

# Further symmetry properties and the Pythagorean Identity

Year 11
Mathematical Methods

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## Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Understand how we can use the Pythagorean Identity to find angles



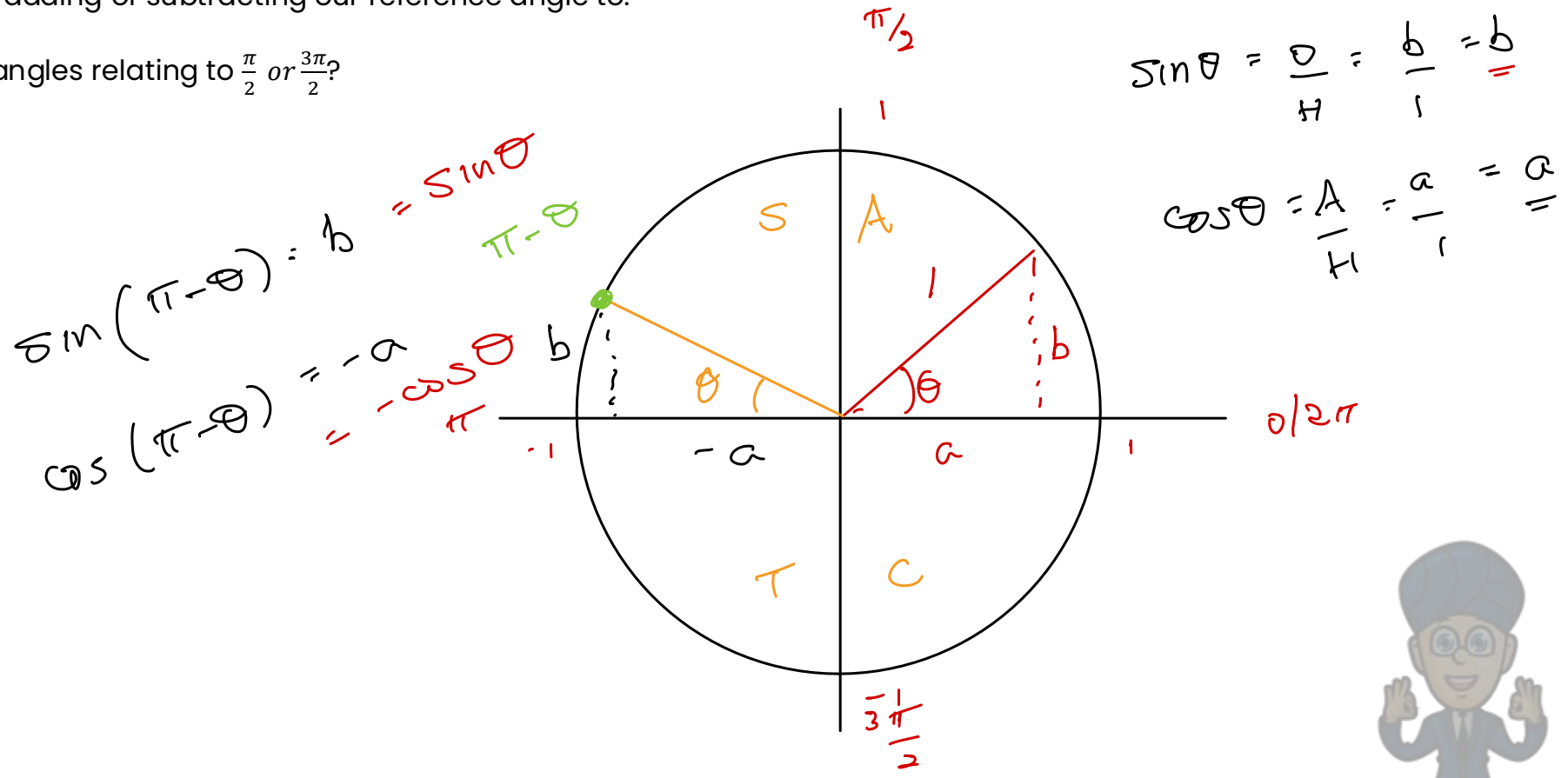
## Recap of past learning

In the past lessons we have been looking at the ideas of Circular functions.

A long time ago, we looked at the idea of Symmetry properties of circular functions which related to using $0 \text{ or } \pi$ as the angle we are adding or subtracting our reference angle to.

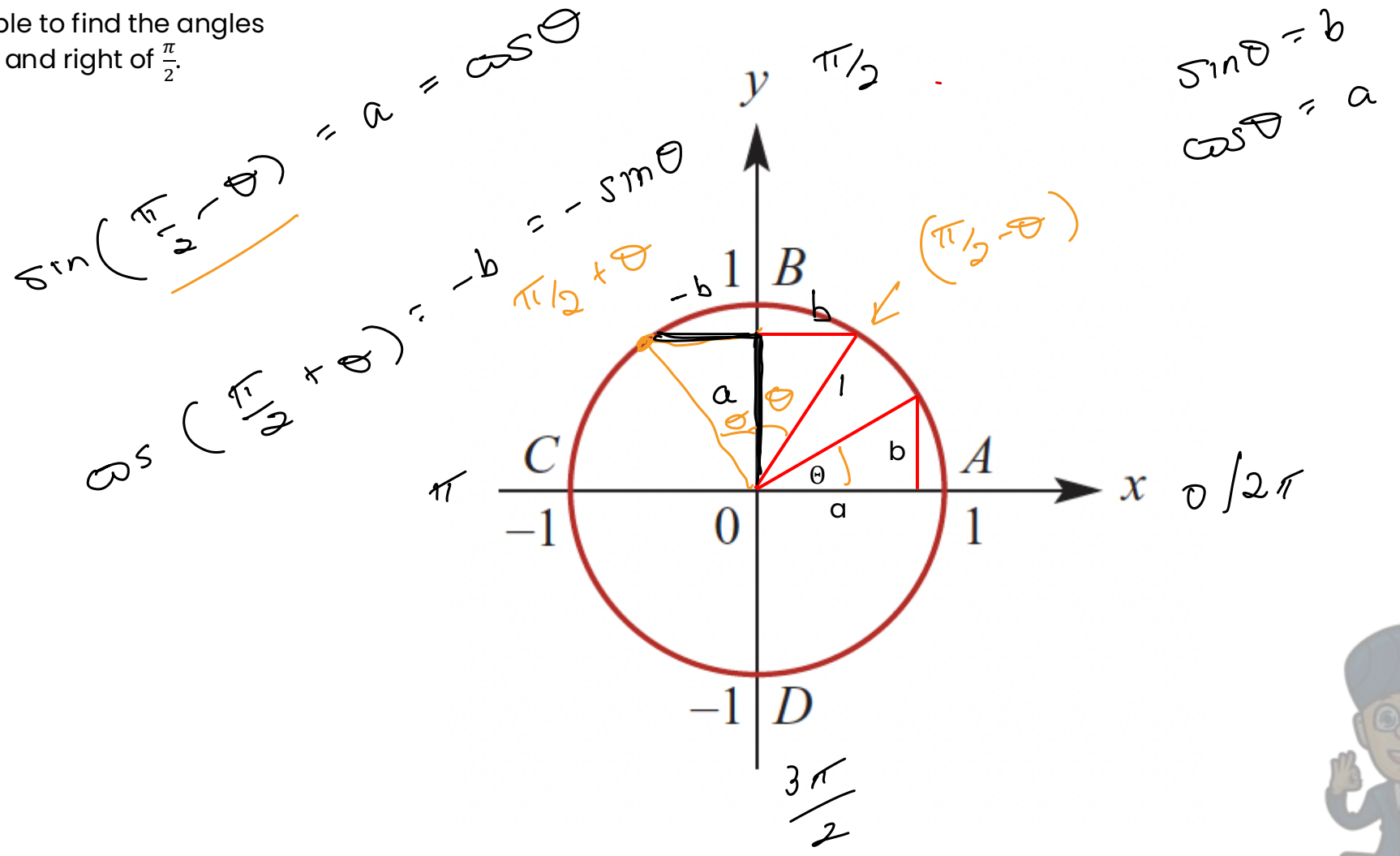
But what if we want to look at the angles relating to $\frac{\pi}{2}$ $or$ $\frac{3\pi}{2}$?

Well, we can do that too!



## Further symmetry properties

Let's look at how we might be able to find the angles which relate to being on the left and right of $\frac{\pi}{2}$.





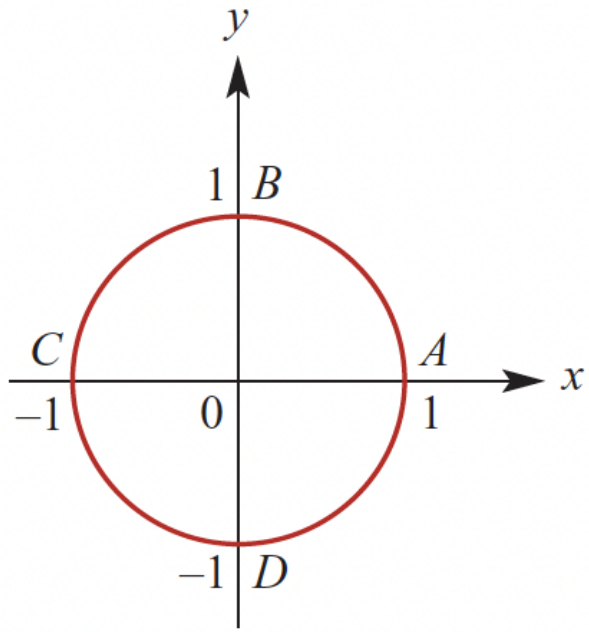

## Examples

If $\sin\theta = 0.3$ and $\cos\alpha = 0.8$, find the values of:

**a** $\sin\left(\frac{\pi}{2} - \alpha\right)$ **b** $\cos\left(\frac{\pi}{2} + \theta\right)$ **c** $\sin(-\theta)$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

$= 0.8$





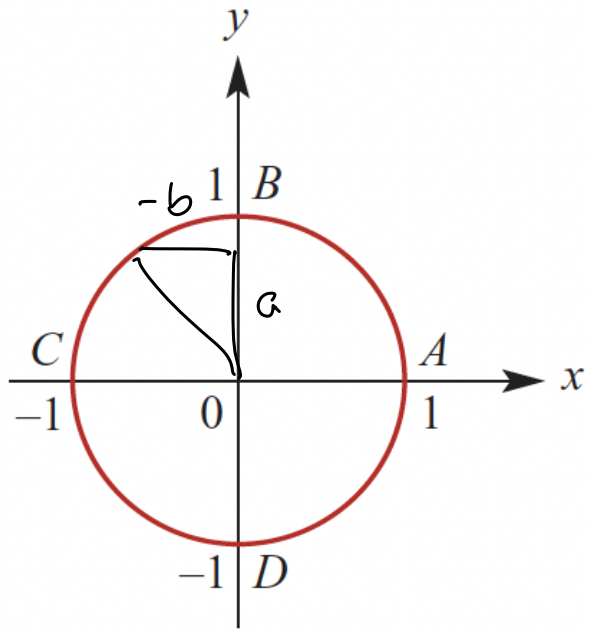

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$\cos\left(\frac{\pi}{2} + \theta\right) = -b = -\sin\theta$

$= -0.3$





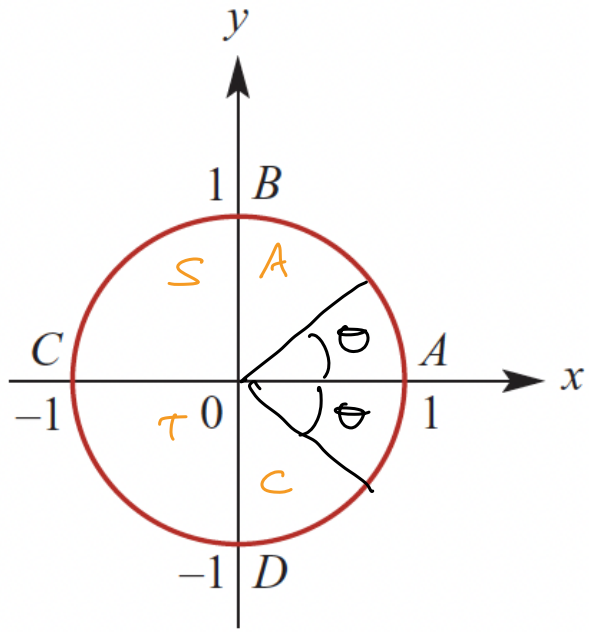

## Examples

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$\sin(-\theta) = -\sin\theta$

$= -0.3$






# Pythagorean Identity

We have met this in a lesson before, but it's interesting to note that this can be really important!

We know that Pythagoras (or, more likely, one of his followers) came up with the following for right angled triangles:

$$c^2 = a^2 + b^2$$

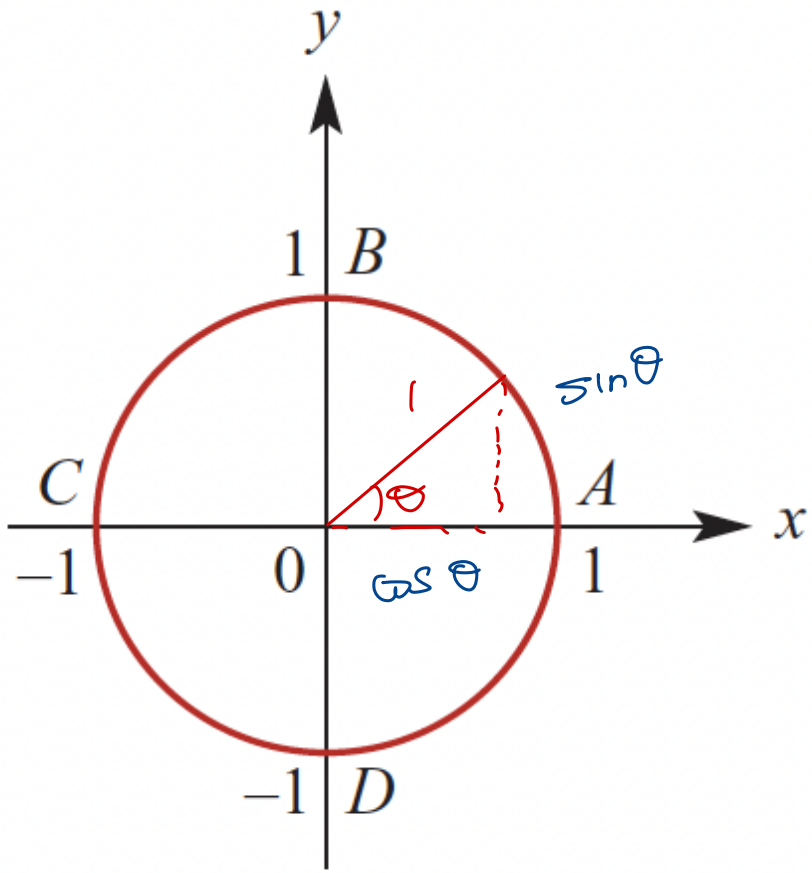
Well, the unit circle is full of right-angled triangles with hypotenuse length of 1.

$c^2 = a^2 + b^2$

$1^2 = (\sin\theta)^2 + (\cos\theta)^2$

$1 = \sin^2\theta + \cos^2\theta$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

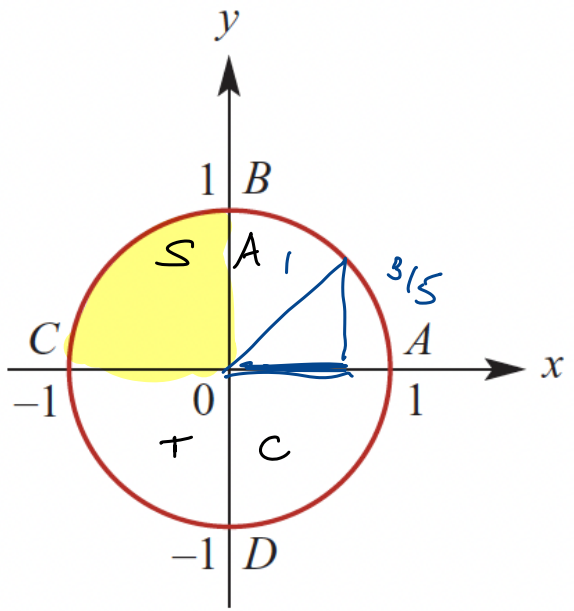





## Examples

Given that $\sin x = \frac{3}{5}$ and $\frac{\pi}{2} < x < \pi$, find:

**a** $\cos x$ **b** $\tan x$




$\sin^2 x + \cos^2 x = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2 x = 1$

$\frac{9}{25} + \cos^2 x = 1$

$\cos^2 x = \frac{16}{25}$

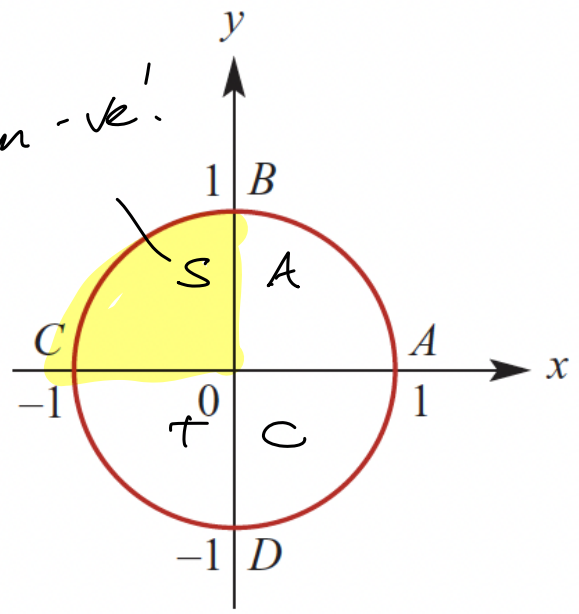
$\therefore \cos x = \pm\frac{4}{5}$ or $\cos x = -\frac{4}{5}$



## Examples

Given that $\sin x = \frac{3}{5}$ and $\frac{\pi}{2} < x < \pi$, find:

**a** $\cos x$ **b** $\tan x$




$\tan x = \frac{\sin x}{\cos x}$

$= \frac{\frac{3}{5}}{-\frac{4}{5}}$

$= \frac{3}{5} \times \frac{5}{-4} = \frac{3}{-4}$ or $\tan x = -\frac{3}{4}$



## Learning Objectives: Reviewed

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- Understand how we can use the Pythagorean Identity to find angles



## Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

**Ex 14J**
Questions: 3, 4 and 5

**Additional Sheets:**
MM213
MM214






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