Rates of change

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Recognise relationships
- Understand what it means to be a rate of change.
- Understand what the following rates of change would look like:
    - Positive
    - Negative
    - Zero

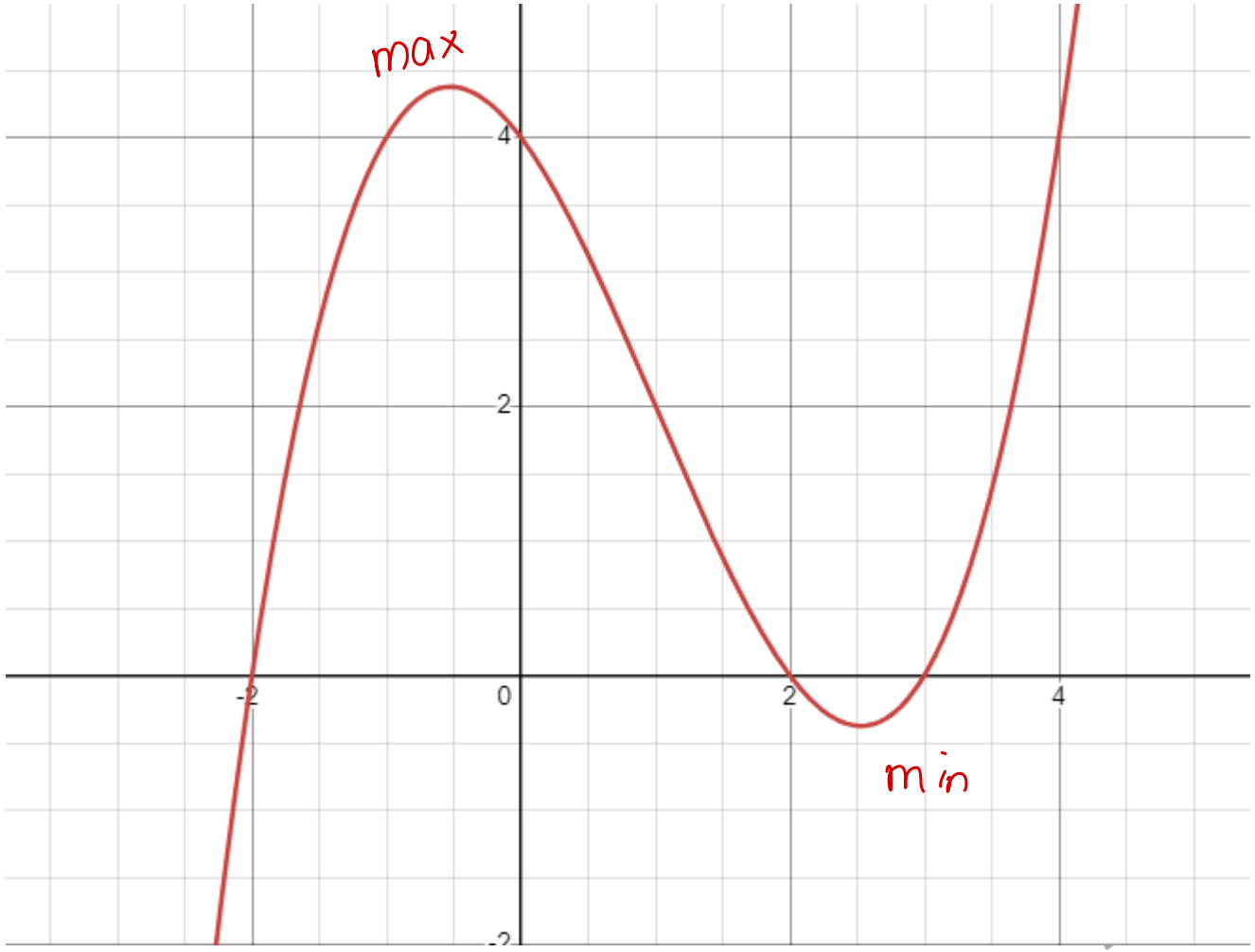
RECAP

This is the first lesson looking at Rates of Change and differentiation and so you might think, having never really heard of these things before, that you had never met them before. But you have! Both in real life AND in the Mathematical Methods course.

When we look at the relationship shown on the right we see that it's a cubic.

We have a number of tools to enable us to sketch this reasonably accurately. The only thing we keep tripping over is how to find the maximum or minimum points of the curve.

$$\frac{1}{3}(x-2)(x+2)(x-3)$$



A game show with an interesting difference

When I was young there was an interesting “game show” which was really a lots of psychological tests trying to find out if there were differences in the way men and women perceive the world and approach problems of team building and problem solving.

One of the question was simply, “Mark a line on the bottle shown to show there the liquid would be if the bottle was half full”.

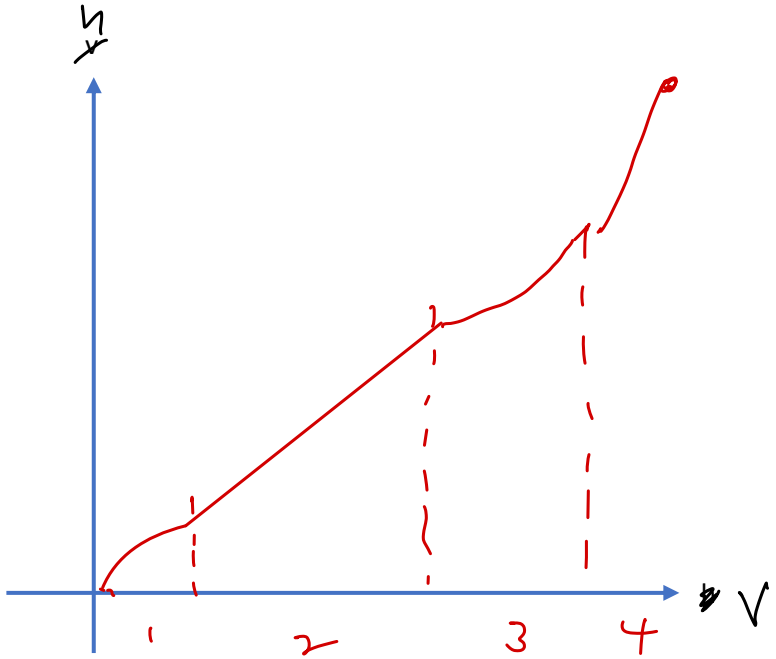
From what I remember, the men were terrible at this! The women were great. They were much more visual thinkers.

Something a little more interesting?

I think a better question would have been, "Draw a graph which would show the relationship between the height of the water and the volume of water in the bottle".

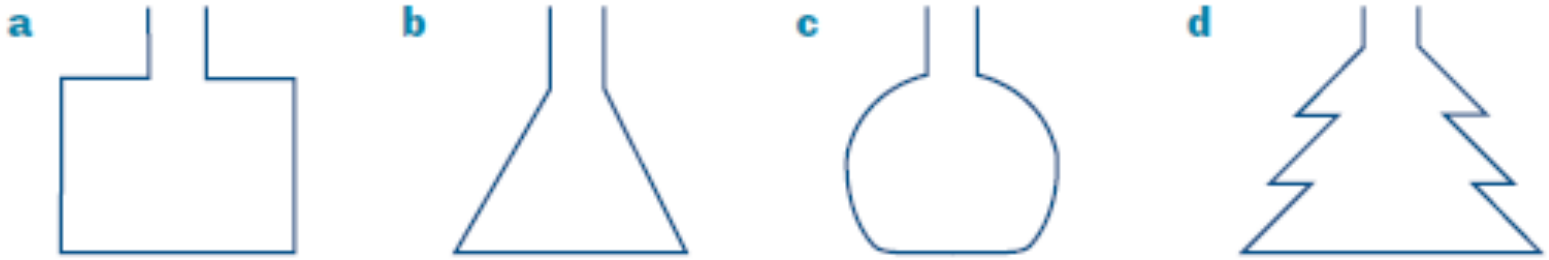
Would we have to take anything into account?

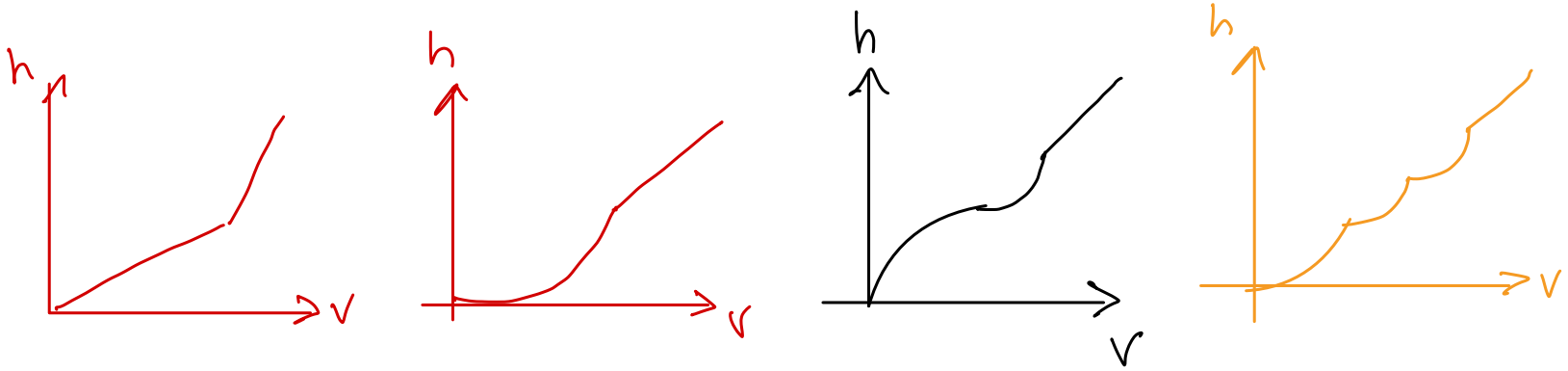
Remember: this is a sketch!



Example 1: Different shaped bottles?

What about the following shaped bottles? What would they look like?





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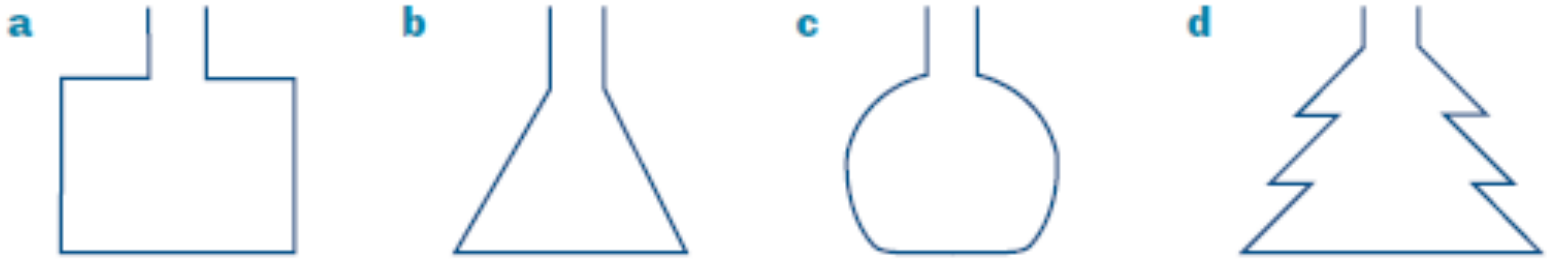
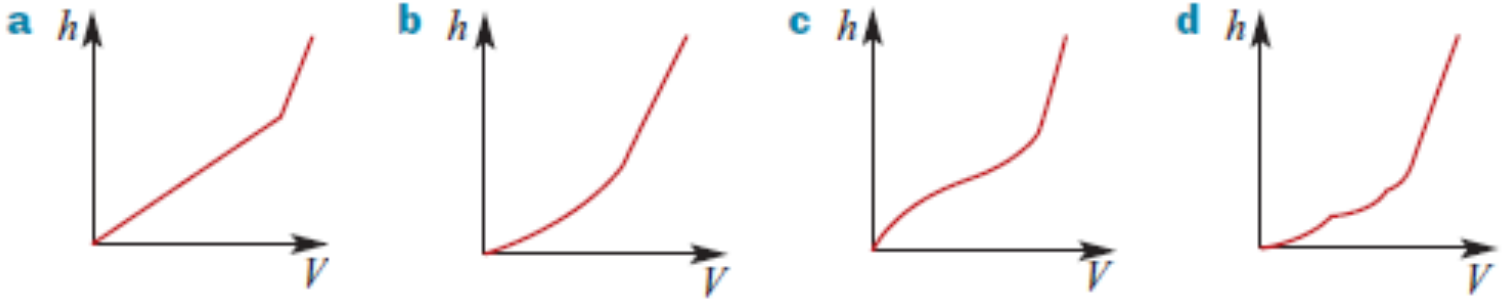




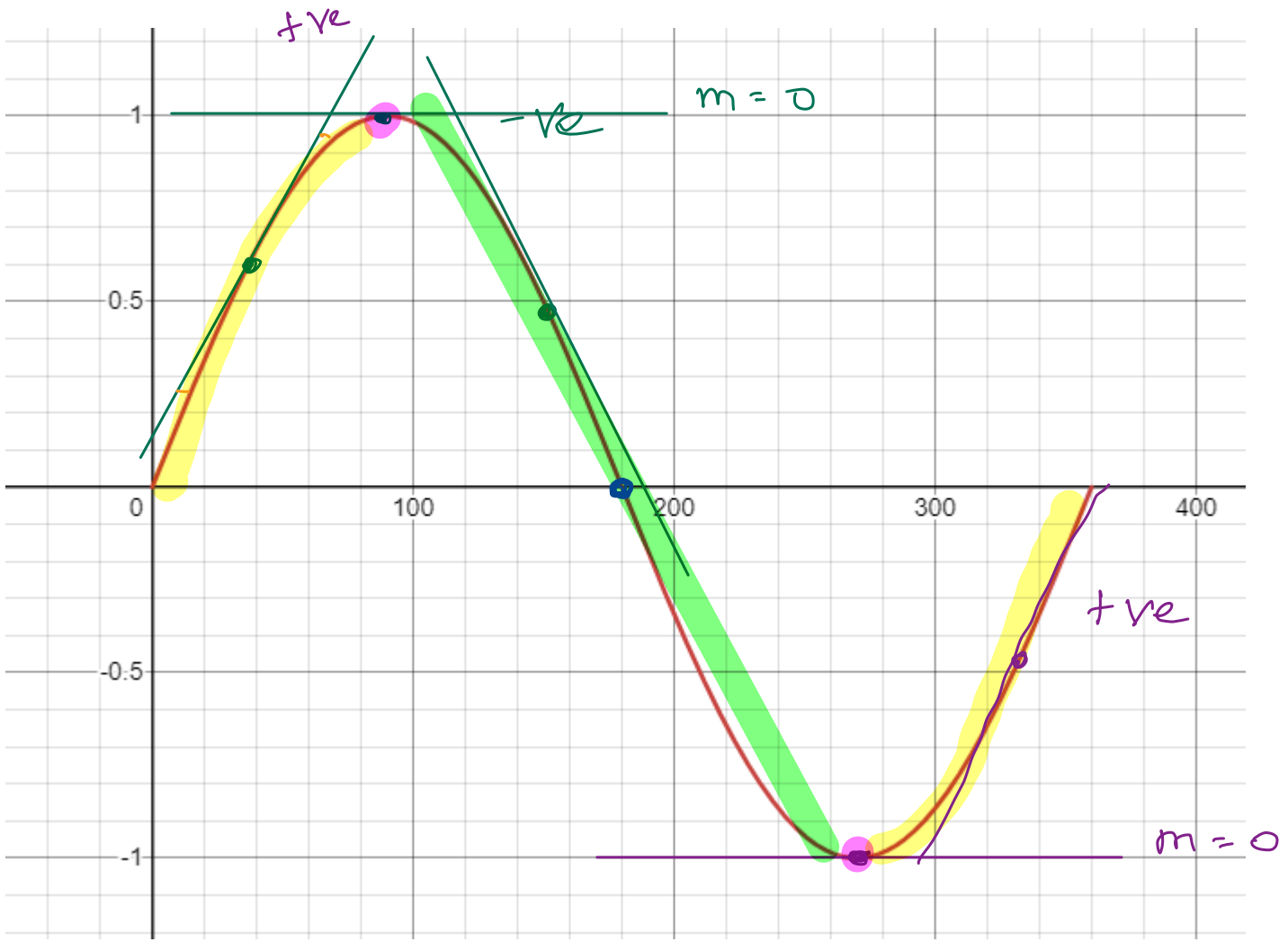
Looking at the rate of change is really important in Mathematics

For example, if we look at the Sine Curve, there are some very interesting things to note.

+ve

−ve

0



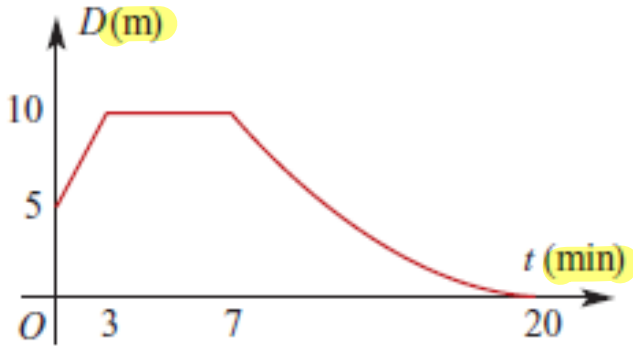
Example 2: Understanding Rates of Change

Rates of change are in every application of Mathematics. Some of the more common ones relate to changes in distance with respect to time and speed with respect to time.

A particle travels in a straight line.

The graph shows the distance, D metres, of the particle from a fixed point O over a period of 20 minutes.

Describe the motion of the particle.



$S/V = \frac{5}{3}$

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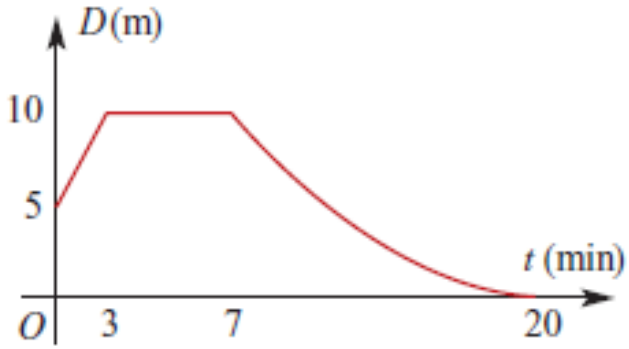
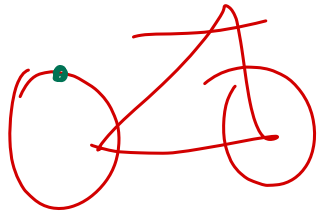


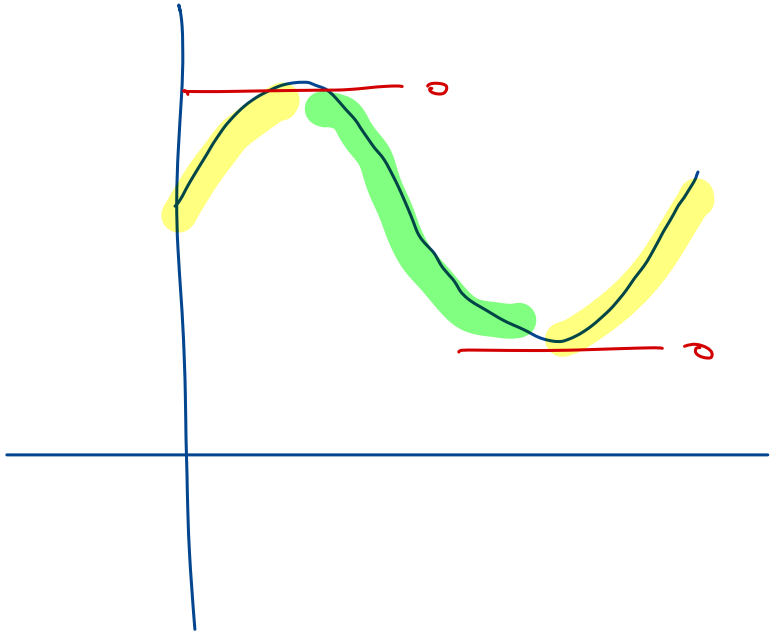
The particle is initially 5 m from O. It travels away from O for 3 minutes at a constant speed of $\frac{5}{3}$ m/min. It then remains stationary at a distance of 10 m from O for 4 minutes, before returning to O at a speed which is gradually decreasing so that it comes to rest at O at time $t = 20$ minutes.

Increasing, decreasing and zero rates of change

It is important to note the following:

- When y is increasing as x increases then we say that the rate of change of y with respect to x is **positive**.
- When y is decreasing when x increases, we say that the rate of change of y with respect to x is **negative**.
- If y stays the same as x increases, we say that the rate of change of y with respect to x is **zero**.



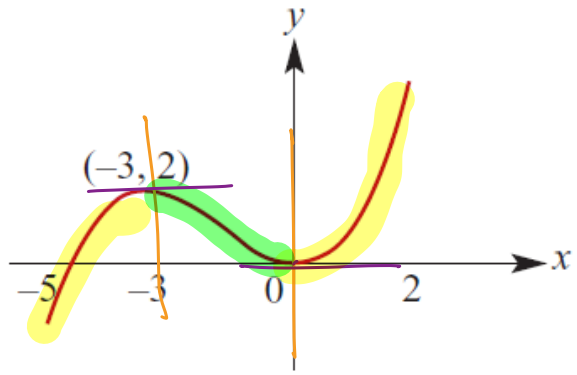


Example 3

For the graph shown on the right for $x \in [-5, 2]$, use interval notation to describe the set of values of x for which:

a the rate of change of y with respect to x is negative

b the rate of change of y with respect to x is positive.



a. $x \in (-3, 0)$

b. $x \in (-5, -3) \cup (0, 2)$

Learning Objectives: Reviewed

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Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 16A
Questions: 1-6, 9, 10, 11

Additional Work:
Worksheet MM141

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