Number Patterns

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to determine a simple rule for a sequence of numbers
- To be able to generate a sequence from a starting number and a simple rule

Recap

This is the first lesson in this topic and so there isn't really much to recap.

Please note that this is a massive topic in Year 12 and, as such, it would be a really good idea to spend time really understanding this work.

What is a sequence?

Numbers can be random or follow some sort of pattern in their creation.

I can use my CAS to come up with a random list of numbers really easily. This might come in handy later on.

A sequence is a list of numbers which follow some sort of order or pattern.

Sequences can be created in relation to the previous number in the list or by their position in the list.

Examples could be:

- 1, 2, 3, 4, 5 …
- 2, 4, 6, 8, 10 …
- 2, 4, 8, 16, 32, 64 …
- 100, 50, 25, 12.5, …
- 1, 1, 2, 3, 5, 8, 13, 21, …

Sequences don’t have to just be whole numbers or decimal numbers

They can be fractional too!

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$$

Sequences can be random or created through **recursion**.

Recursion is a repeated process using a rule.

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Language of sequences

We can see that some sequences will continue to **increase** or be **increasing** with no end.

Eg. 2, 4, 6, 8, 10 ...

Some sequences can be see to **decrease** or be **decreasing** with no limit to when it ends.

e.g. 100, 50, 25, 12,5, 6,25, ...

Some will simply oscillate between two or more values

e.g. 1, −1, 1, −1, 1, −1, ...

Some will, eventually, have a **limiting value** where the sequence will approach a particular value.

Some might even just have a **constant value**.

e.g. 4, 4, 4, 4, 4, 4, ...

Examples of sequences

Consider the following sequences and identify their behaviour as increasing, decreasing, constant or oscillating. Also state whether the sequence has a limiting value.

- −3,−3,−3,−3,...
- 100,90,80,70,...
- **1000,100,10,1,0.1,...**
- 1,4,9,16, 25,36,...
- 10,−8,10,−8,...

Note: Be careful with the highlighted one. There are two answers!

Recursion: An introduction

Recursion basically describes a pattern which is used to form a sequence of numbers.

It might be an “add 3” rule which means you add three to one number in the list to get to the next number.

e.g. 4, 7, 10, 13, 16, 19

Note: Sequences don’t have to start with 1 or, in fact, be positive!

Example: Finding the recursive rule

Look for a pattern or rule in each sequence and find the next number.

- 2,8,14,20,...
- 5,15,45,135,...
- 7,4,1,−2,...

Where does a sequence start?

As I said in a previous slide, sequences can start and end wherever they like! Or … where they are told to start and end.

In the previous examples, I started where I wanted (as I wasn't given any instructions to do so otherwise). But, you can be told not only the rule but where to start.

Example: Write down the first five terms of the sequence with a starting value of 5 and the rule 'add 3 to each term'.

Doing recursion on the CAS

The CAS is great at doing recursion for us.

The most important key on the CAS would have to be the ANS key as it stores the last number in this special place. Knowing this means we can use the CAS to find sequences for us.

Note: The two CAS calculators operate in the same way and so I will only show the instructions for one of them.

Example: Using the CAS, generate the first six terms of the arithmetic sequence: 2, 7, 12, …