Arithmetic sequences using recursion

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to generate an arithmetic sequence using a recurrence relation
- To eb able to use the rule for the nth term to solve problems involving arithmetic sequence

Recap

In the previous lesson we looked at how we can write a term to term rule as a recurrence relation.

A recurrence relation tells us what number to start a sequence on and how to get from one term to the next.

This lesson is going to look at how we can create arithmetic sequences using recursion.

$6 \quad 9 \quad 12 \quad 15 \quad \ldots \qquad +3$

$100 \quad 50 \quad 0 \quad -50 \quad -100 \qquad -50$

$t_0 = 3, \quad t_{n+1} = t_n + 3$

A general form for the recurrence relation

We will come to find that there are general forms for all recurrence relations.

In the previous video we used the following. But this will only ever give us the same sequence.

$$t_0 = 2, \qquad t_{n+1} = t_n + 2$$

Add two

Start number

To get to the next number

Take the current term

To describe all arithmetic sequence we can write it in another way:

$$t_0 = a, \qquad t_{n+1} = t_n + D$$

Where ‘a’ is the start number and ‘D’ is the common difference.

Example: Using a recurrence relation to generate an arithmetic sequence

Generate and graph the first five terms of the arithmetic sequence defined by the recurrence relation:

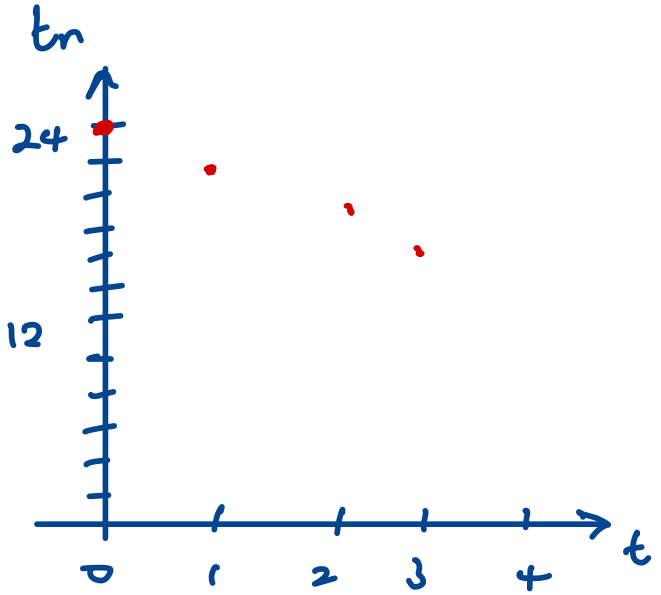
$$t_0 = 24, \qquad t_{n+1} = t_n - 2$$

Subtract two

'a' for this sequence is 24

To get to the next number

Take the current term



24 22 20 18 16

Example: Using a recurrence relation to generate an arithmetic sequence

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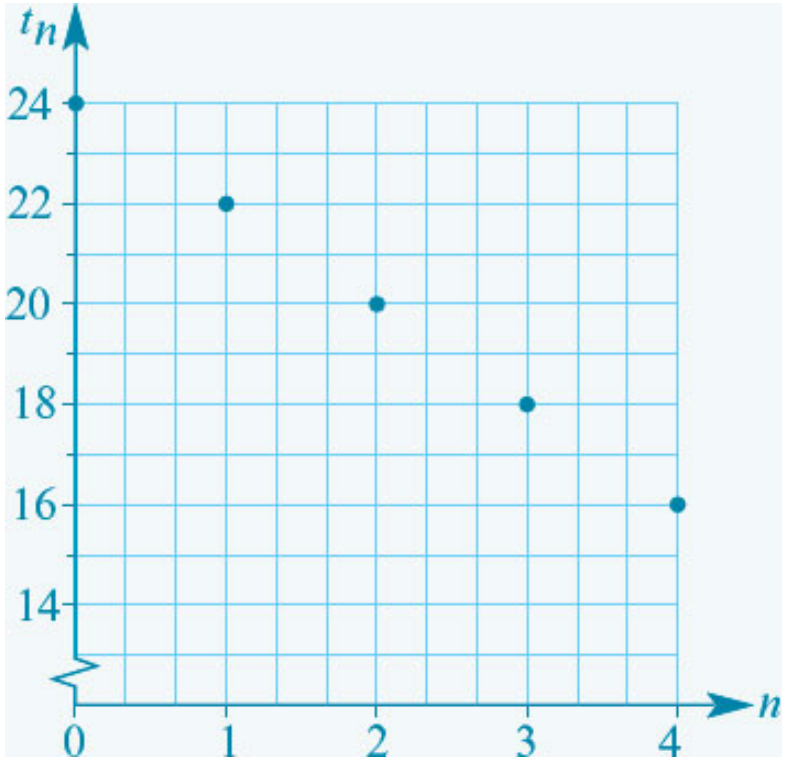

Finding the nth term of an arithmetic sequence

If we look at the previous example where we were asked to generate and graph the **first five terms** of the arithmetic sequence defined by the recurrence relation:

$$t_0 = 24, \qquad t_{n+1} = t_n - 2$$

rule $t_n =$

This was nice and easy. But what if I wanted to find t_{100}?

I don't want to sit there all day adding two to the previous number until I get to the 101st term!!!

So we can turn a recurrence relation into a rule.

Firstly, some theory as to how it works …

Finding the nth term of an arithmetic sequence

Let's start nice and simple:

$$t_0 = 2, \qquad t_{n+1} = t_n + 2$$

t_4

2 4 6 8

t_0 2

t_1 2 + 2 $= 2 + 1 \times 2$

t_2 2 + 2 + 2 $= 2 + 2 \times 2$

t_3 2 + 2 + 2 + 2 $= 2 + 3 \times 2$

t_4 2 + 2 + 2 + 2 + 2 $= 2 + 4 \times 2$

$t_n = 2 + n \times 2$

$t_n = t_0 + 2 \times n$

$t_4 = 2 + 2 \times 4$

$= 2 + 8 = 10$

Formula for the nth term from a recurrence relation

If we have a recurrence relation of the form:

$$t_0 = a, \qquad t_{n+1} = t_n + D$$

The rule for this sequence will be:

$$\boldsymbol{t_n = a + nD}$$

What is really important to note is the sign of D. If D is negative, you must make the formula:

$$t_n = a - nD$$

$t_n = t_0 + n \times D$

$t_n = a + nD$

$t_n = a - nD$

Example: Finding the nth term

Consider the recurrence relation:

$$t_0 = 21, \qquad t_{n+1} = t_n - 3$$

Find t_{20}

$t_n = a - nD$

$t_n = 21 - n.3$

$t_{20} = 21 - 20 \times 3$

$= 21 - 60$

$= -39$

Example: Finding the nth term

Consider the recurrence relation:

$$t_0 = 54, \qquad t_{n+1} = t_n + 4$$

Find t_{50}

$$t_n = t_0 + n.D$$

$$t_n = 54 + n.4$$

$$t_{50} = 54 + 50 \times 4$$

$$= 54 + 200$$

$$= \underline{\underline{254}}$$

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