complete for this lesson are:

Exercise 3D: Pythagoras in three dimensions
Questions: 1, 2b, 3c, 5, 6, 10, 11

Extension Questions (optional)
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