Graphs of Sine and Cosine

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Understand how to sketch standard graphs of sine and cosine
- Know what it means by amplitude and period
- Know how to dilate sine and cosine graphs from the x-axis
- Know how to translate graphs both in the positive and negative direction of the x-axis.
- Know key values for sine and cosine graphs

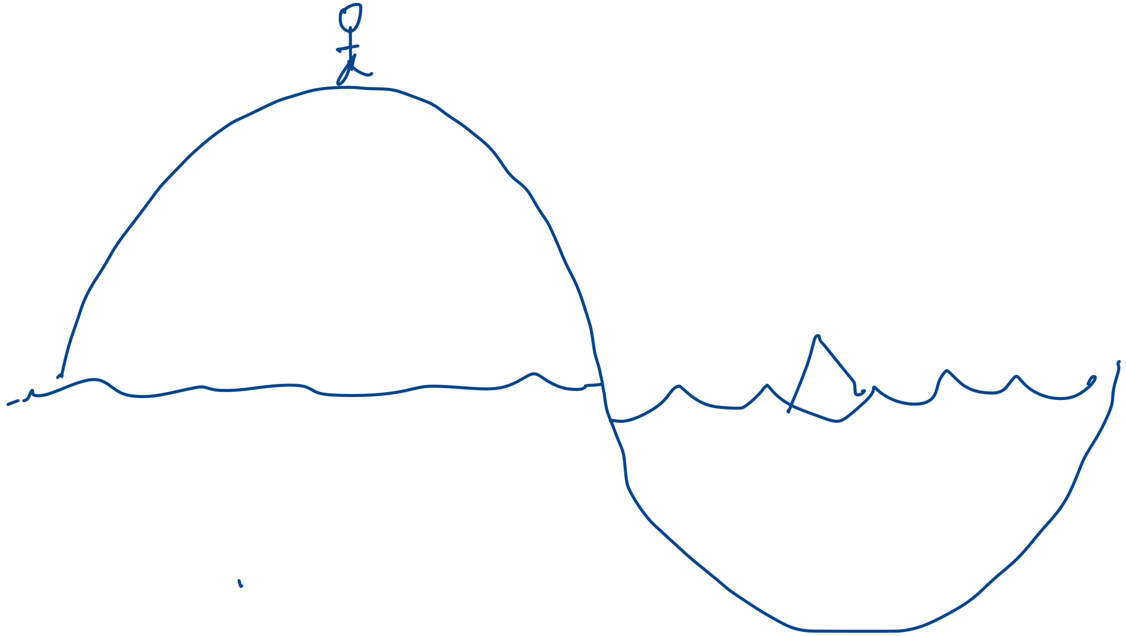
Recap of past learning

In the previous lessons we have looked at degrees and radians. We have looked at defining circular functions relating to sine and cosine. We then looked at tan.

We have spent a lot of time looking at how to use the Unit Circle. We know how to find all manner of angles if we know the reference angle. We know how to use the "two triangles" to help us find the most common angles.

These are all tools to help us with what this chapter is really all about – sketching graphs AND finding solutions to various trigonometric equations.

It's important to know how to draw/sketch the graphs of sine, cosine and tangent and use them to help us solve a vast array of questions.



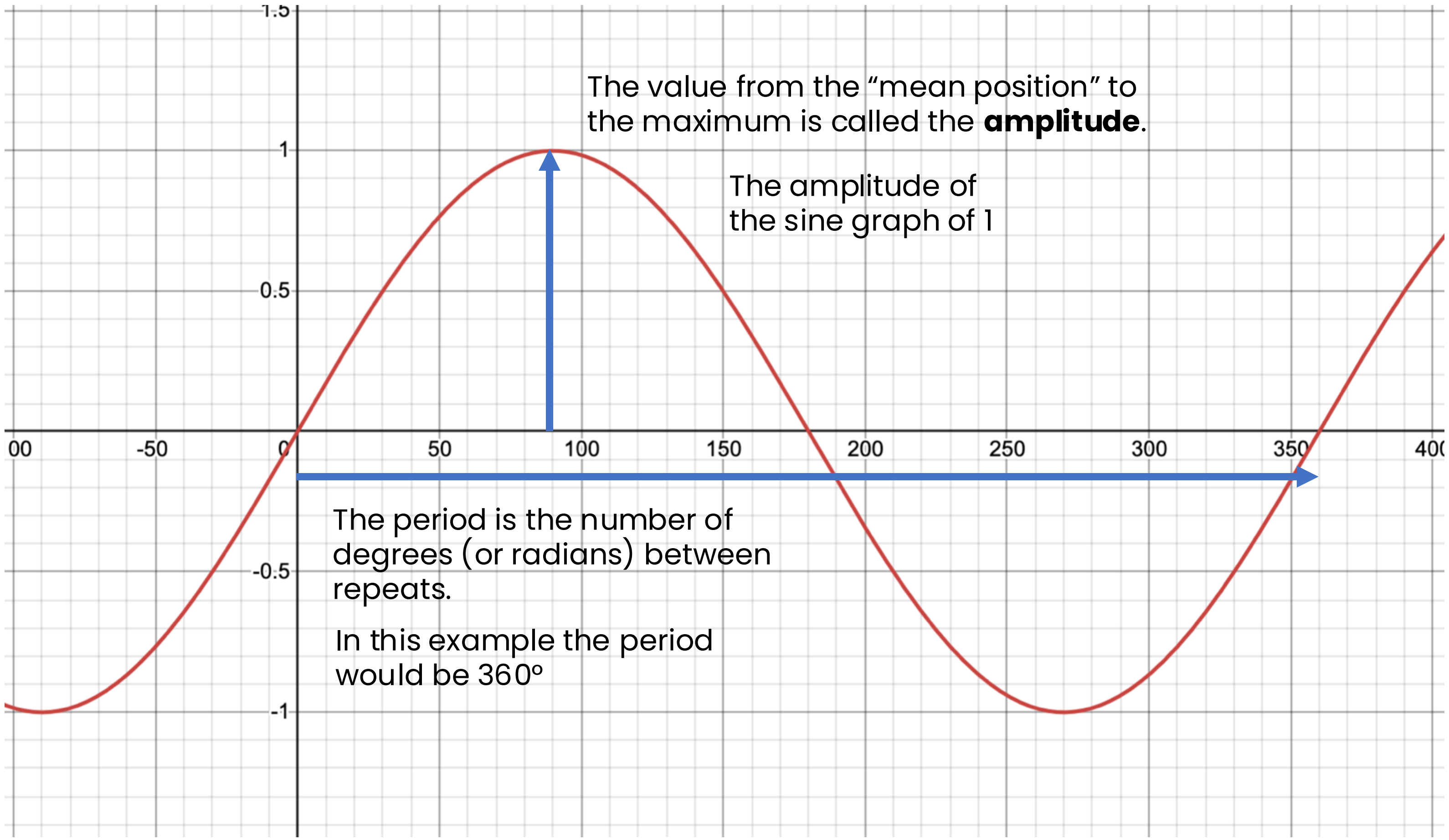
Graph of the sine function

$y = \sin(x)$

The first thing to note about this graph from Desmos.com is that the units are in degrees. But this will be fine for the moment.

The most important things to note are:

- Period
- Amplitude



The value from the "mean position" to the maximum is called the **amplitude**.

The amplitude of the sine graph of 1

The period is the number of degrees (or radians) between repeats.

In this example the period would be 360°

Graph of the sine function

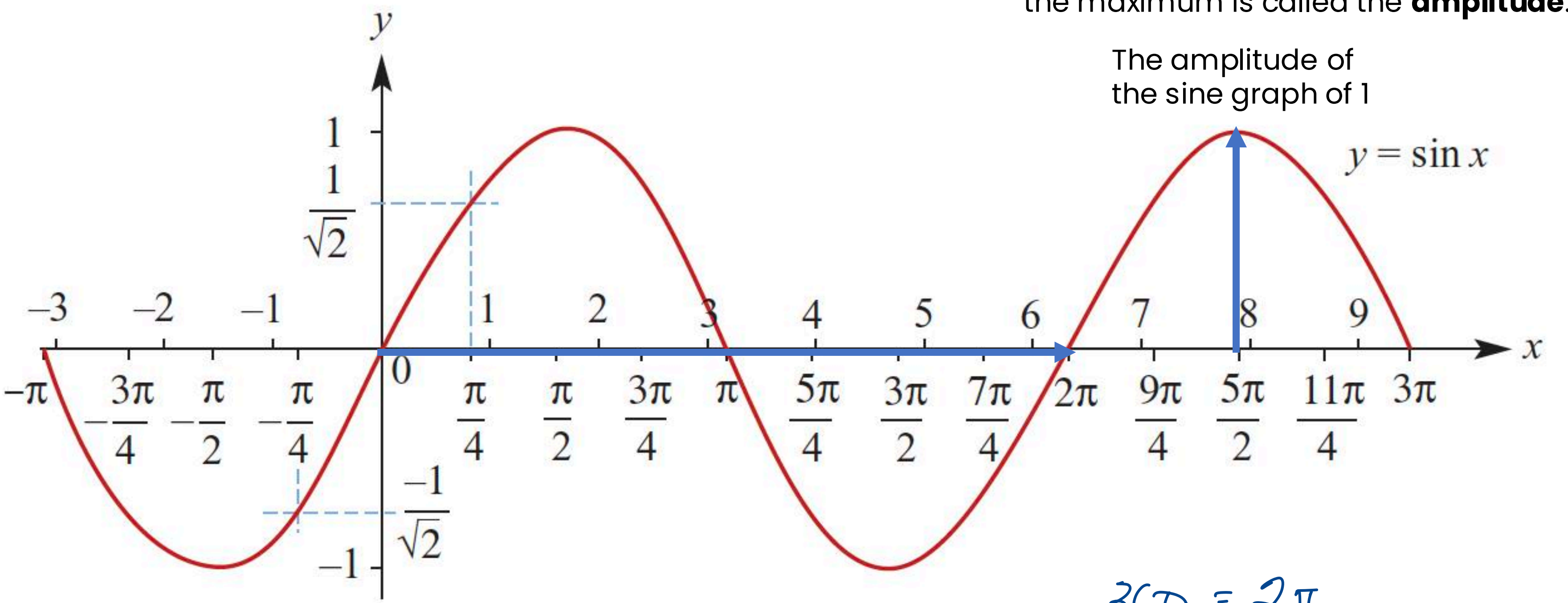
It is more usual to look at the scale in terms of radians.

Hint: Learn these like it's your alphabet. It's going to be massively important.

Note: The denominators of the fractions are all different. I tend to make them all the same until the final calculation or sketch.

The value from the "mean position" to the maximum is called the **amplitude**.

The amplitude of the sine graph of 1



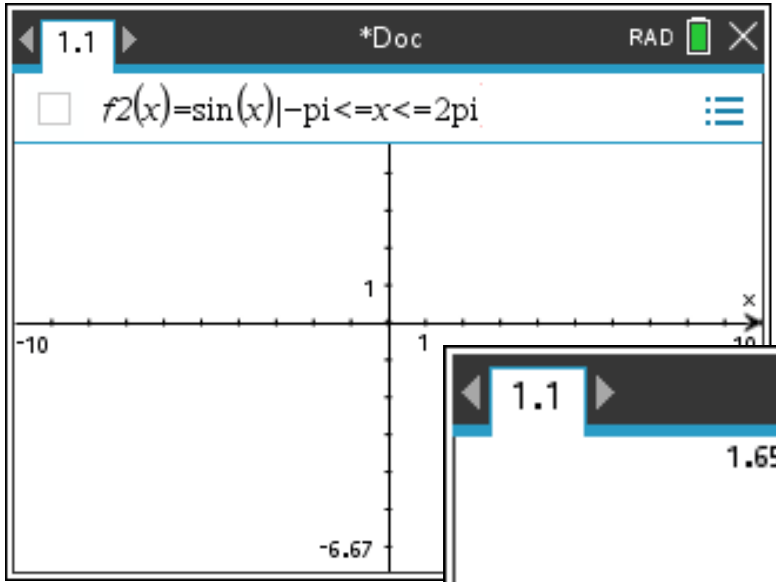
$360 = 2\pi$

The period is the number of degrees (or radians) between repeats.

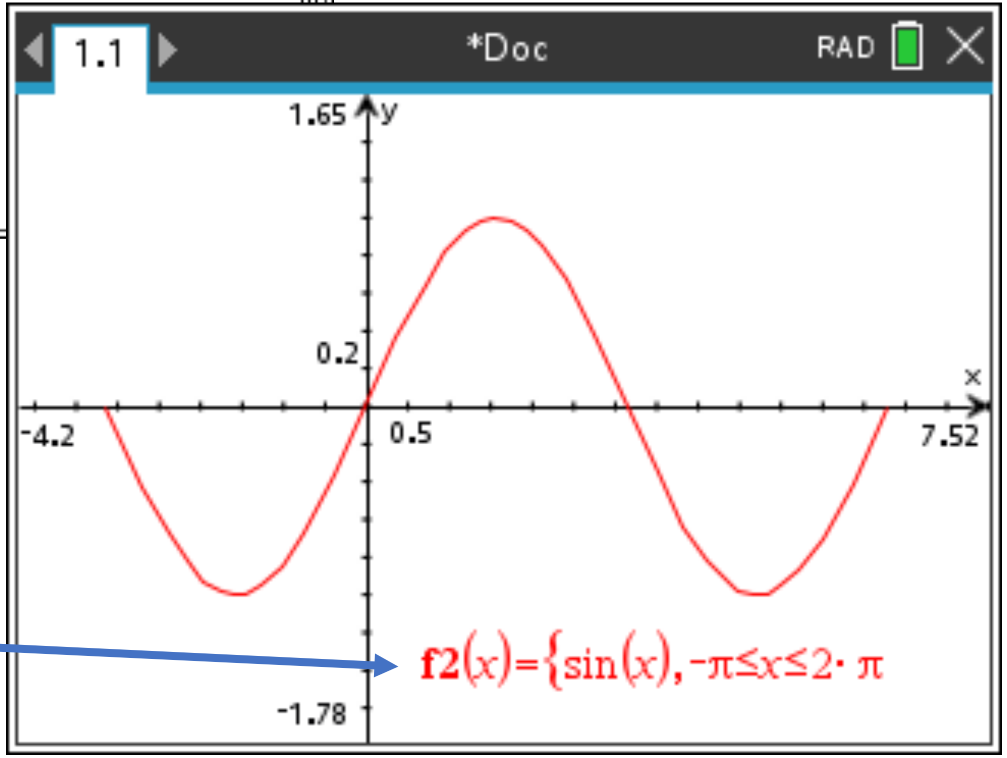
In this example the period would be 360°

Graph of the sine function

You can also use the CAS to draw a graph of the sine functions.



Note that the CAS also doesn't show nice values for the x-axis in terms of radians

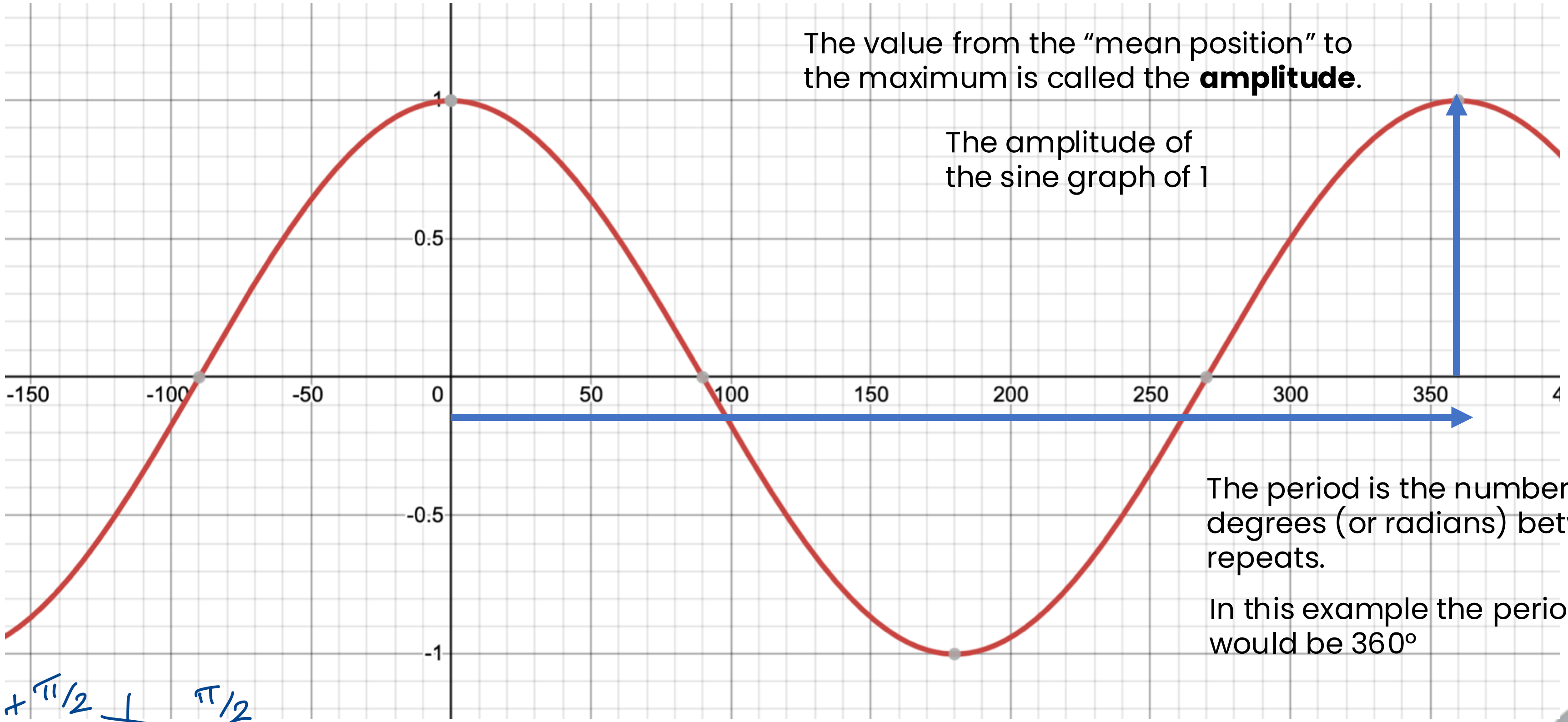


It's important to know how to limit the domain for trig functions as this will save a LOT of time later.

Graph of the cosine function

We can see that the period and the amplitude are the same as the sine curve.

The only difference between the two is that the cosine curve has been translated 90° to the left.

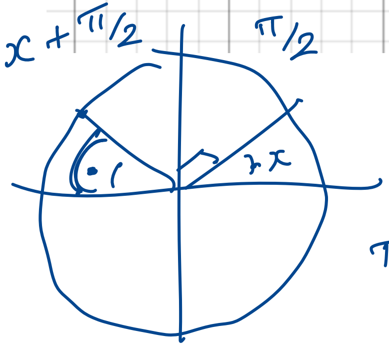


The value from the "mean position" to the maximum is called the **amplitude**.

The amplitude of the sine graph of 1

The period is the number of degrees (or radians) between repeats.

In this example the period would be 360°



$$\cos(x) = \sin\left(x + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - x\right)$$

$$\pi - \left(x + \frac{\pi}{2}\right) = \pi - x - \pi/2$$
$$= \pi/2 - x$$

Transformations: The Return

Everything you're going to deal with from now will be transformations of graphs. It is going to be really important to be able to look at a function and know exactly which parts represent:

- Dilations
- Translations
- Reflections

This might be a good time and go back and make sure you remember how to do this.

$$y = 3\sin\left(x + \frac{\pi}{2}\right) + 1$$

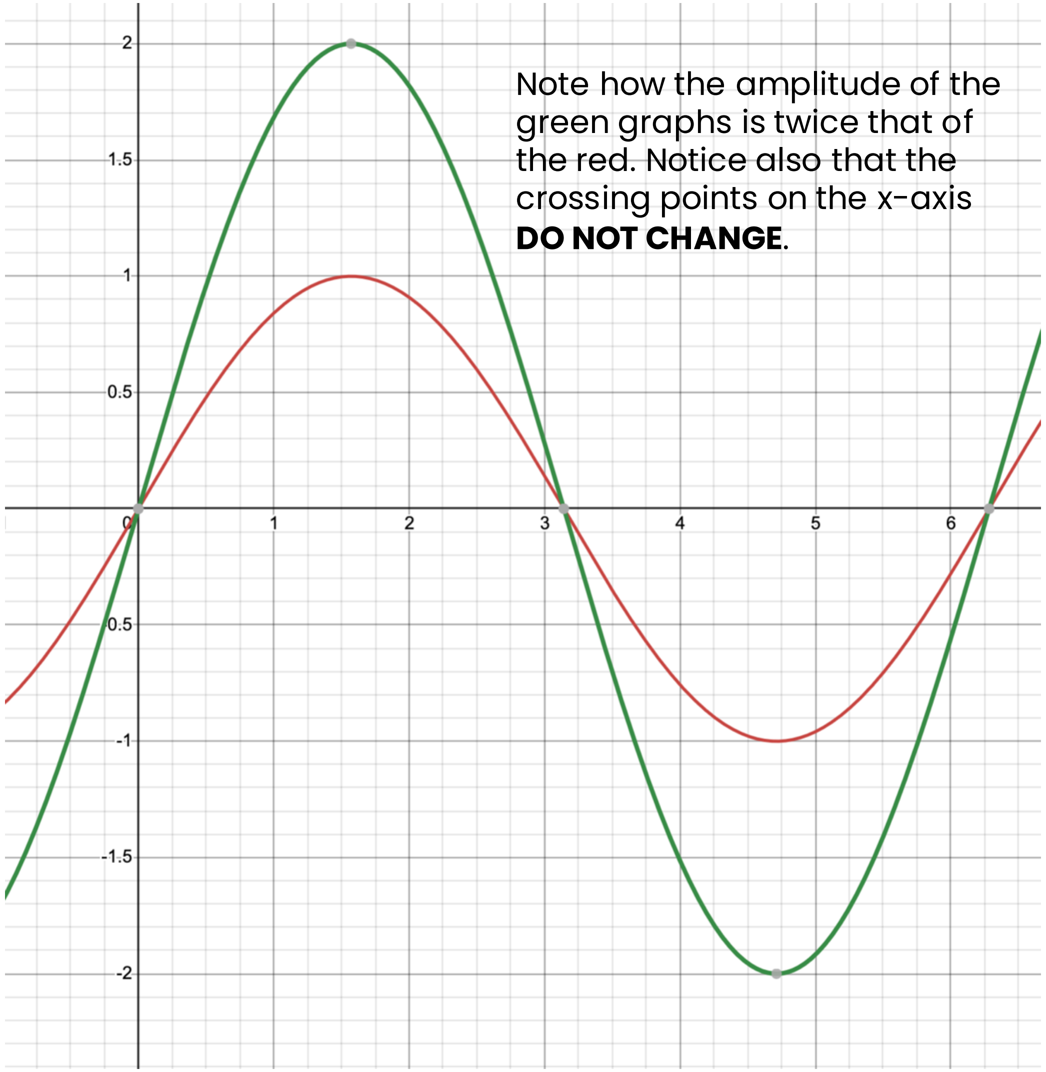
$$y = \sqrt{3}\sin\left(2x - \frac{\pi}{2}\right) + 1$$

$$y = \sqrt{3}\sin 2\left(x - \frac{\pi}{4}\right) + 1$$

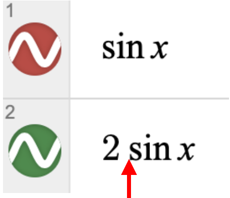
These are just two of the examples which will be used and each contains a number of tricks!

Dilating away from the x-axis

When we dilate away from the x-axis are are stretching the graphs vertically. It makes sense, therefore, that this will change the amplitude



Note how the amplitude of the green graphs is twice that of the red. Notice also that the crossing points on the x-axis **DO NOT CHANGE**.



This number is **generally** the amplitude of the function

Dilating away from the y-axis

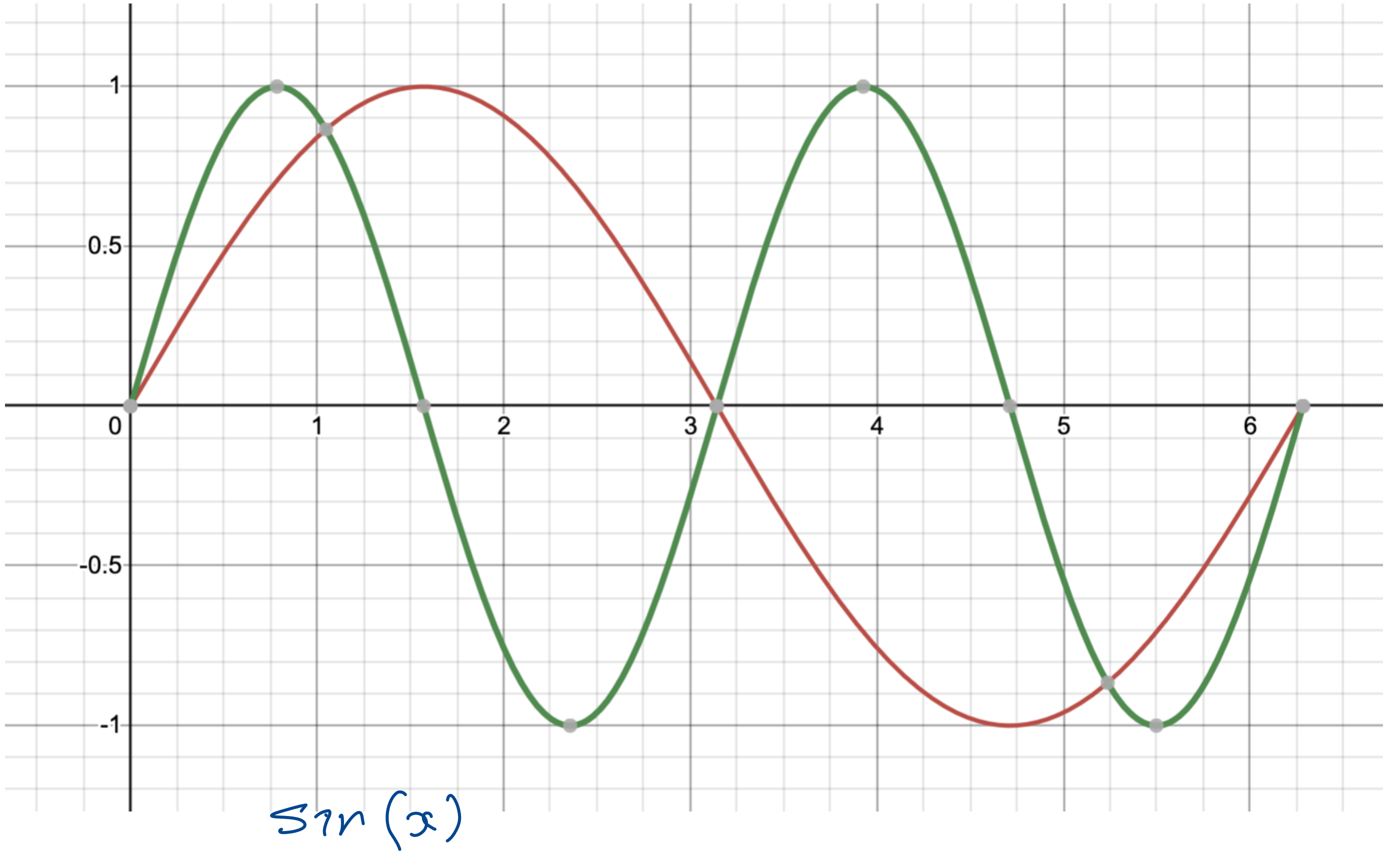
$x \to 2x$ $\quad \frac{x}{1/2}$

$x \to \frac{x}{a}$

$y \to \frac{y}{a}$

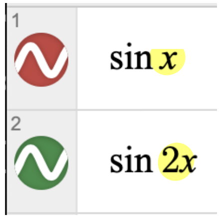
When we dilate away from the y-axis, we will be changing the period of the graph.

It is vitally important that you remember the period of a **standard sine and cosine graph is** 2π.



This might look confusing but not that the green graph repeats every π now.

Note the relationship between the numbers inside the function and the impact it has on the period.

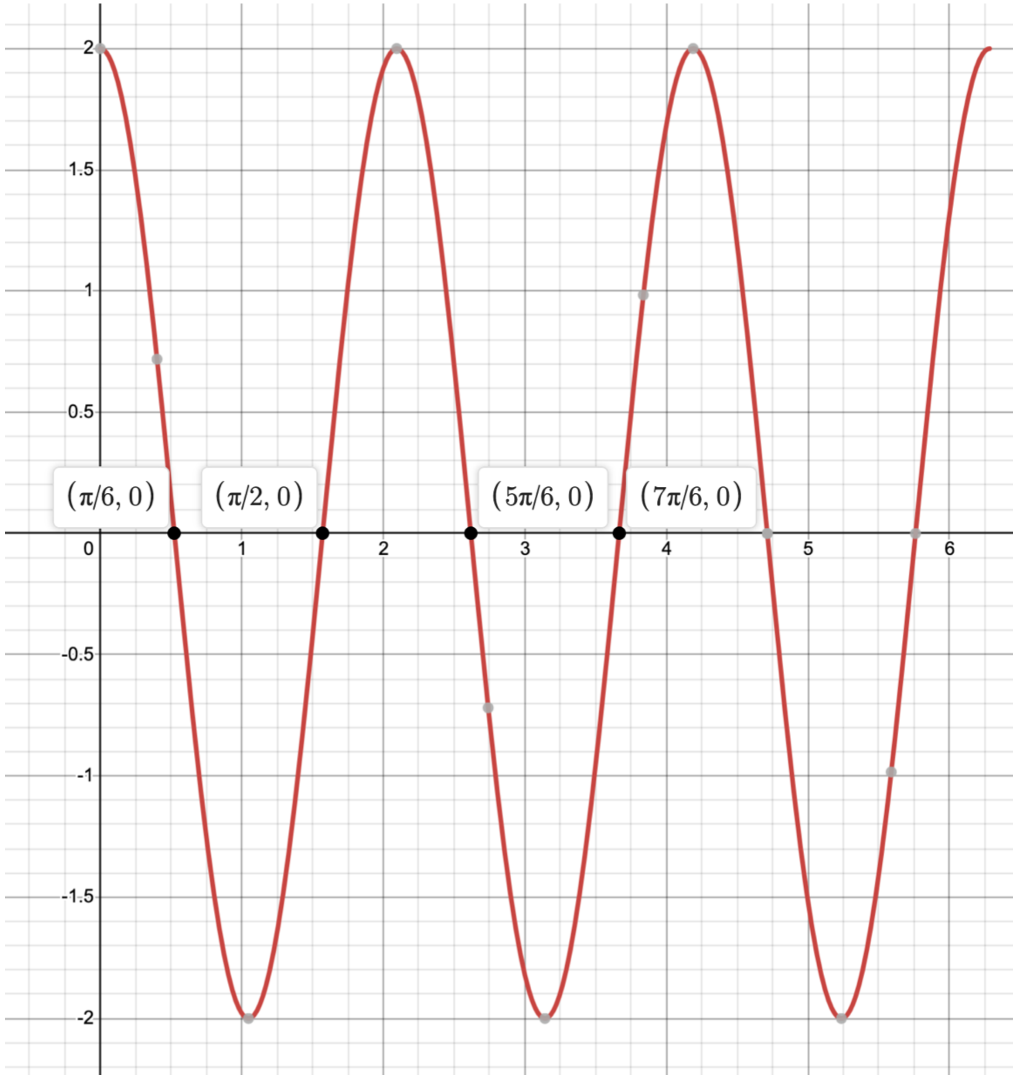


Combining transformations

This shows the graph of:

$$y = 2\cos 3x$$

Note that the amplitude is 2 and the period is $\frac{2\pi}{3}$



We can do so many of these that we can see rules start to appear.

The first, and foremost, is that:

Period = $\frac{2\pi}{n}$

But what is 'n'?

$2\pi \times \frac{1}{3} = \frac{2\pi}{3}$

General forms of sine and cosine

The following are the general forms of sine and cosine:

Note: Normally we use the letter 't' in place of the 'x'. I have no idea why!

$$y = \text{a}\sin(nt) \qquad\qquad y = \text{a}\cos(nt)$$

period $= \dfrac{2\pi}{n}$

amplitude

This 'n' affects the amplitude. In the opposite way you are expecting!

There is a massive algebra trick relating to the value of 'n' which cannot be understated and relates to translations. But this comes in another lesson.

Examples

For each of the following functions with domain $\mathbb{R}$, state the amplitude and period:

a $f(t) = 2\sin(3t)$ **b** $f(t) = -\frac{1}{2}\sin\left(\frac{t}{2}\right)$ **c** $f(t) = 4\cos(3\pi t)$

Note: Always make sure you take note of the domains and any restrictions!

$p = \frac{2\pi}{n}$

a. period $= 2$

amp $= \frac{2\pi}{3}$

b. period $= \frac{1}{2}$

amp $= \frac{2\pi}{\frac{1}{2}}$

$= \underline{\underline{4\pi}}$

c. amp $= 4$

period $= \frac{2\cancel{\pi}}{3\cancel{\pi}}$

$= \underline{\underline{\frac{2}{3}}}$

Examples

For each of the following, give a sequence of transformations which takes the graph of $y = \sin x$ to the graph of $y = g(x)$, and state the amplitude and period of $g(x)$:

a $g(x) = 3\sin(2x)$

b $g(x) = 4\sin\left(\frac{x}{2}\right)$

D

R

T

$y = \sin x$

$\frac{y}{3} = \sin x$

$\therefore$ Dil factor 3 from x-axis

$y = 3\sin x$

$y = 3\sin\left(\frac{x}{\frac{1}{2}}\right)$

$\therefore$ Dil factor $\frac{1}{2}$ from y-axis

$y = 3\sin 2x$

Examples

For each of the following, give a sequence of transformations which takes the graph of $y = \sin x$ to the graph of $y = g(x)$, and state the amplitude and period of $g(x)$:

a $g(x) = 3\sin(2x)$

b $g(x) = 4\sin\left(\frac{x}{2}\right)$

b.

$y = \sin x$

$\frac{y}{4} = \sin x$ $\quad\therefore$ Dil factor 4 from x-axis

$y = 4\sin x$

$y = 4\sin\left(\frac{x}{2}\right)$ $\quad\therefore$ Dil factor 2 from y-axis

Examples

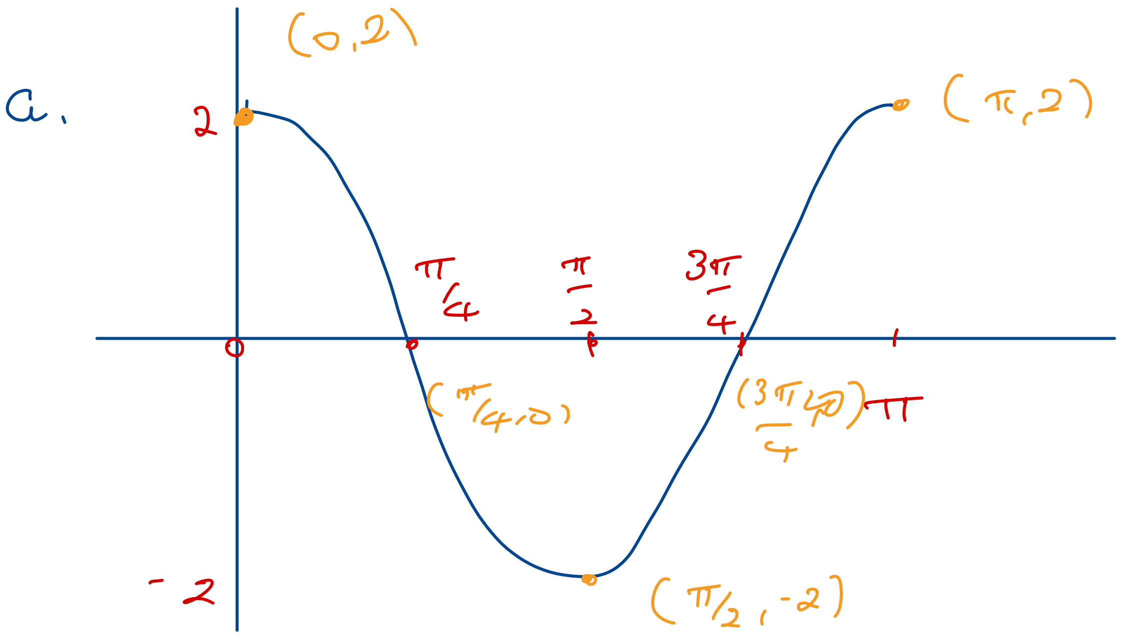
Sketch the graph of each of the following functions:

a $y = 2\cos(2\theta)$

b $y = \frac{1}{2}\sin\left(\frac{x}{2}\right)$

In each case, show one complete cycle.

$D = \frac{2\pi}{2} = \pi$



$\frac{\pi}{2} \times \frac{1}{2} = \frac{\pi}{4}$

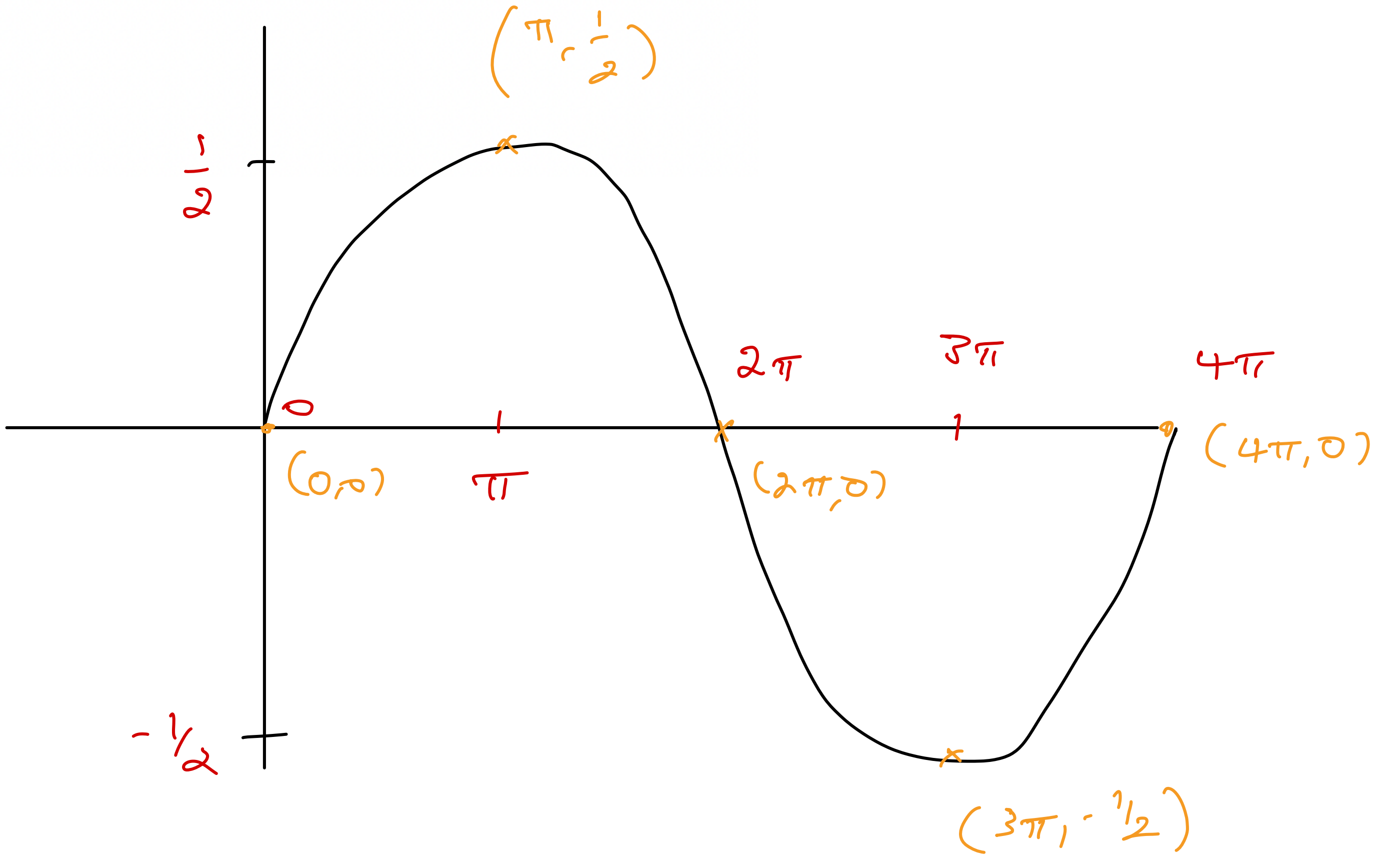
Examples

Sketch the graph of each of the following functions:

a $y = 2\cos(2\theta)$

b $y = \frac{1}{2}\sin\left(\frac{x}{2}\right)$

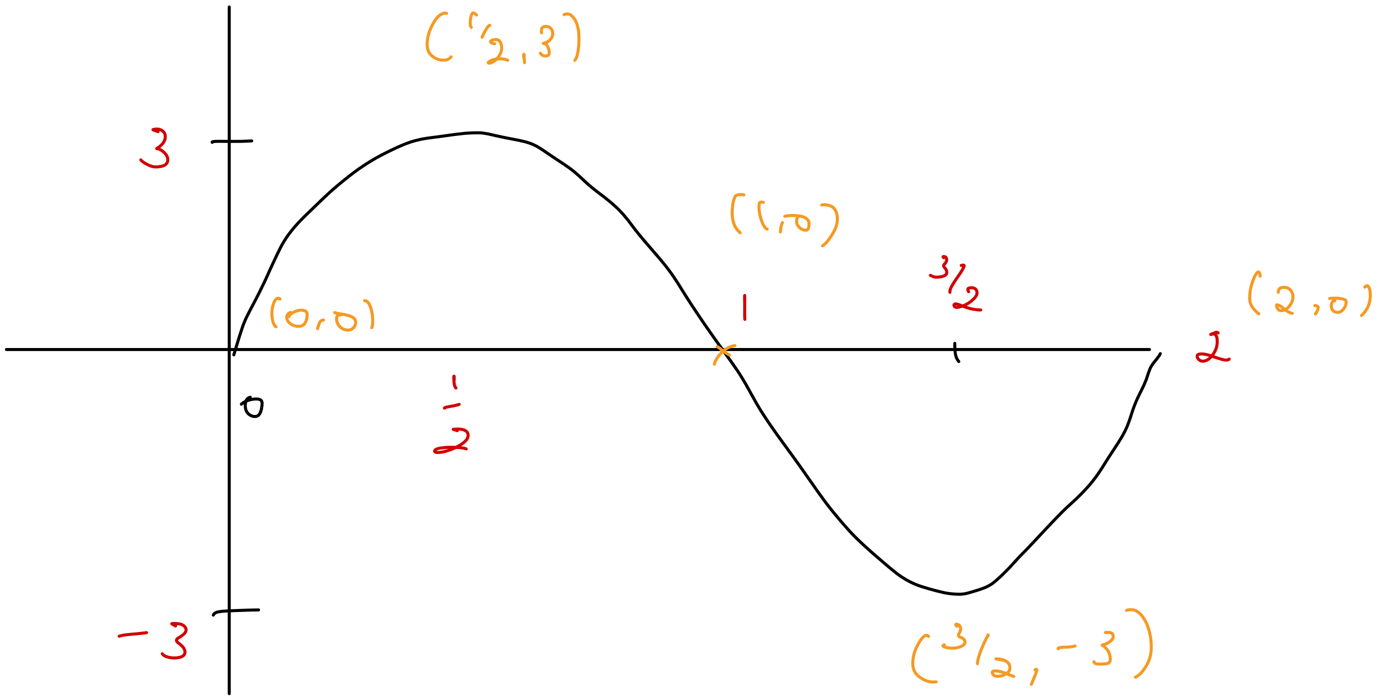
In each case, show one complete cycle.



Examples

Sketch the graph of $f: [0, 2] \to \mathbb{R}$, $f(t) = 3\sin(\pi t)$.

$P = \frac{2\cancel{\pi}}{\cancel{\pi}} = 2$



Examples

Sketch the following graphs for $x \in [0, 4\pi]$:

a $f(x) = -2\sin\left(\frac{x}{2}\right)$

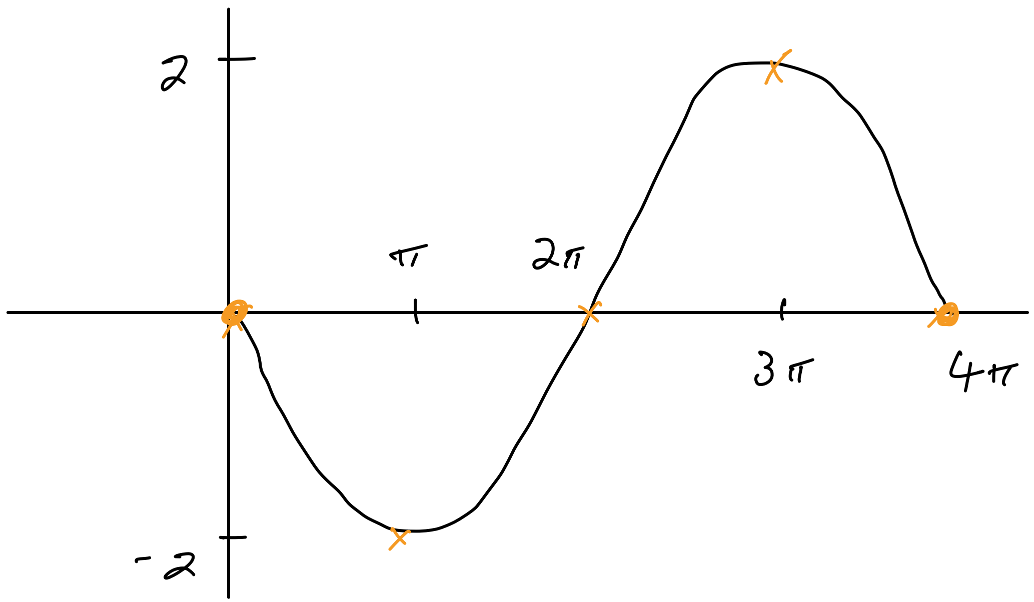
b $y = -\cos(2x)$

D

R

T

a.



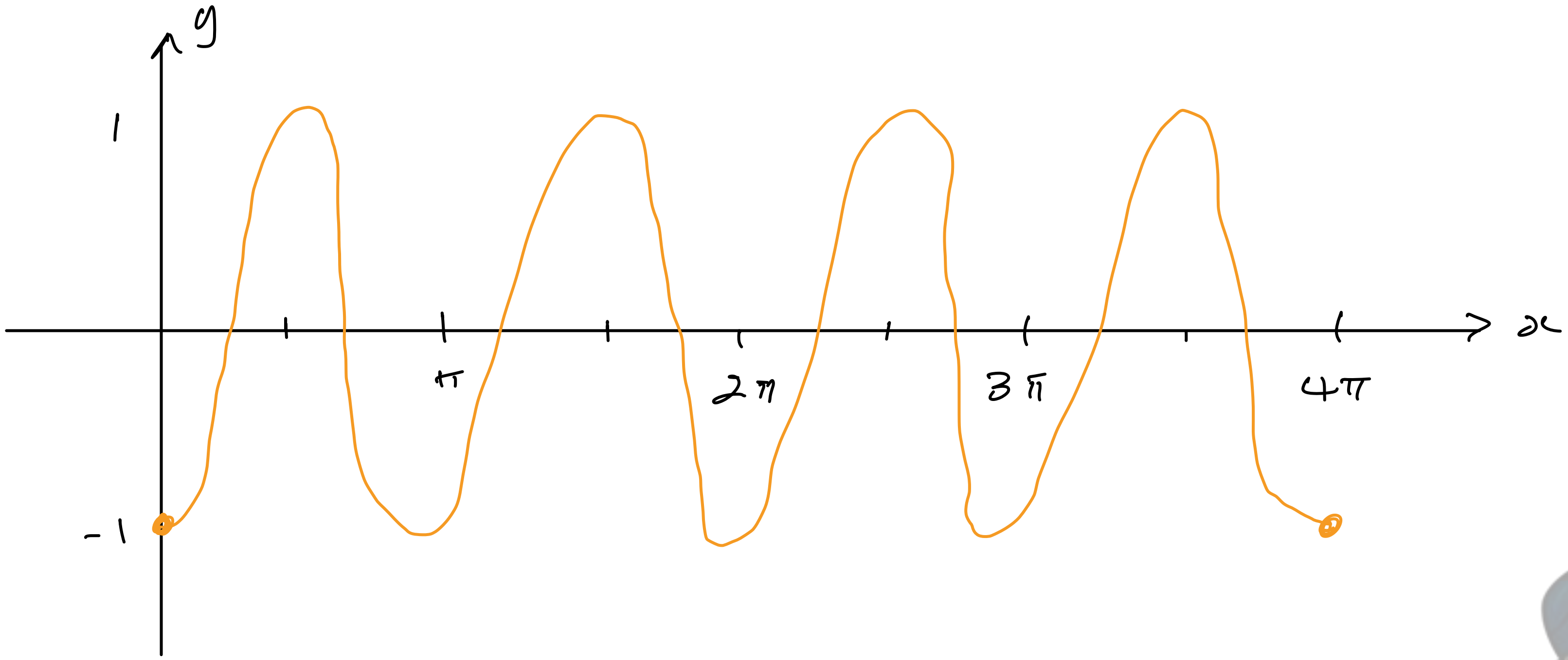
Examples

$$P = \frac{2\pi}{2} = \pi$$

Sketch the following graphs for $x \in [0, 4\pi]$:

a $f(x) = -2\sin\left(\frac{x}{2}\right)$ **b** $y = -\cos(2x)$

b.

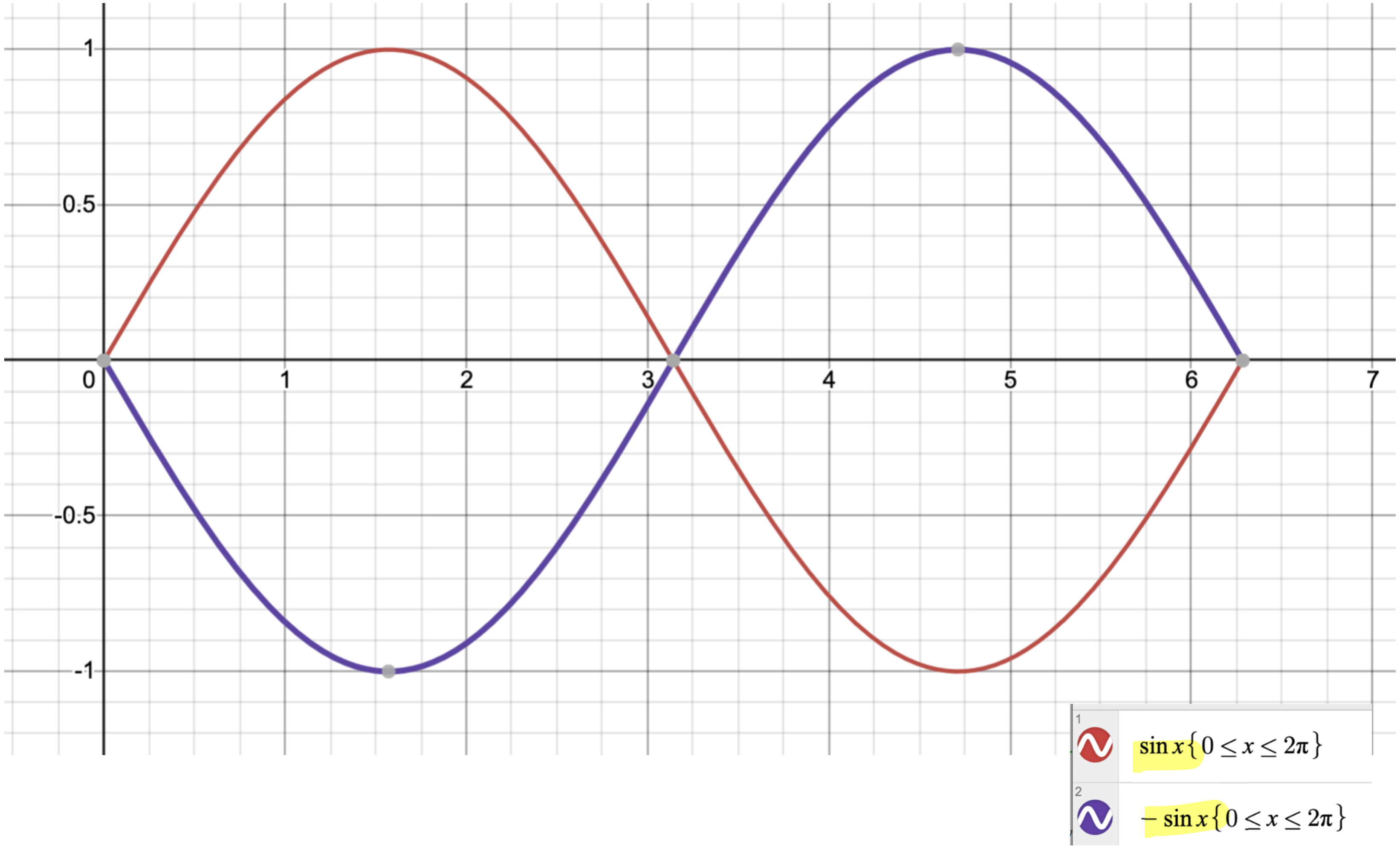


Reflections in the x- and y-axes

It's important to also look for how to reflect in the x- and y-axes and what this does to the graphs!

A reflection in the x-axis is shown to the right.

Notice that the function is fully negative.



Reflections in the x- and y-axes

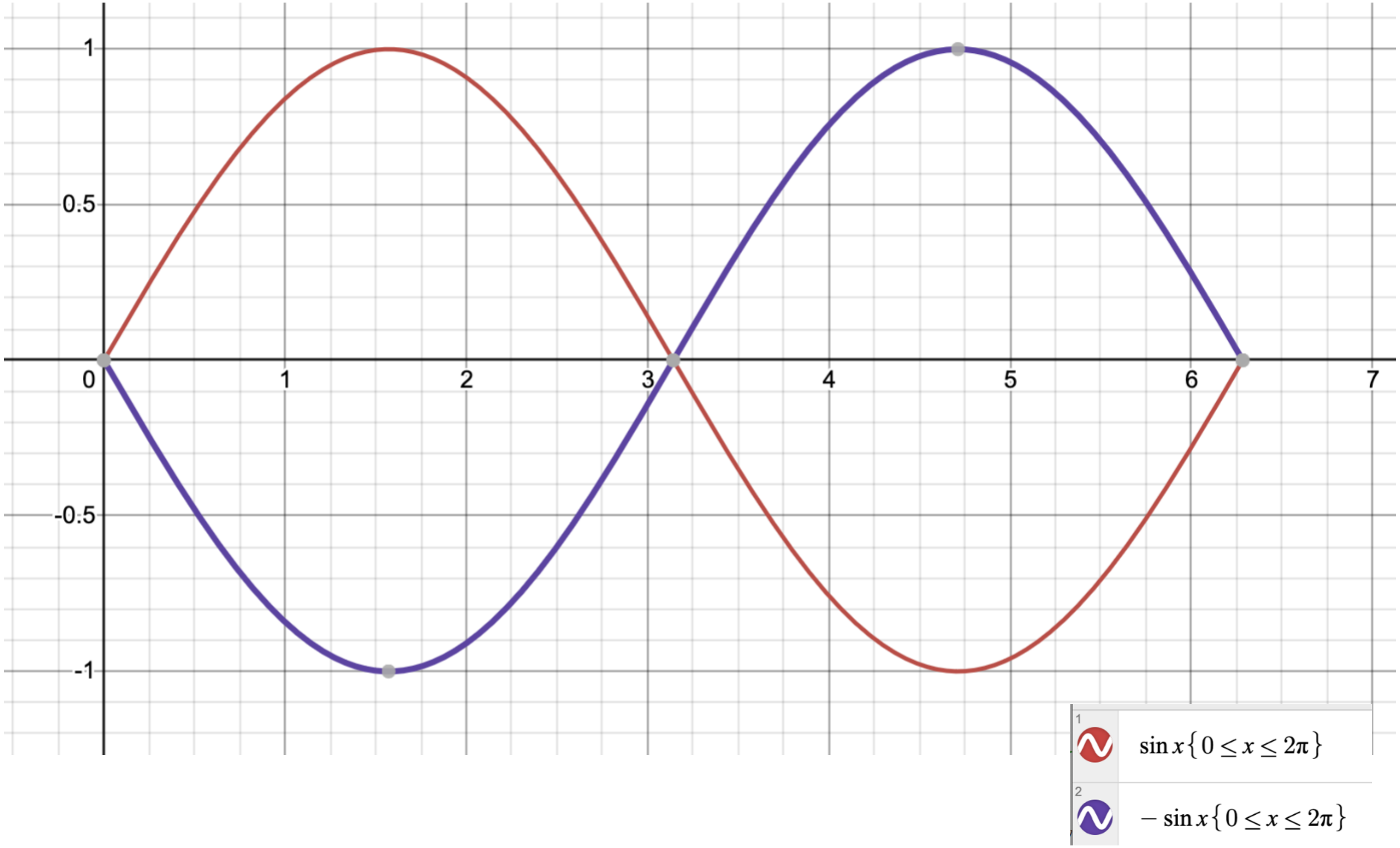
$-\sin(x) = \sin(-x)$

It's important to also look for how to reflect in the x- and y-axes and what this does to the graphs!

A reflection in the x-axis is shown to the right.

This is also a reflection in the y-axis!

Notice that the function is fully negative.



Learning Objectives: Revisited

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 Mathematical Methods course.

- Understand how to sketch standard graphs of sine and cosine
- Know what it means by amplitude and period
- Know how to dilate sine and cosine graphs from the x-axis
- Know how to translate graphs both in the positive and negative direction of the x-axis.
- Know key values for sine and cosine graphs

Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Questions to complete:
Ex 14F q1-7
Ex 14H q1,2,3,5
Ex 14I q1-3 LHS

Additional Sheets:
MM216

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