

Graphs of Sine and Cosine with horizontal translations

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Know how to read and translate graphs of sine and cosine parallel to the x-axis.
- Remember that DRT still applies to transformations of sine and cosine curves.



Recap of past learning

In the previous lesson we looked at how we can dilate and reflect the graphs of sine and cosine. We learned that we can (generally) read an equation and find, pretty quickly, the dilation factor from both the x-axis and the y-axis.

But what if we want to translate a function parallel to the x-axis.

There will be lots of examples used both now and in Year 12 where a function like the one shown below will be used.

$$y = 3\sin 2\left(t - \frac{\pi}{4}\right), \qquad \frac{\pi}{4} \leq t \leq \frac{5\pi}{4}$$

There is a trick which will be employed to try and confuse you.

The restricted domain is really important. You are expected to draw only the part(s) of the function which fall between the intervals.



Amplitude and Period: Recap

Remember:

$$Period = \frac{2\pi}{n}$$

The whole reason we are doing this is to sketch these functions. To sketch you need to know:

- Maximums
- Minimums
- Crossing Points (roots)
- Intercepts (y-axis)

This can all be found if you know what you're looking for

$$y = 3\sin 2\left(t - \frac{\pi}{4}\right), \qquad \frac{\pi}{4} \leq t \leq \frac{5\pi}{4}$$

Amplitude: Distance from mean line to the highest point.

Period: The number of times a function will appear in the space of 2π



Translations of sine and cosine functions

Much like the translations for quadratics (and other polynomials) we look to what is inside the brackets of the sine and cosine functions and remember "the reverse of the inside is true".

Remember:

$$Period = \frac{2\pi}{n}$$

This is a horizonal translation of $\frac{\pi}{4}$ units in the positive direction of the x-axis.

$$y = 3\sin 2\left(t - \frac{\pi}{4}\right), \qquad \frac{\pi}{4} \leq t \leq \frac{5\pi}{4}$$

Amplitude: Distance from mean line to the highest point.

Period: The number of times a function will appear in the space of 2π



Tricks, tricks and more tricks

The way I teach this is that the coefficient of the 't' must always be 1. If there is any other coefficient then you need to move it outside of a set of brackets.

The chances are; to trick you, the previous equation would have been given to you as:

$$y = 3\sin\left(2t - \frac{\pi}{2}\right), \qquad \frac{\pi}{4} \leq t \leq \frac{5\pi}{4}$$

Written this way you get the wrong horizontal translation

Remember:

$$Period = \frac{2\pi}{n}$$

This is a horizonal translation of $\frac{\pi}{4}$ units in the positive direction of the x-axis.

$$y = 3\sin 2\left(t - \frac{\pi}{4}\right), \qquad \frac{\pi}{4} \leq t \leq \frac{5\pi}{4}$$

Amplitude: Distance from mean line to the highest point.

Period: The number of times a function will appear in the space of 2π



Example

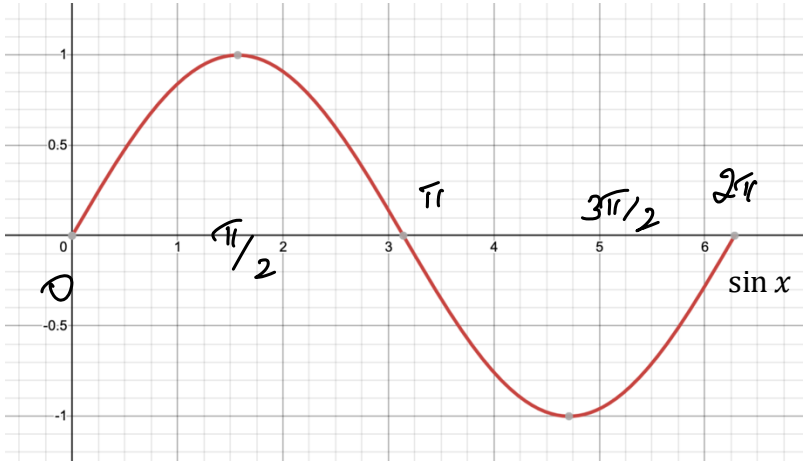
On separate axes, draw the graphs of the following functions. Use a calculator to help establish the shape. Set the window appropriately by noting the range and period.

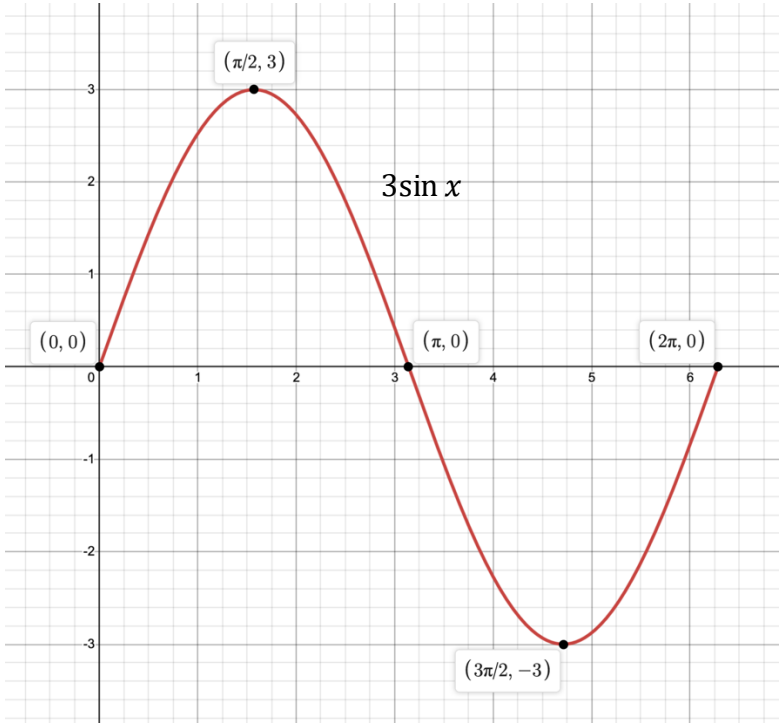
a $y = 3\sin 2\left(t - \frac{\pi}{4}\right), \quad \frac{\pi}{4} \leq t \leq \frac{5\pi}{4}$

b $y = 2\cos 3\left(t + \frac{\pi}{3}\right), \quad -\frac{\pi}{3} \leq t \leq \frac{\pi}{3}$

Hint: When doing these questions always draw a basic graph and then adapt accordingly.

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T







Example

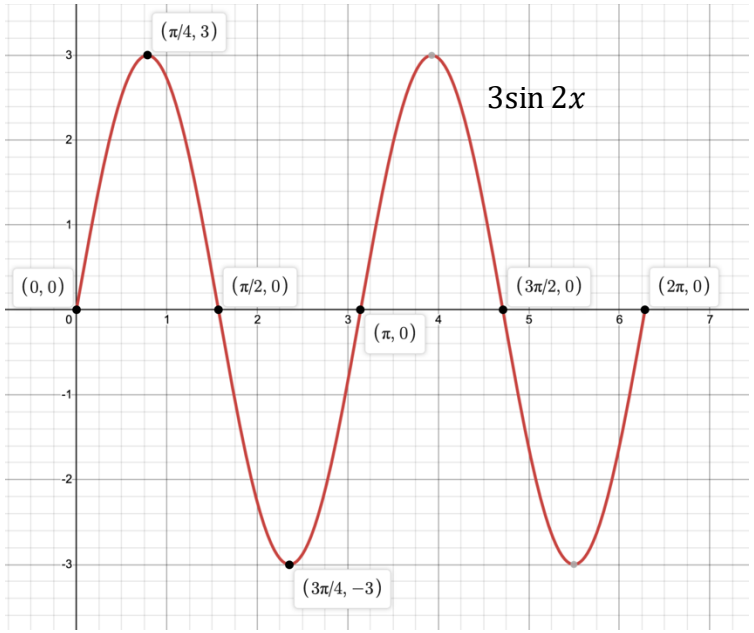
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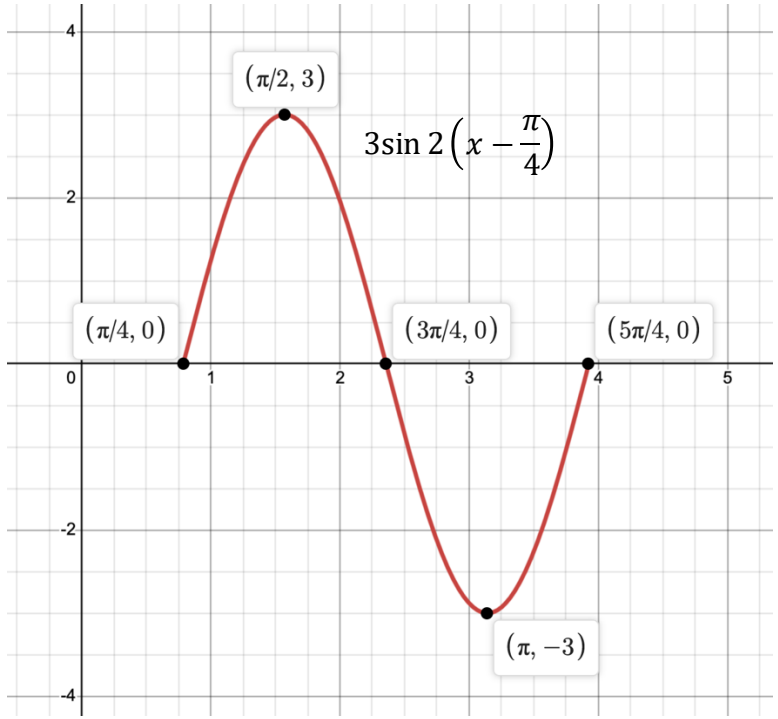
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Note: When drawing make sure you look for the domain and the inequalities used. Make sure you draw the correct endpoints as open or closed circles.

$$T = \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$$





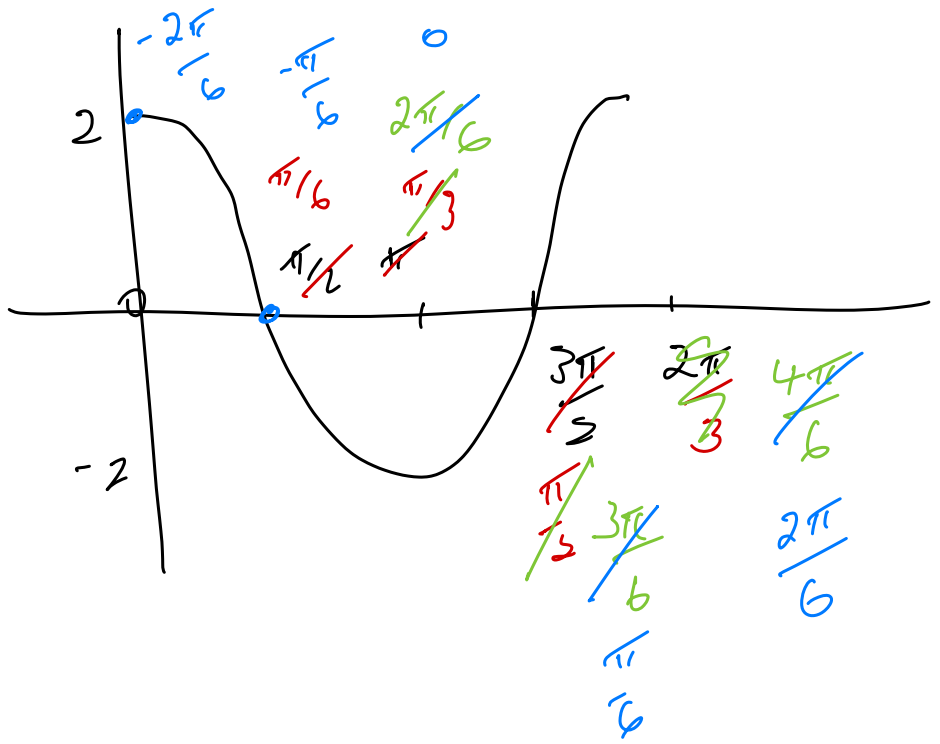


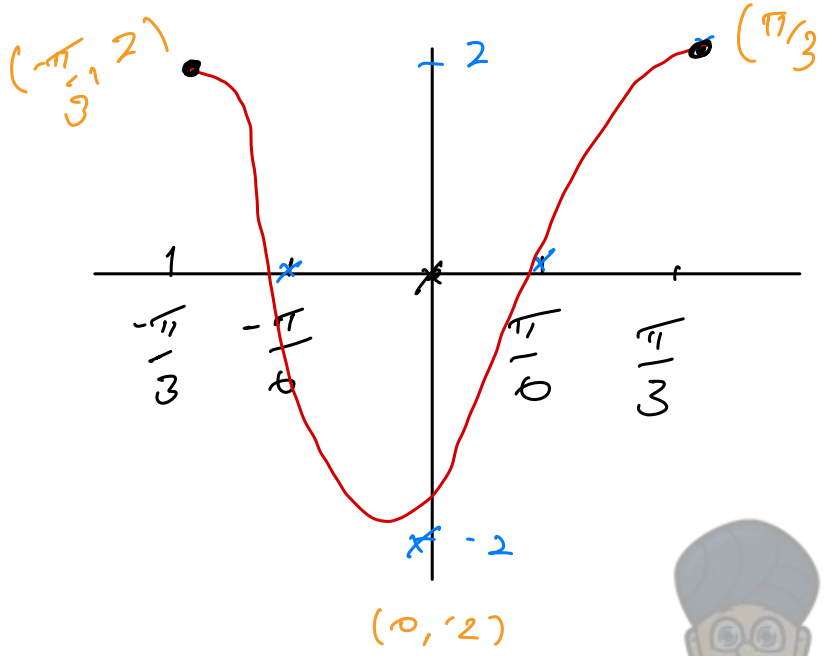
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Hint: When doing these questions always draw a basic graph and then adapt accordingly.



Learning Objectives: Reviewed

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Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Questions to complete:
Ex 14F q1-7
Ex 14H q1,2,3,5
Ex 14I q1-3 LHS

Additional Sheets:
MM216





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