

Dijkstra's algorithm (Updated)

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Further Mathematics course.

- Understand what it means by “shortest path problems”
- Know how to apply Dijkstra's algorithm

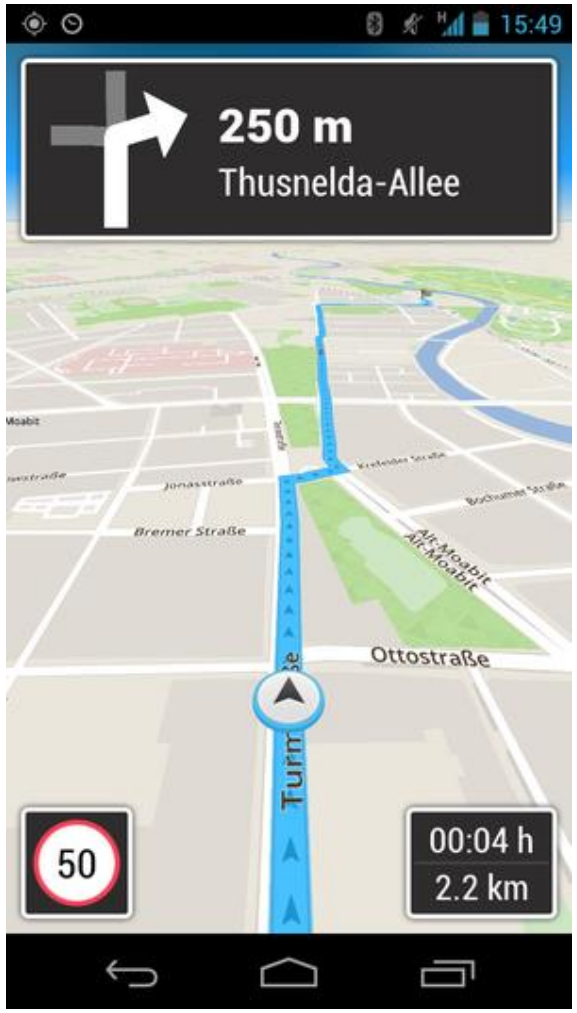


Recap of past learning

In previous lessons we have looked at drawing graphs and how they are effectively used to show connections between vertices. In all the examples we have been looking at the edges didn't have any numerical data associated with them. This isn't really modelling a real world situation.

Towns are connected by roads which will have a distance associated with them. Computers connected on the internet have a time taken for transmission of information.

This lesson is going to look at how we can use graphs with numbers to help us solve some real world problems.





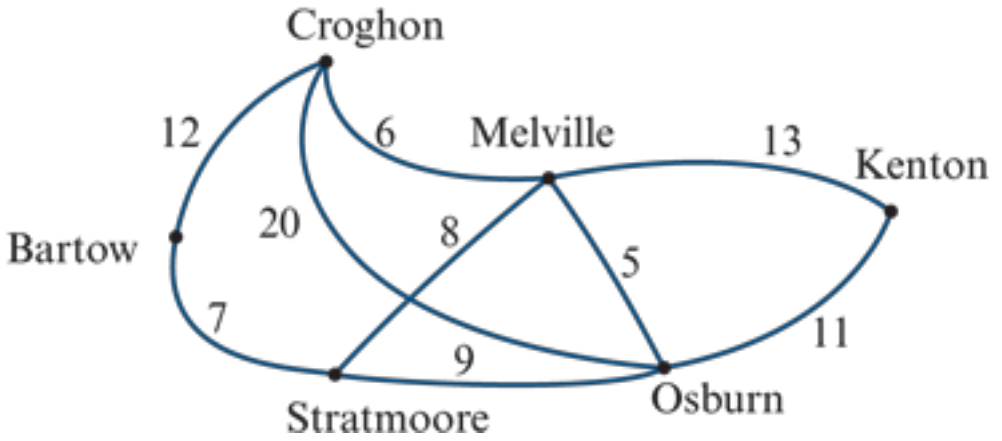
Shortest path

In networks that have only a few vertices, it is often easy to find the shortest path between two vertices by inspection.

Example: Find the shortest path from Bartow to Kenton in the network shown on the right.

Steps to solve this:

- List all the routes
- Add all the distances
- Find the shortest one!



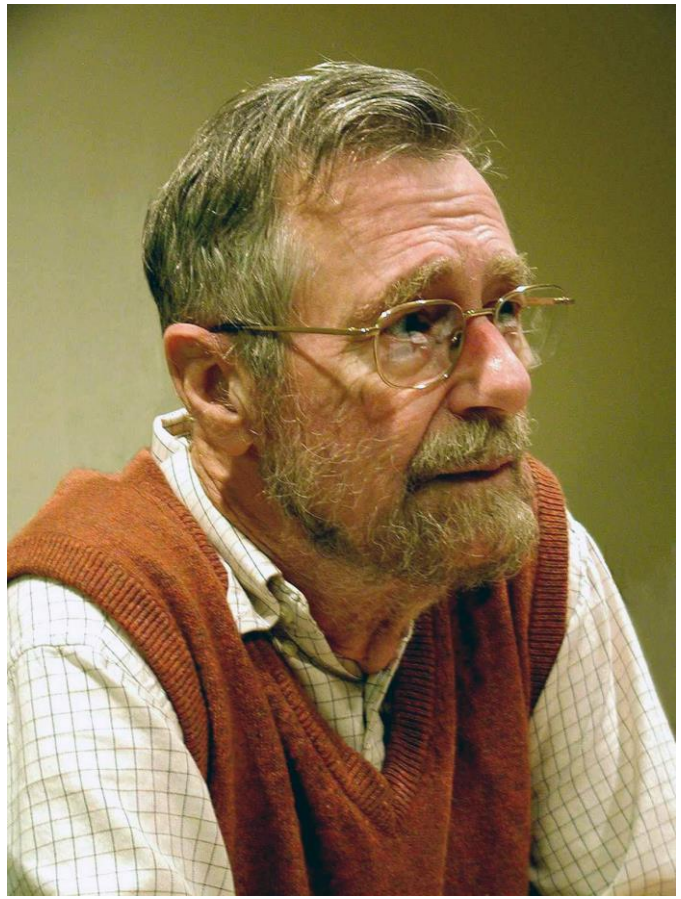


Dijkstra's algorithm

OK! Another person who had too much time on their hands and an over excitement about Mathematics.

Edsger Wybe Dijkstra
11 May 1930 – 6 August 2002)
A Dutch computer scientist, programmer, software engineer, systems scientist, science essayist,[9][10] and pioneer in computing science.[11] A theoretical physicist by training, he worked as a programmer at the Mathematisch Centrum (Amsterdam) from 1952 to 1962. A university professor for much of his life, Dijkstra held the Schlumberger Centennial Chair in Computer Sciences at the University of Texas at Austin from 1984 until his retirement in 1999. He was a professor of mathematics at the Eindhoven University of Technology (1962–1984) and a research fellow at the Burroughs Corporation (1973–1984). In 1972, he became the first person to win the Turing Award who was neither American nor British.

Extracted from Wikipedia





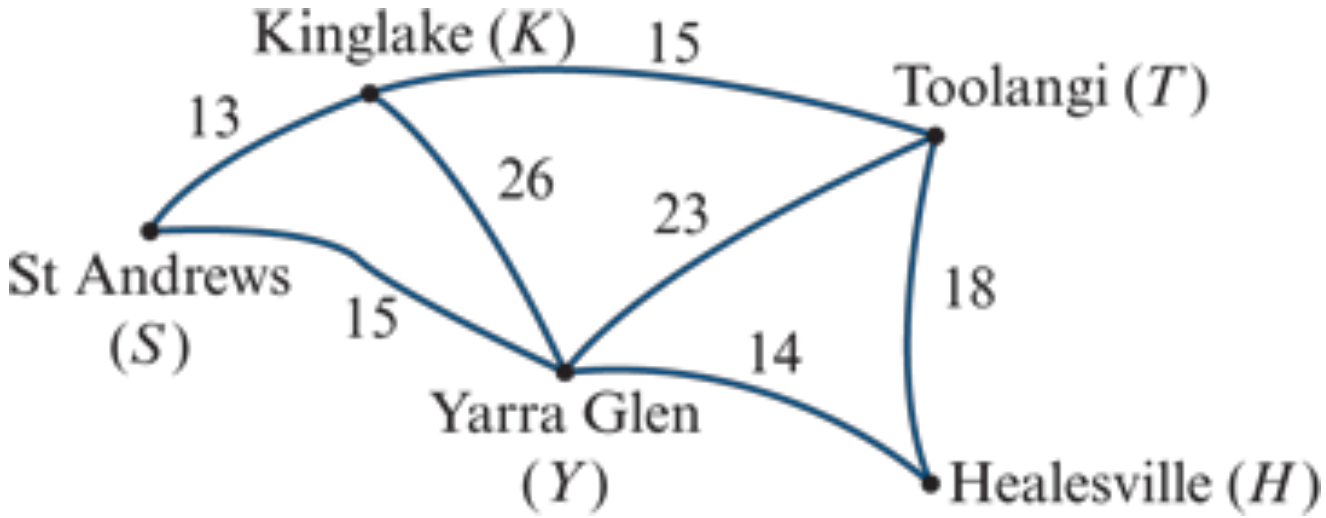
Dijkstra's algorithm

When there are many vertices and many edges, a systematic method, called an algorithm, can be used to find the shortest path.

Dijkstra's algorithm can be used to find the shortest path through a network, from a starting vertex to a destination vertex.

Let's work through this with an example.

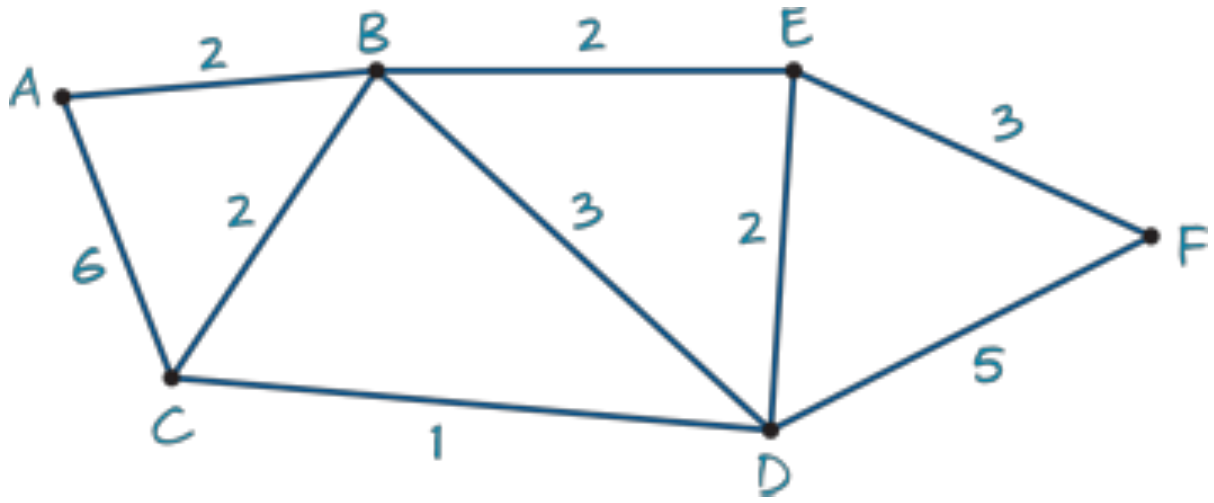
The algorithm will be used to find the shortest path between St Andrews (S) and Toolangi (T).





Another example of Dijkstra's algorithm

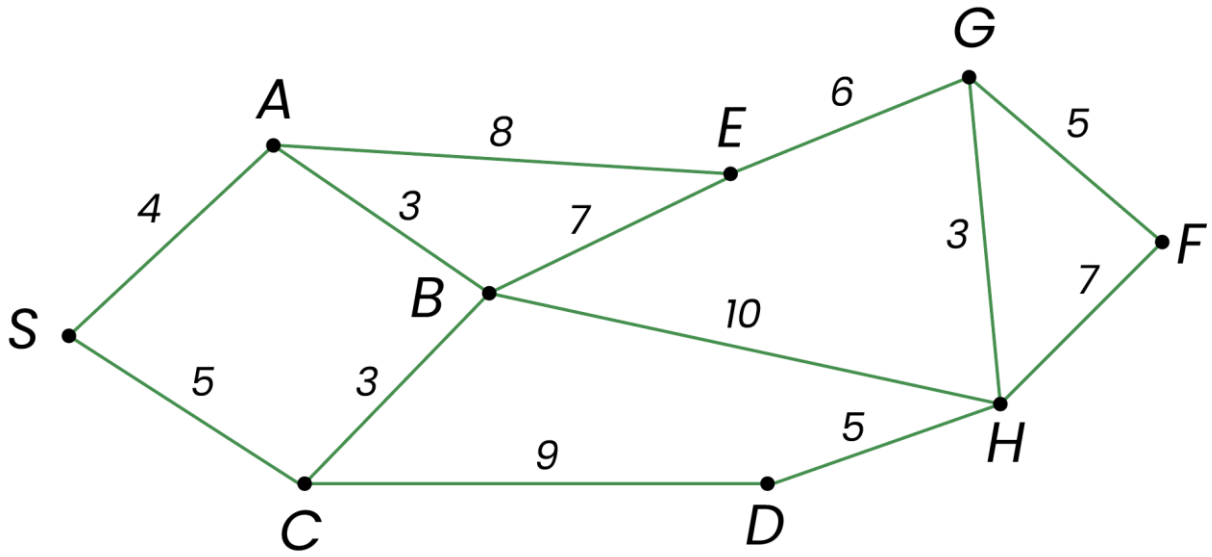
Find the shortest path from A to F in the weighted graph shown on the right





One more example of Dijkstra's algorithm

Find the shortest path from *S* to F in the weighted graph shown on the right





VCAA Exam Question

Year: 2021
Paper: 2
Question: 2
Percent correct: 85%

Question 2 (2 marks)

George lives in Town G and Maggie lives in Town M.

The diagram below shows the network of main roads between Town G and Town M.

The vertices G, H, I, J, K, L, M, N and O represent towns.

The edges represent the main roads. The numbers on the edges indicate the distances, in kilometres, between adjacent towns.

a. What is the shortest distance, in kilometres, between Town G and Town M?

