Average rates of change

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be an average rate of change
- Understand what it means to be an average speed.
- Know how to find the average rate of change for a function

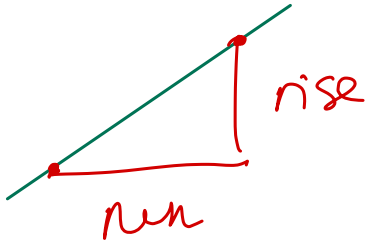
RECAP

tangent

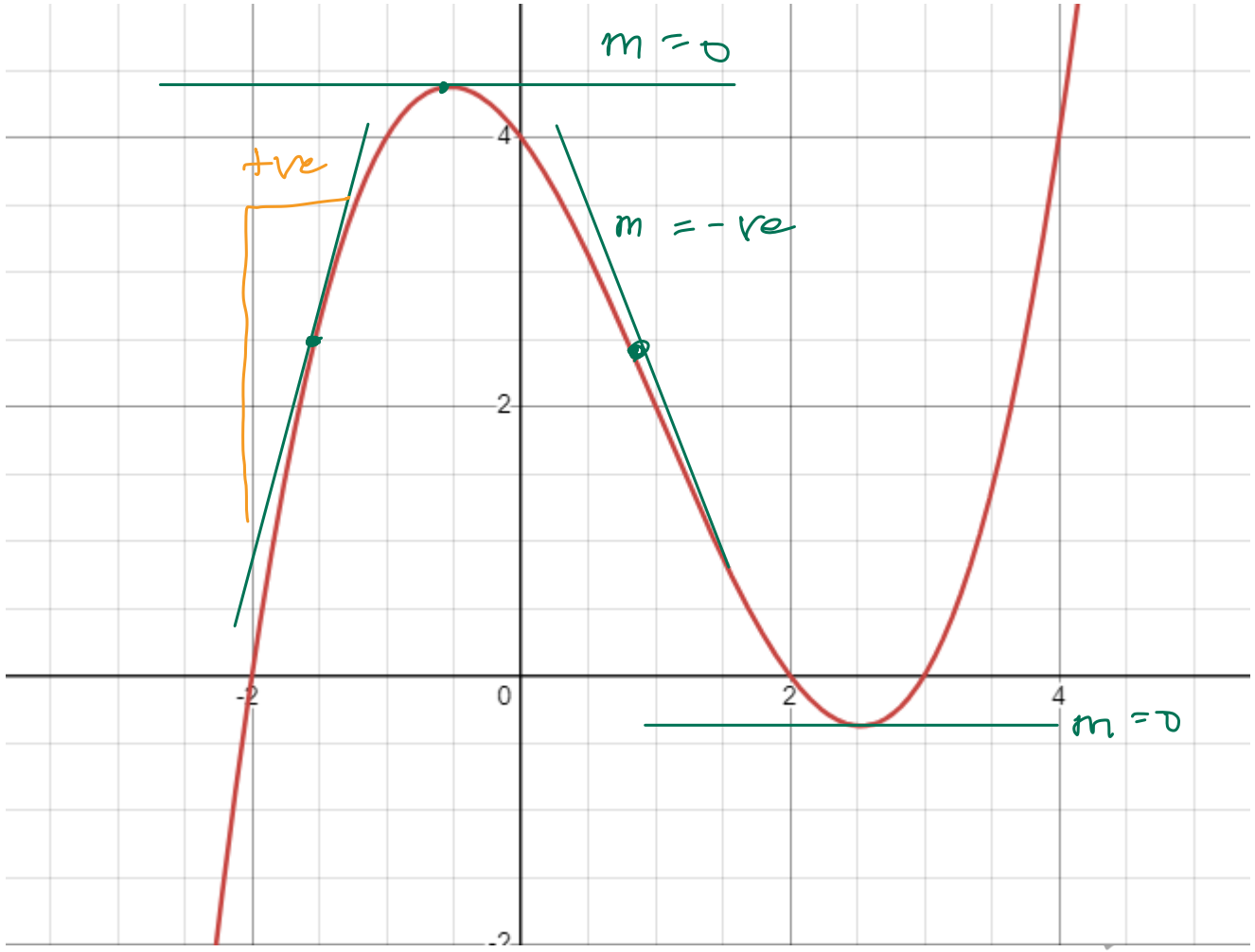
In the previous lessons we have looked at the idea of relationships and rates of change (including constant rates of change).

Everything we seem to be working towards relates to finding gradients. Not just of straight lines but now of curves.

The basics hold true! To find the gradient between two points you join them with a straight line and find the gradient of the line. This is going to be an important distinction between finding the average gradient and an instantaneous gradient.



$\frac{1}{3}(x-2)(x+2)(x-3)$



Average speed

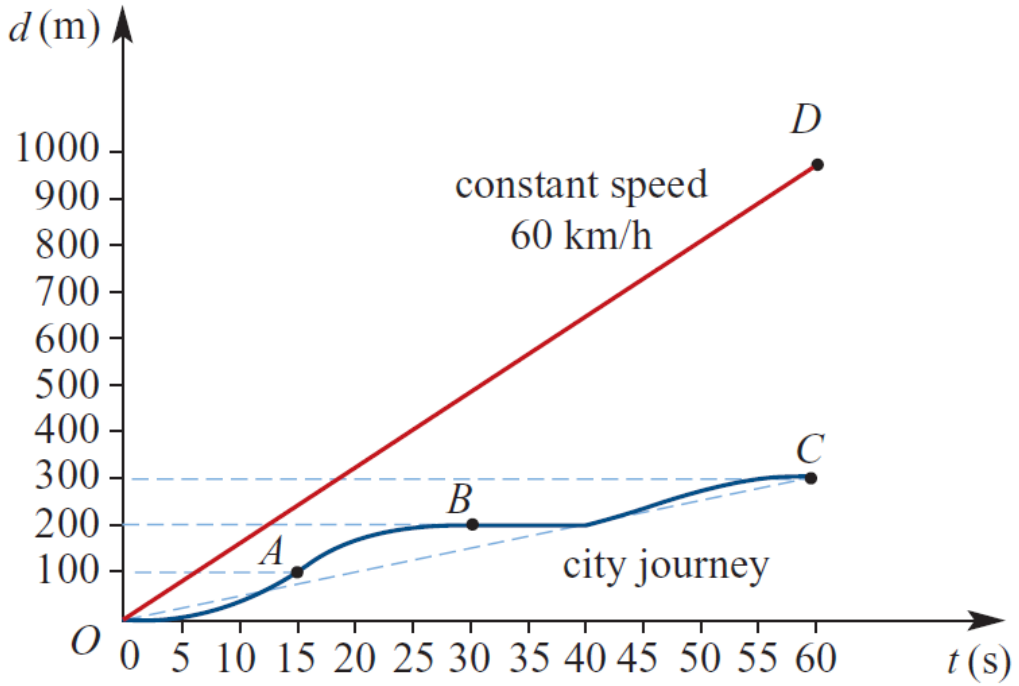
This is a formula we have used a number of times in both Mathematics and Science:

$$average\ speed = \frac{total\ distance\ travelled}{total\ time\ taken}$$

If we know a distance travelled and a time taken, we can find the average speed.

This does not need to just be for the whole journey. We could find the average speed for an interval of time over that journey.

We can find the average speed, for example, between A and B.

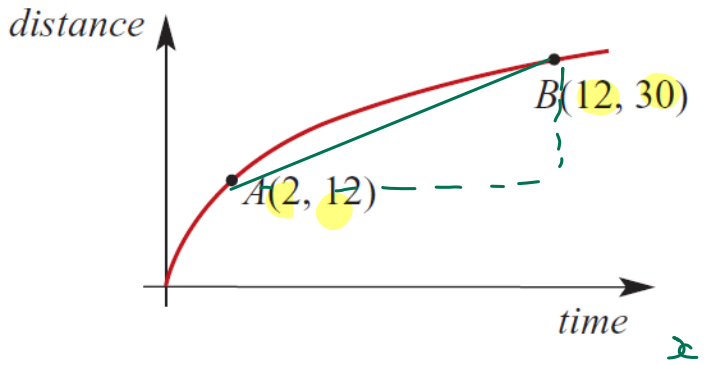


Question 1

The graph of distance travelled (metres) against time (seconds) for the motion of an object is shown.

Find the average speed of the object in m/s over the interval from $t = 2$ to $t = 12$.

x_1 x_2



Co-ord

(12, 30)

(2, 12)

$A.S = \frac{TDT}{TTT}$

$A.S = \frac{30-12}{12-2} = \frac{18}{10}$

$= 1.8 \text{ m/s}$

$m = \frac{rise}{run} \Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$

Average rate of change for a function

A line which passes through two points on a curve is called a **secant**.

A line segment joining two points on a curve is called **chord**.

Notation, notation, notation:

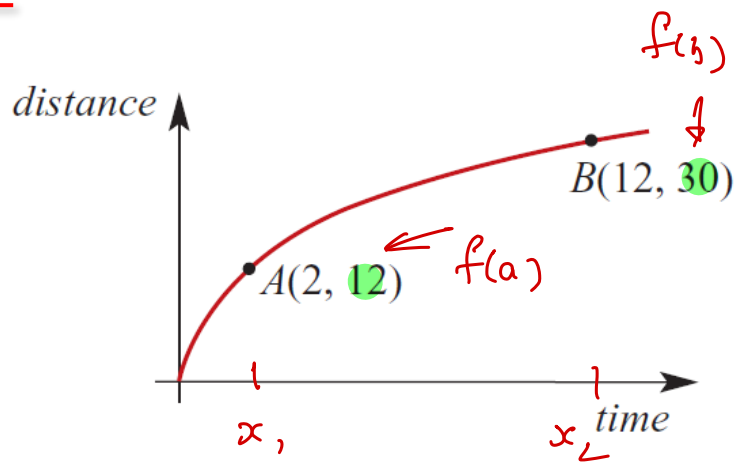
$$average\ rate\ of\ change = \frac{f(b) - f(a)}{b - a}$$

This looks confusing but it's really another way of writing **rise** over **run**!

Average rate = grad

$= \frac{rise}{run}$

$= \frac{y_2 - y_1}{x_2 - x_1}$



Question 2

Find the average rate of change of the function with rule $f(x) = x^2 - 2x + 5$ as x changes from 1 to 5.

$$\text{arc} = \frac{f(b) - f(a)}{b - a}$$

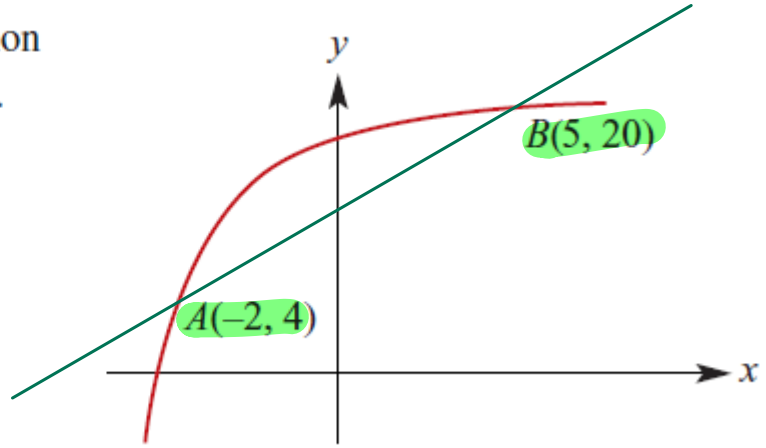
$$= \frac{f(5) - f(1)}{5 - 1}$$

$$= \frac{20 - 4}{5 - 1}$$

$$= \frac{16}{4} = 4$$

Question 3

Find the average rate of change of the function depicted in the graph for the interval $[-2, 5]$.



chord...

secant

$$m = \frac{20 - 4}{5 - (-2)}$$

$$= \frac{16}{7}$$

Question 4

The air temperature, T°C, at a weather station on a particular evening is modelled by the equation $T = \frac{600}{t^2 + 2t + 30}$, where t is the time in hours after 6 p.m.

a Find the temperature at 6 p.m.

b Find the temperature at midnight.

c Find the average rate of change of the air temperature from 6 p.m. until midnight.

a. 6pm $t = 0$ $T = 20$

b. $t = 6$ $T = 7.6923$

c. $\text{arc} = \frac{7.6923 - 20}{6 - 0} = -2.05$ °C/hrs

Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what a rate of change means
- Understand what a constant rate of change means

Learning Objectives: Revisited

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be an average rate of change
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Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 16C
Questions: 1-7

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