



Instantaneous rate of change

Year 11
Mathematical Methods

www.maffsguru.com

Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be an instantaneous rate of change
- Know how to find the instantaneous rate of change using the tangent line at a point.

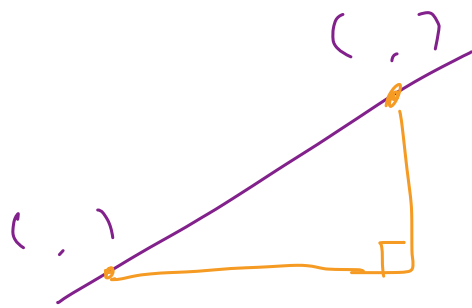


RECAP

We have spent some time so far looking at rates of change (gradients) of straight lines and functions. In the last lesson we looked at finding average speeds by taking two coordinate points and drawing a line between them. This line is either a chord or a secant.

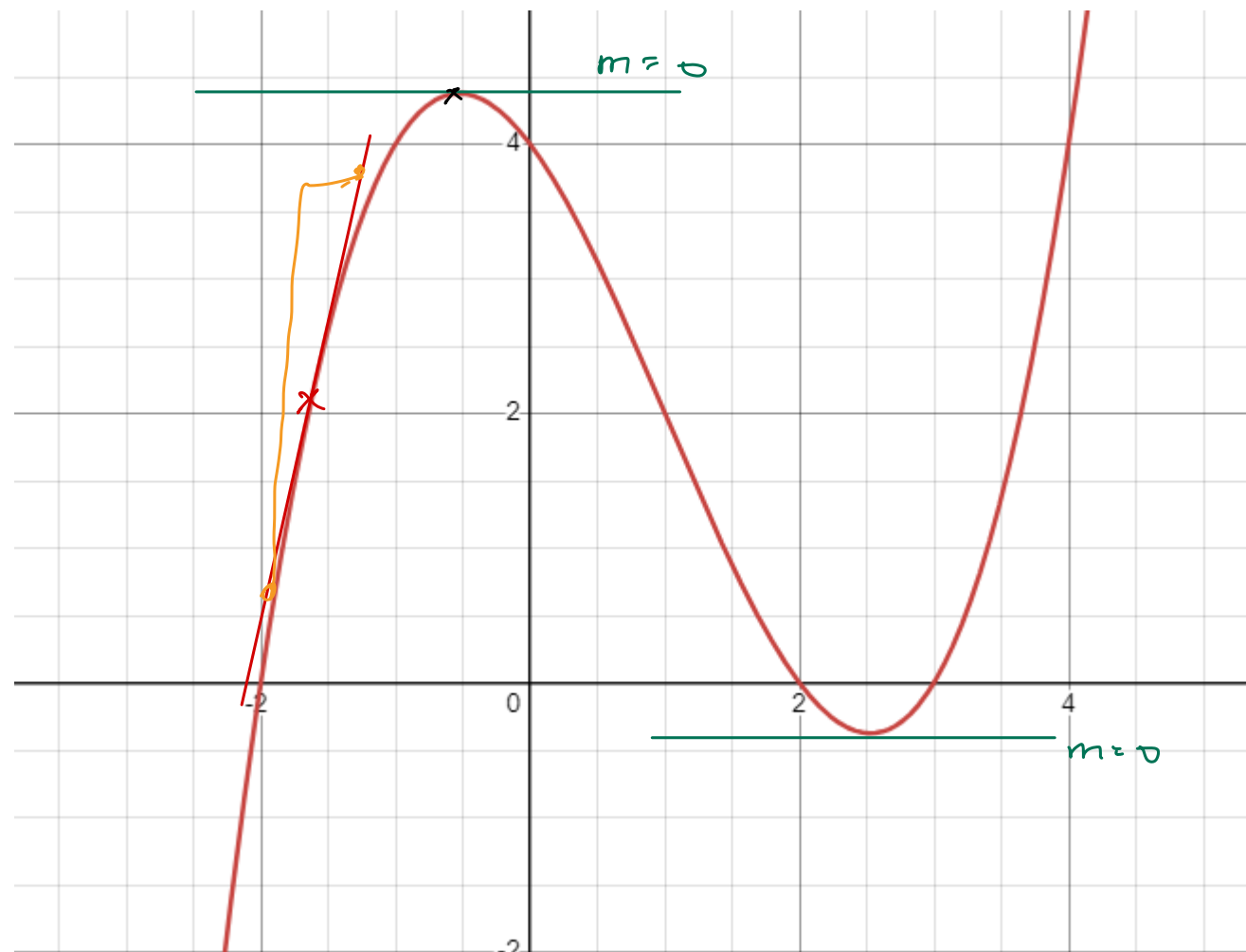
Whenever we have a straight line, we can find the gradient of the line. This would then be the average speed (or quantity) between those two points.

Question: What happens when the two points we are using to find the gradient get closer and closer to each other?


$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\text{rise}}{\text{run}}$$

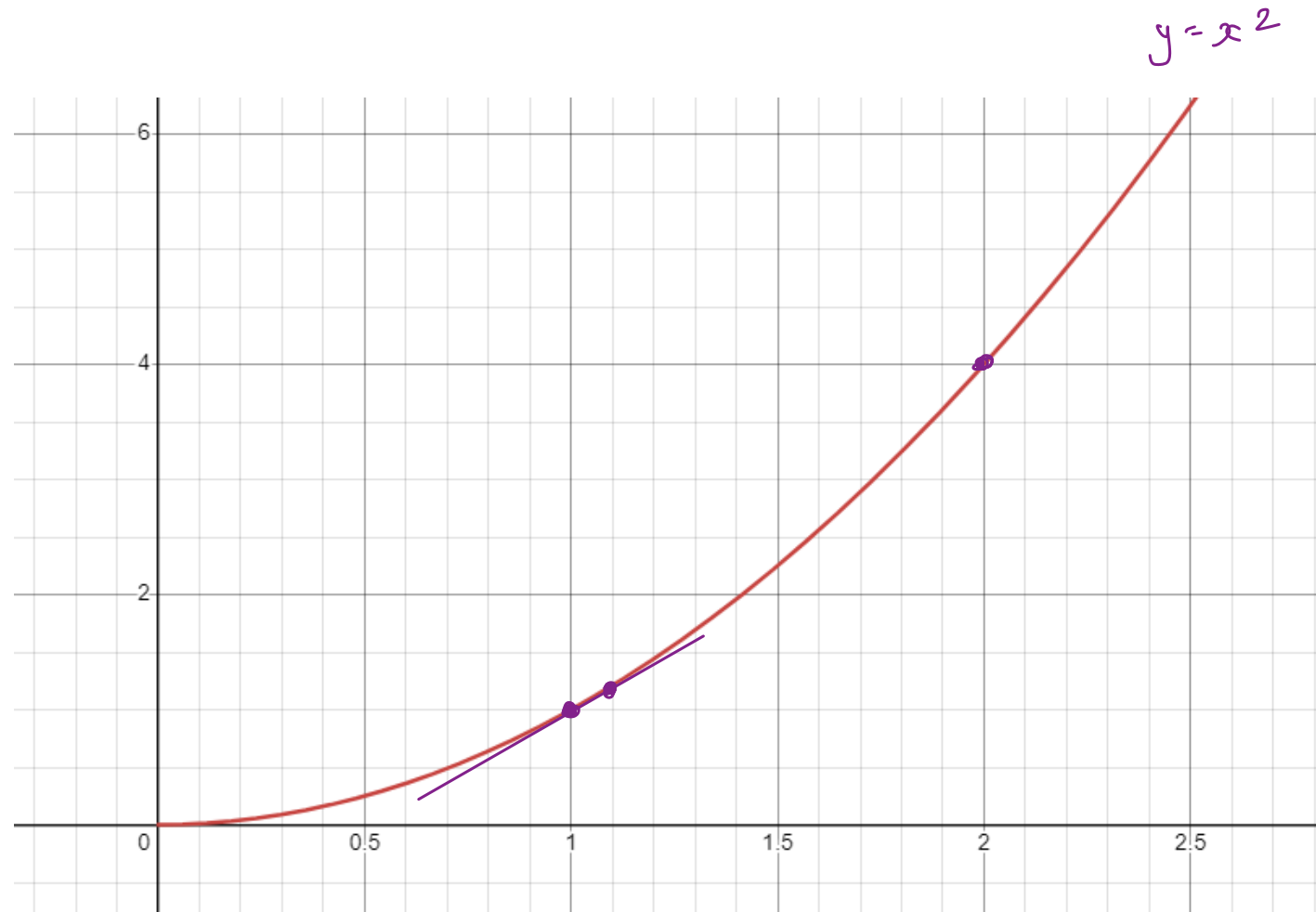
$$\frac{1}{3}(x-2)(x+2)(x-3)$$



Secant between two points on $y = x^2$

If we superimpose on a section of the graph of $y = x^2$ and look at what happens when we draw secants which get closer and closer to one point, something interesting happens!

Note: We will use the value of $(1, 1)$ as one of the points for every gradient.



Recap: Finding the gradient between two points

Methods uses a particular language which might be designed to confuse more than it helps.

We need to remember that we can write the average rate of change, or gradient, using a lot of different notations.

An example is shown on the right.

$$y = f(x)$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a}$$

$$(x, y)$$

$$(x, f(x))$$

$$(b, f(b))$$

Here, a function will already have been defined.

So, $f(b)$ means the y value when ' b ' is put into the function.

$$(a, f(a))$$



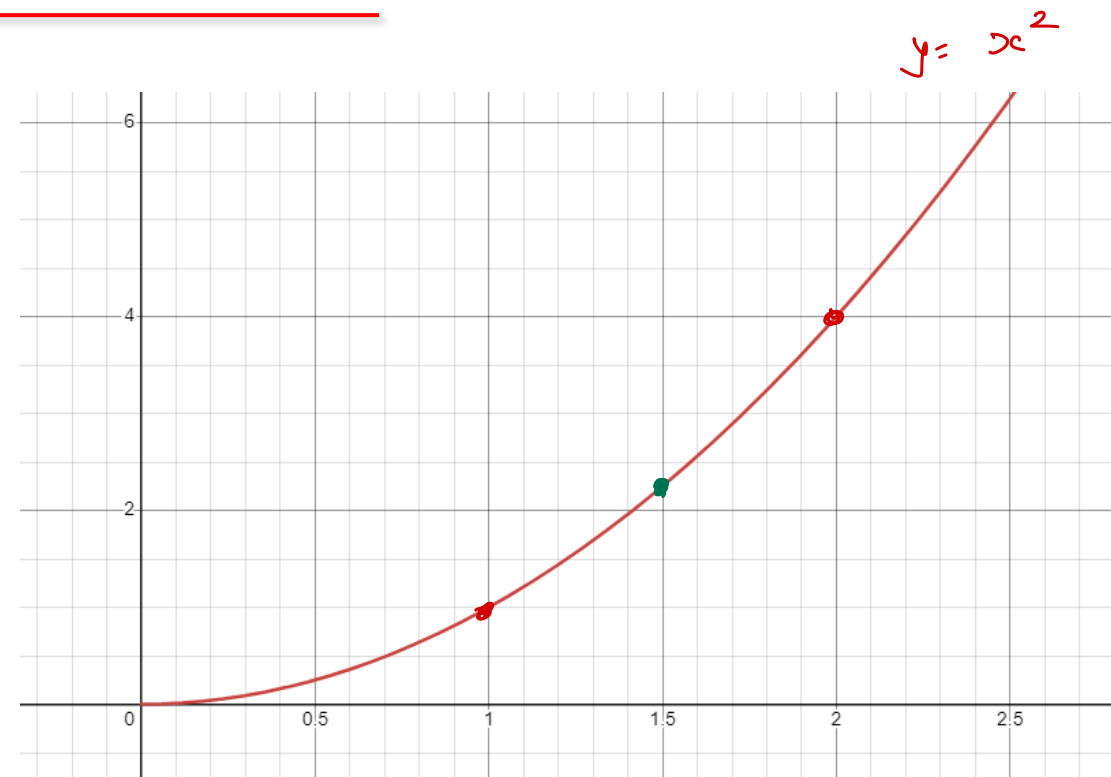
Looking at the values of the gradients

If we use rise over run and function notation for the CAS, we can see that something interesting is happening with the values of the gradients.

Question: Does it appear that the gradient is approaching a particular value?

1.1	*Doc	DEG	Done
Define $f(x)=x^2$			
$\frac{f(2)-f(1)}{2-1}$		3.	
$\frac{f(1.5)-f(1)}{1.5-1}$		2.5	
$\frac{f(1.25)-f(1)}{1.25-1}$		2.25	

1.1	*Doc	DEG		
$\frac{f(1.25)-f(1)}{1.25-1}$				
		2.25		
$\frac{f(1.1)-f(1)}{1.1-1}$				
		2.1		
$\frac{f(1.05)-f(1)}{1.05-1}$				
		2.05		
				



1, 1.00001



Another way to write gradient

Later in this course you will see the notation shown on the right being used.

This relies on the idea that you can write a coordinate in the following way:

$$(x, f(x))$$

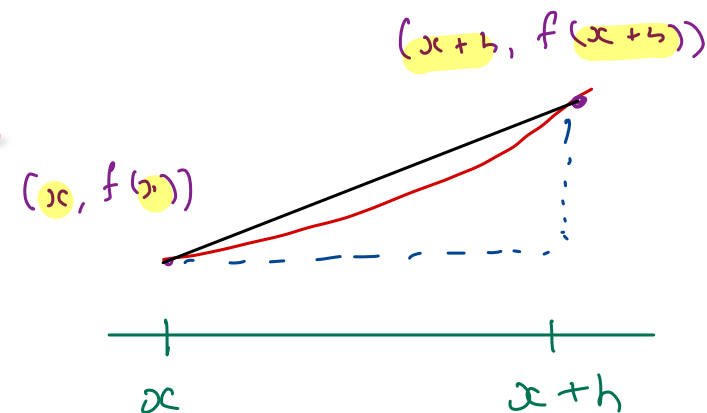
It should make sense, when we move a small distance from 'x' that we can write a coordinate as:

$$(x + h, f(x + h))$$

$$\text{gradient} = \frac{f(x + h) - f(x)}{x + h - x}$$

This will simplify to ...

$$\text{gradient} = \frac{f(x + h) - f(x)}{h}$$



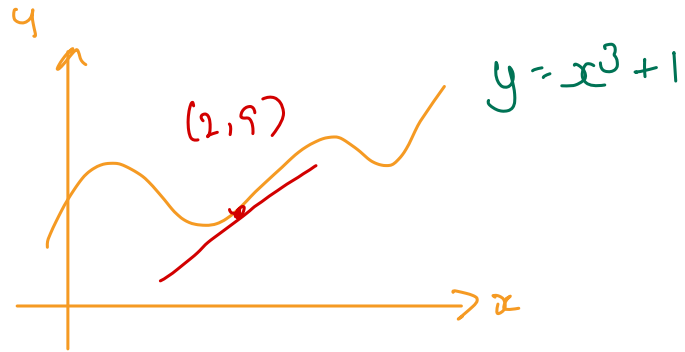
$$y = f(x)$$

$$\frac{f(x+h) - f(x)}{\cancel{x} + h \cancel{+ x}}$$



Example 1

Estimate the instantaneous rate of change of y with respect to x at the point $P(2, 9)$ on the curve $y = x^3 + 1$ by considering the secant PQ , where $Q = (2.01, (2.01)^3 + 1)$.



$$(x, y)$$
$$(2.01, (2.01)^3 + 1)$$
$$(2, 9)$$

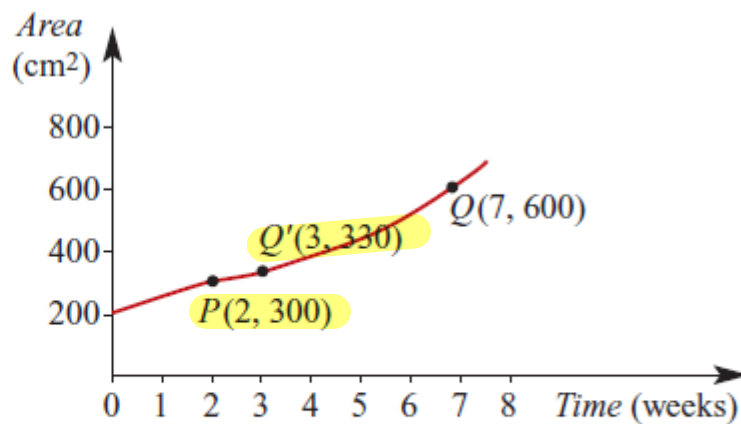
A calculator screenshot showing the calculation of the instantaneous rate of change. The expression $\frac{(2.01)^3 + 1 - 9}{2.01 - 2}$ is entered, and the result 12.0601 is displayed.



Example 2

The graph represents the area covered by a spreading plant. Area is measured in square centimetres and time in weeks.

- a Find the gradient of the secant PQ .
- b The point Q' has coordinates $(3, 330)$. Find the average rate of change of area with respect to time for the interval $[2, 3]$, and hence estimate the instantaneous rate of change of the area of the plant at $t = 2$.



$$\begin{aligned} \text{b. } m_{PQ'} &= \frac{330 - 300}{3 - 2} \\ &= \underline{\underline{30 \text{ cm}^2/\text{week}}} \end{aligned}$$

$$\begin{aligned} \text{a) } m_{PQ} &= \frac{600 - 300}{7 - 2} \\ &= \frac{300}{5} = \underline{\underline{60 \text{ cm}^2/\text{week}}} \end{aligned}$$



Example 3

Consider the curve $y = 2^x$.

- a** Using the secant through the points where $x = 3$ and $x = 3.1$, estimate the instantaneous rate of change of y with respect to x at the point where $x = 3$.
- b** Repeat for the points where $x = 3$ and $x = 3.001$.

$$a) \quad m = \frac{f(3.1) - f(3)}{3.1 - 3}$$

$$b) \quad m = \frac{f(3.001) - f(3)}{3.001 - 3}$$

1.1	*Doc	DEG	✕
Define $f(x) = 2^x$		Done	
$\frac{f(3.1) - f(3)}{3.1 - 3}$		5.7418770029	
$\frac{f(3.001) - f(3)}{3.001 - 3}$		5.5470997007	



Learning Objectives: Revisited

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- ✓ Understand what it means to be an instantaneous rate of change
- ✓ Know how to find the instantaneous rate of change using the tangent line at a point.



Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 16D

Questions: 1-4, 11



Live Streamed Year 12 General Maths on Twitch and YouTube: [Click here for information](#)

Making Maths Easy, Engaging Educational, Entertaining



Navigation: [Home](#)

- Latest uploads
- Years 6 to 10
- VCE Courses
- Exam Solutions
- Buy Merchandise

Why choose MaffsGuru?

I hate talking about myself.
So, here are some of the amazing comments I receive about the videos and content I produce followed by reasons to use the resource:

“I wish I watched your videos before naplan”
— Overjoyed Cherry (youtube)



VCAA exam questions

VCE lessons, where possible, include the use of past VCAA exam questions to



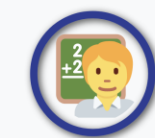
Professional Development

This resource isn't just meant for students. I hope it will be useful for teachers both new



Downloadable notes

Every lesson has downloadable notes. Whatever I write on the screen, you can download for



Respected Presenter

I currently present for Cambridge University Press and Nelson - as well as produce my own content for