Graphs of exponential functions

Year 11

Mathematical Methods

www.maffsguru.com

Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be growing exponentially.
- Understand what a graph of an exponential function looks like.
- Understand the important parts of an exponential function.
- Know how to apply transformations to exponential functions

RECAP

In previous parts of this course we have looked at graphs of common functions including:

- Square root
- Hyperbola
- Truncus
- Cubic
- Quadratic

There is another form of graph which now features very heavily in this course.

Graphs of the form $y = a^x$

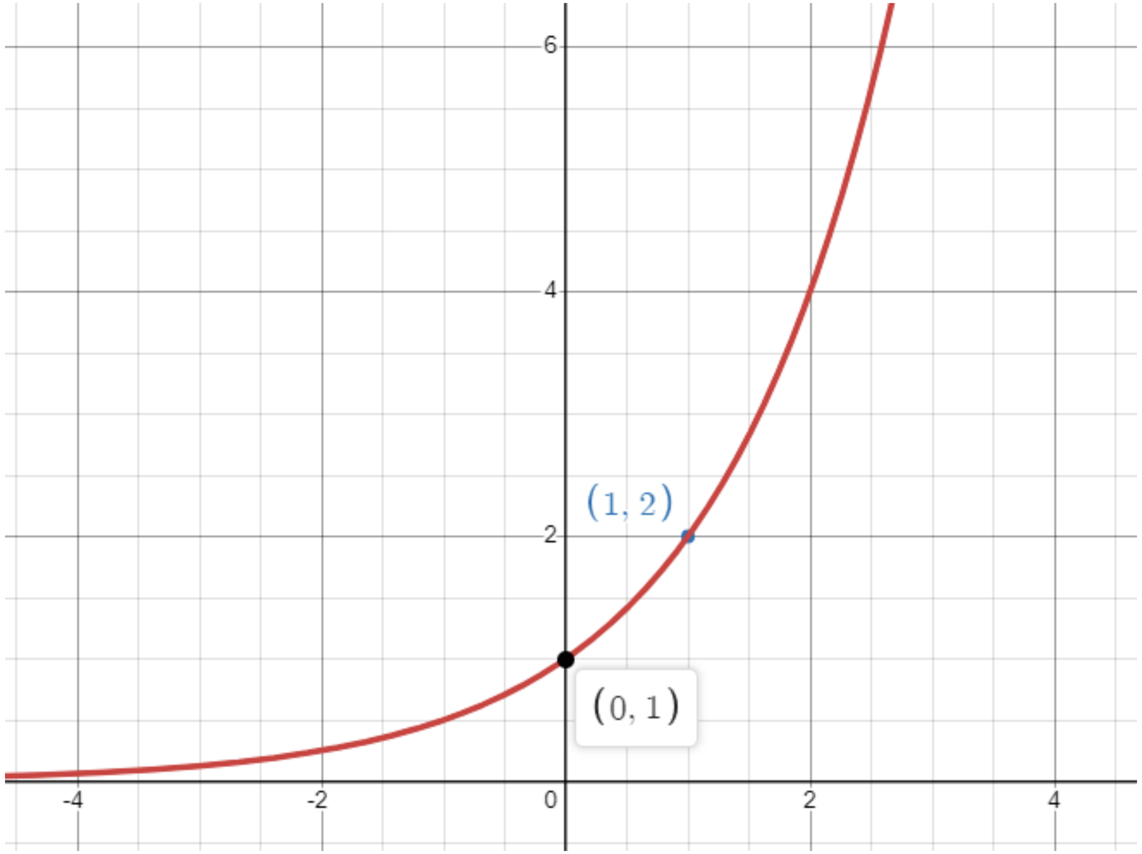
$y = 2^0 = 1$

$y = 2^1 =$

Firstly, we need to be careful and aware that the value of *a* can take various values. For now, we will look at the values of *a* being greater than 1.

So, let's start with the graph of $y = 2^x$

$y = 2^{-3}$



Note where the graph crossed the x-axis.

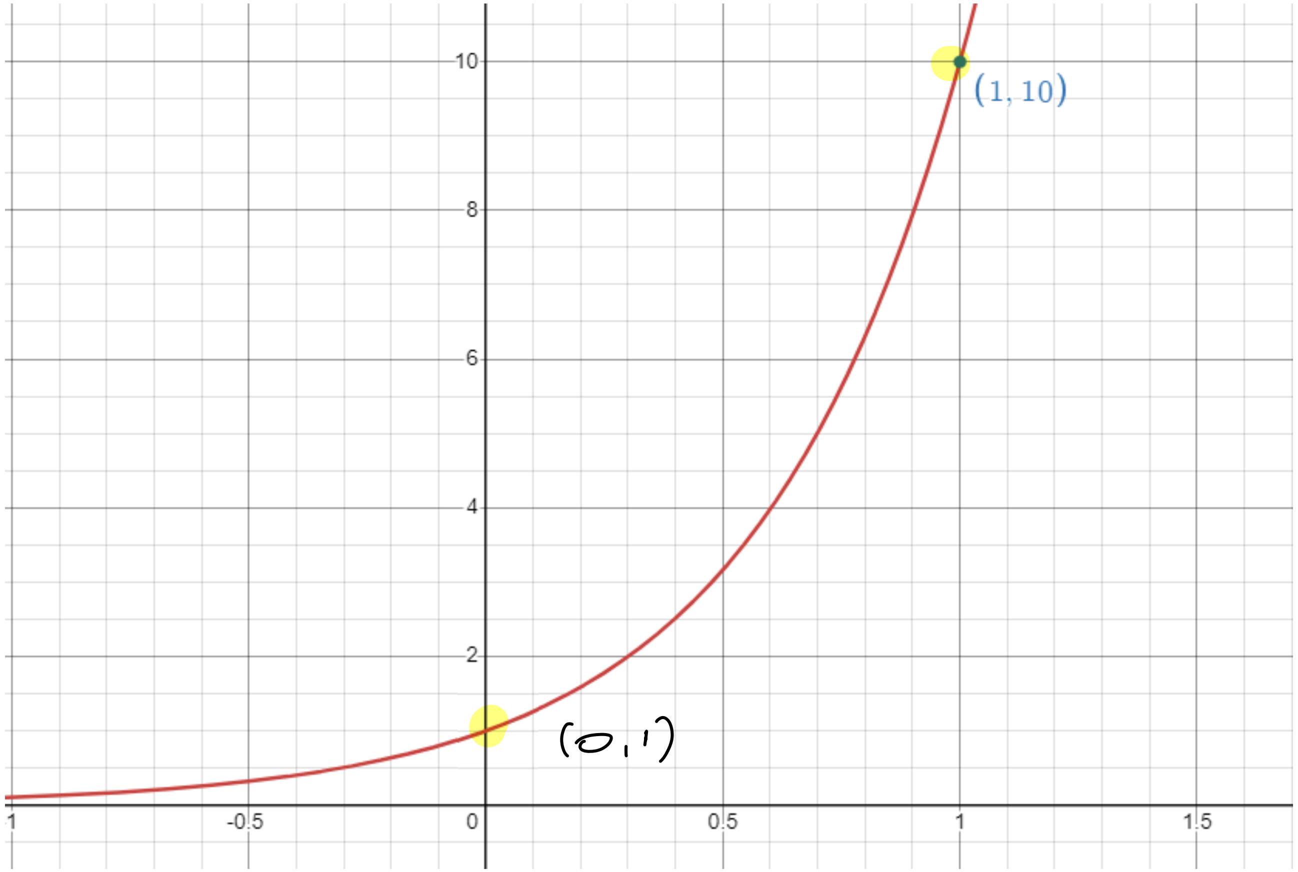
Questions:
Why would this graph have a coordinate of (1,2)?
Could we have predicted this?
Would there be an asymptote with this graph?
What is the domain of the function?
What is the range of the function?

Graphs of the form $y = a^x$

Let's look at the graph of $y = 10^x$

$y = 10^0 = 1$

$y = 10^1 = 10$



Note where the graph crossed the x-axis.

Questions:

Why would this graph have a coordinate of (1, 10)?

Could we have predicted this?

Would there be an asymptote with this graph?

What is the domain of the function?

What is the range of the function?

Important features for graphs of the form $y = a^x$

We can now tie all this learning together with the following statements.

- The x-axis is an asymptote
- The y-axis intercept will be (1, 0).
- The point (1, a) will sit on the line
- We would be able to describe this graph as exponential in terms of growth.
- The domain will be $\mathbb{R}$
- The range will be $\mathbb{R}^+$
- As $x \to -\infty, y \to 0^+$

Dilations and more dilations

All graphs of the form $y = a^x$ are related to each other by a dilation from the y-axis.

$x \div \frac{1}{k}$

$x \times k$

$y = 2^x$

$y = 10^x$

$y = (2^k)^x$

$2^{\frac{x}{1/k}}$

$y = 2^{kx}$

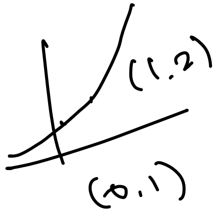
Graphs of the form $y = a^x$ where $0 < a < 1$

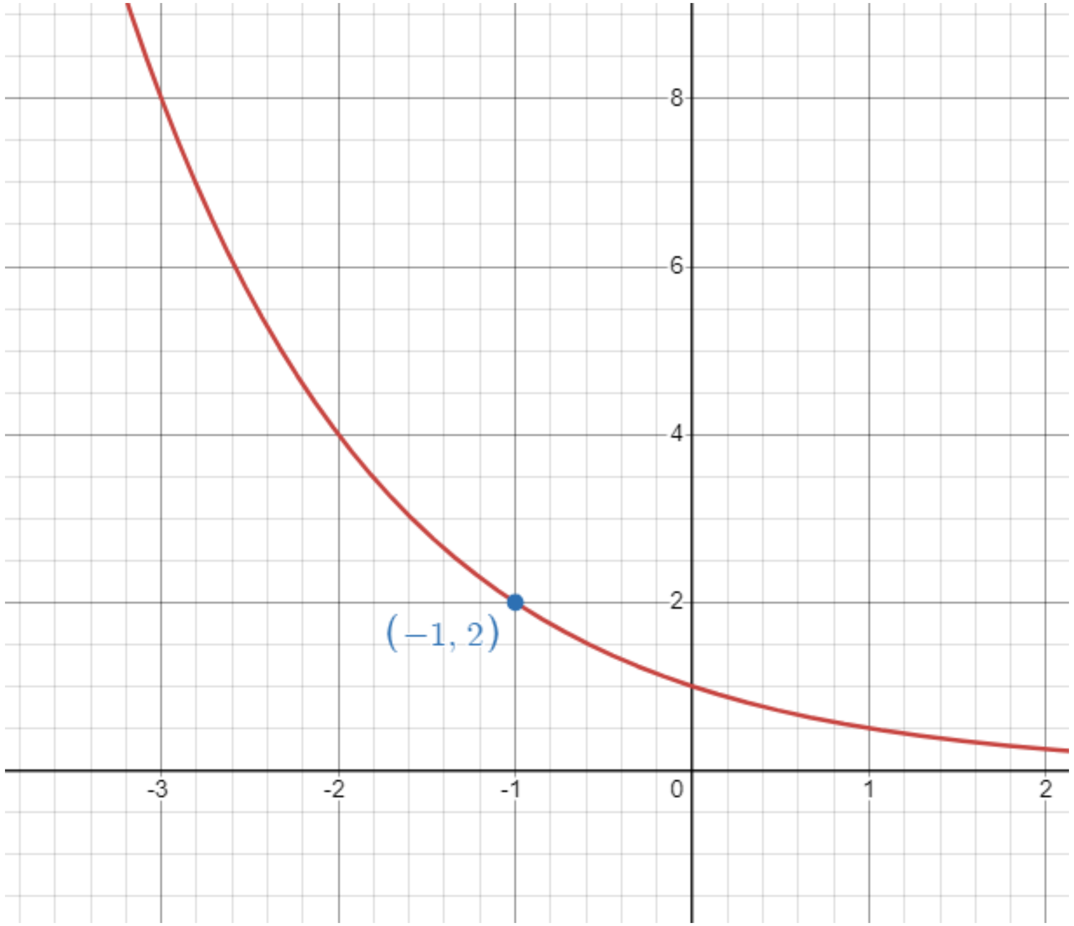
$y = 2^x$ 1,2

Let's look at see what the graph of $y = \left(\frac{1}{2}\right)^x$ looks like.

$y = (2^{-1})^x$

$= 2^{-x}$





Remember we can use index notation to change the equation to look like:

$$y = (2^{-1})^x$$

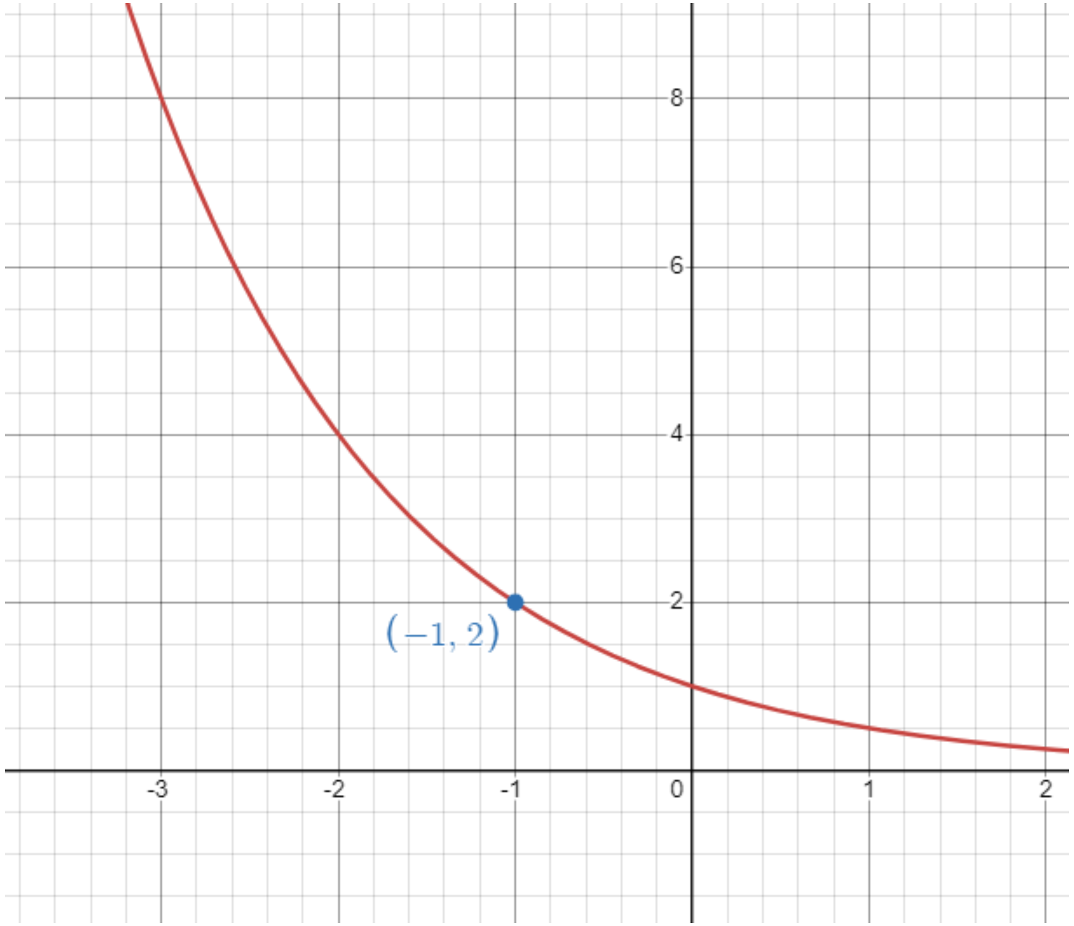
$$y = 2^{-x}$$

Using transformation theory from earlier, when we change $x \to -x$ we are reflecting in the y-axis. Which seems to be what has happened here!

Important information relating to graphs of the form $y = a^x$ where $0 < a < 1$

We can see the following is going to be true for the graphs of the form shown above.

- The x-axis is an asymptote
- As $x \to \infty, y \to 0^+$
- The y-axis intercept is $(0, 1)$
- The domain of the function is $\mathbb{R}$
- The range of the function is $\mathbb{R}^+$



Using a CAS

Plot the graph of $y = 2^x$ on a CAS calculator and hence find (correct to three decimal places):

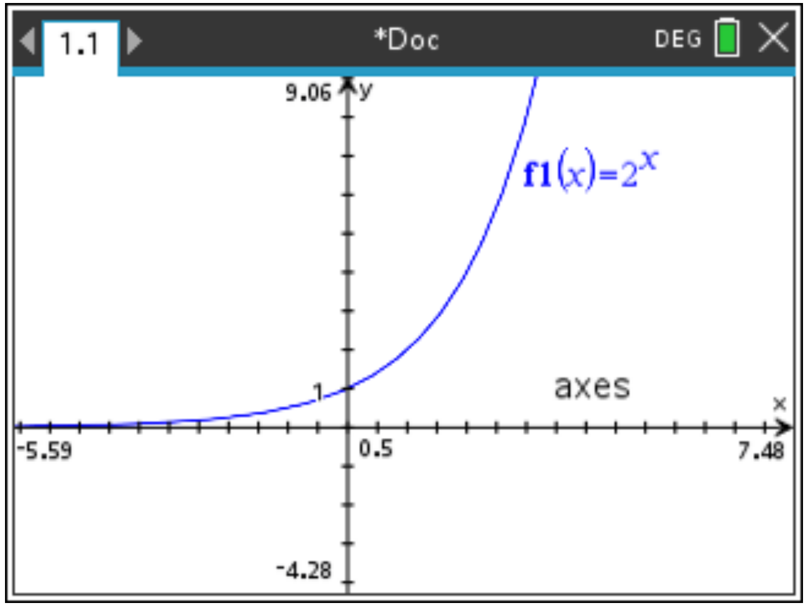
a the value of y when $x = 2.1$

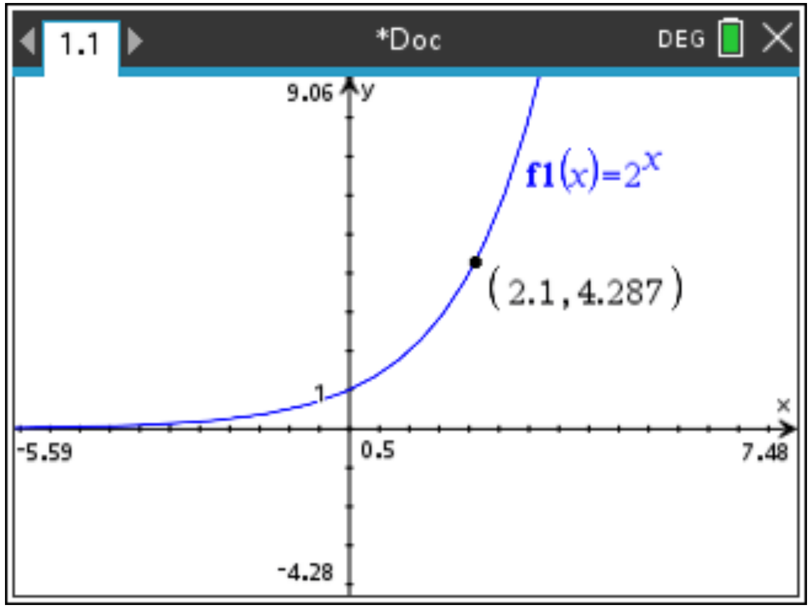
b the value of x when $y = 9$.

Using a CAS

Plot the graph of $y = 2^x$ on a CAS calculator and hence find (correct to three decimal places):

a the value of y when $x = 2.1$ **b** the value of x when $y = 9$.

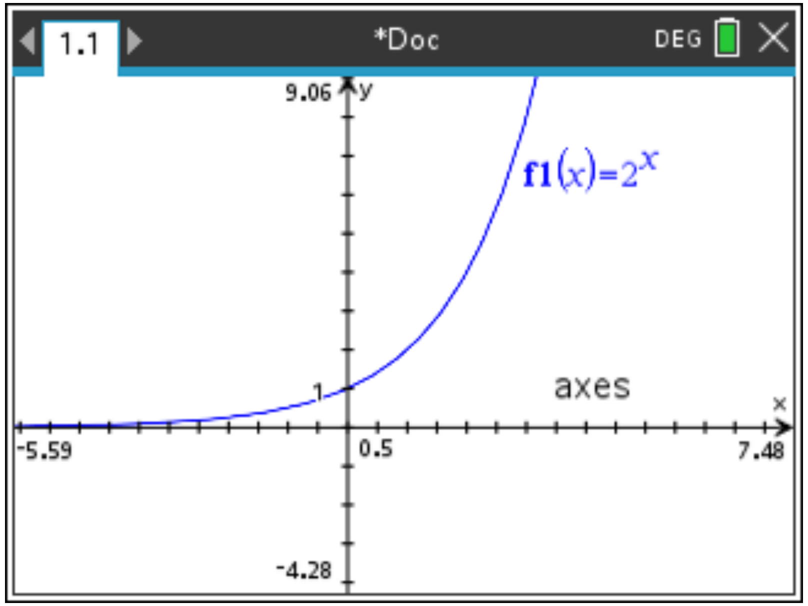


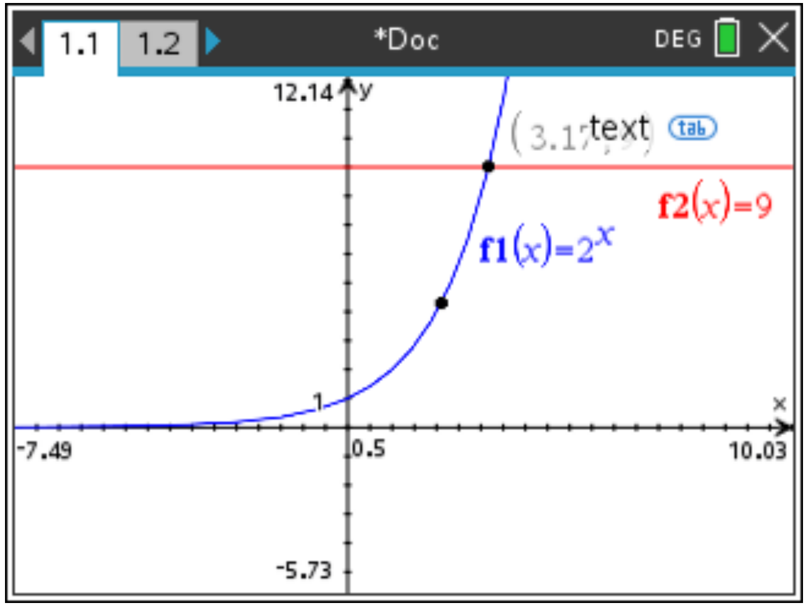


Using a CAS

Plot the graph of $y = 2^x$ on a CAS calculator and hence find (correct to three decimal places):

a the value of y when $x = 2.1$ **b** the value of x when $y = 9$.



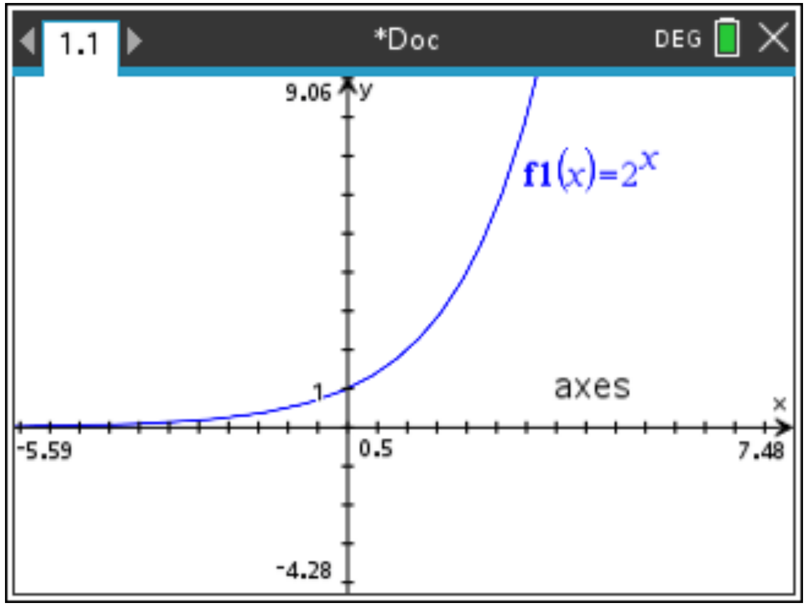


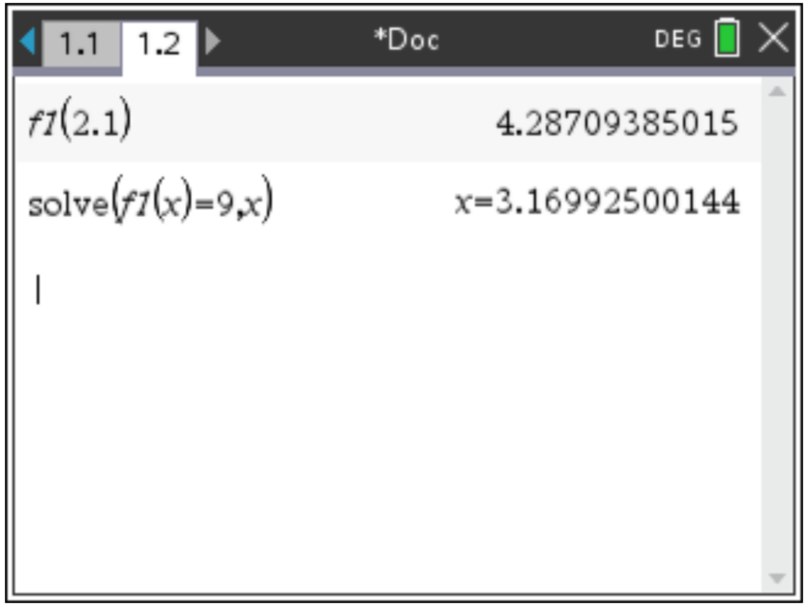
Do you notice the problem with using the CAS and graph screen?

Using a CAS

Plot the graph of $y = 2^x$ on a CAS calculator and hence find (correct to three decimal places):

a the value of y when $x = 2.1$ **b** the value of x when $y = 9$.





Now you have the full range of decimal places.

Transformations of exponential graphs

We have looked extensively at transformations in previous sections and have seen them used when looking at the graphs of $y = \left(\frac{1}{2}\right)^x$ and $y = 2^{-x}$.

Let's have a look at how we can transform graphs of the form $y = a^x$.

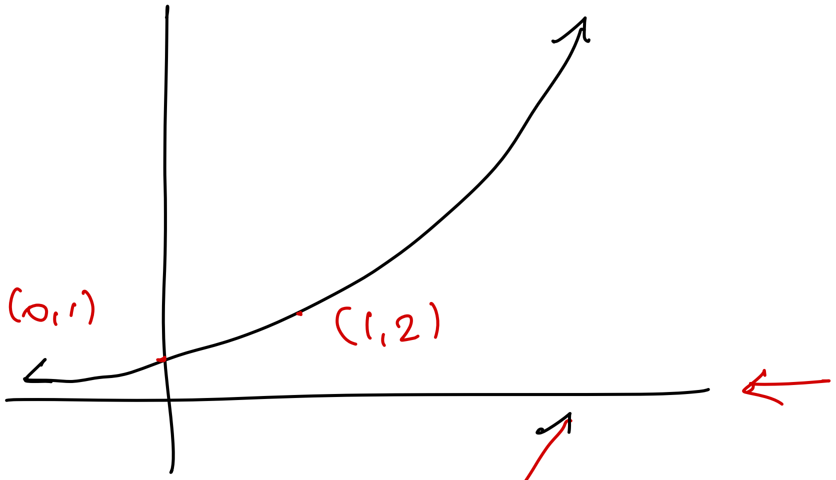
Example 1

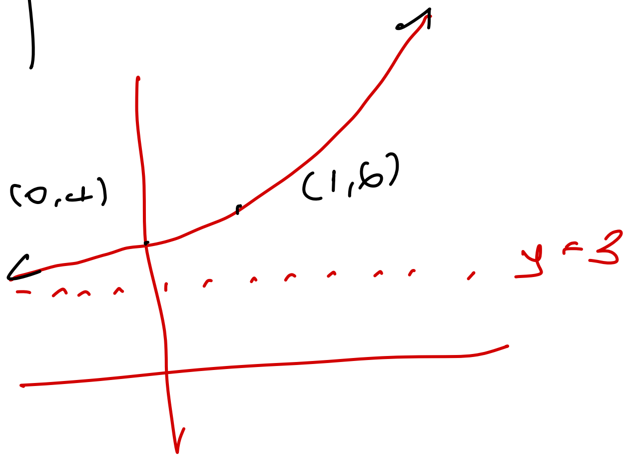
Sketch the graphs of each of the following pairs of functions. For the second function in each pair, state the equation of the asymptote, the y-axis intercept and the range. (The x-axis intercepts need not be given.)

a $f: \mathbb{R} \to \mathbb{R}, f(x) = 2^x$ and $g: \mathbb{R} \to \mathbb{R}, g(x) = 2^x + 3$

b $f: \mathbb{R} \to \mathbb{R}, f(x) = 3^x$ and $g: \mathbb{R} \to \mathbb{R}, g(x) = 2 \times 3^x + 1$

c $f: \mathbb{R} \to \mathbb{R}, f(x) = 3^x$ and $g: \mathbb{R} \to \mathbb{R}, g(x) = -3^x + 2$





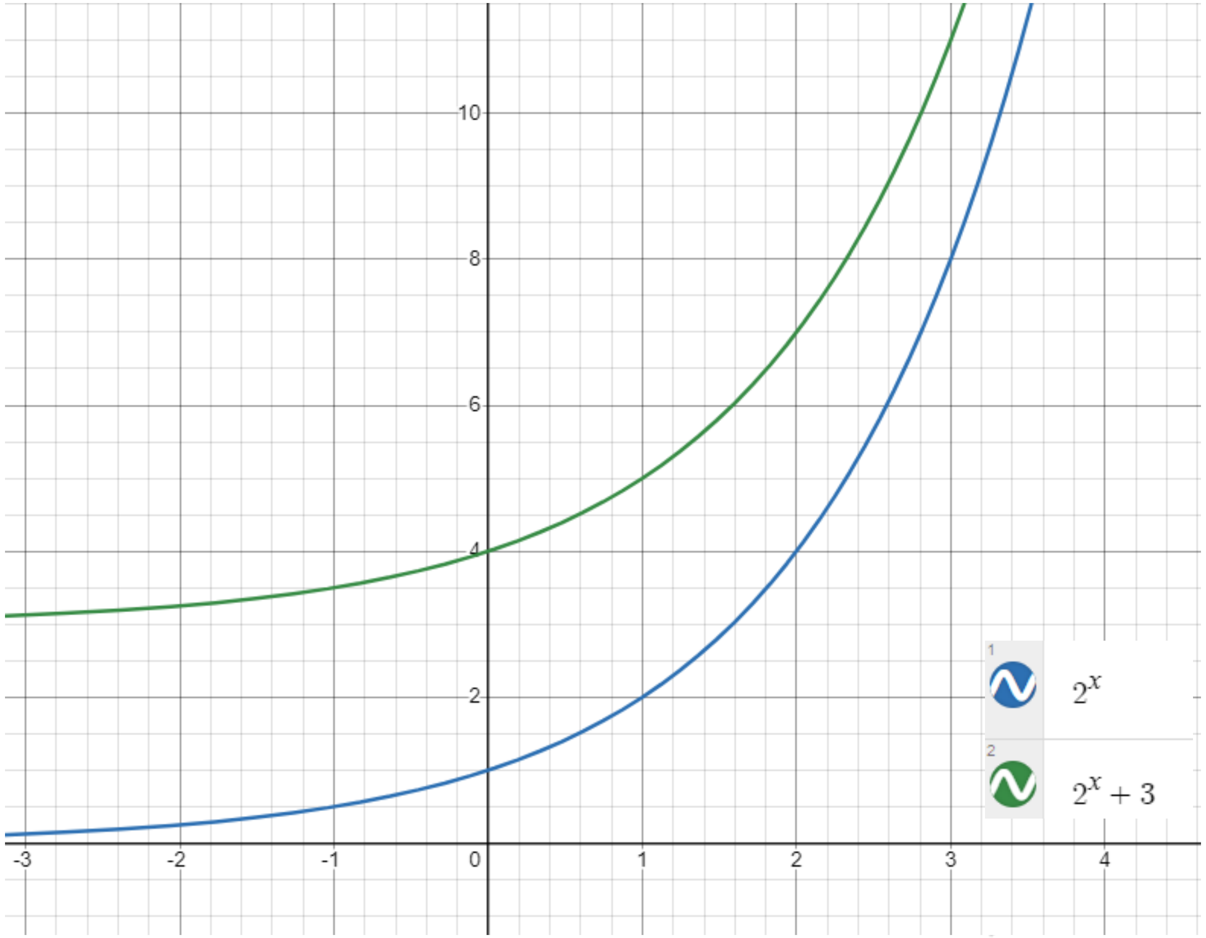
Example 1

Sketch the graphs of each of the following pairs of functions. For the second function in each pair, state the equation of the asymptote, the y-axis intercept and the range. (The x-axis intercepts need not be given.)

a $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 2^x$ and $g : \mathbb{R} \to \mathbb{R}$, $g(x) = 2^x + 3$

b $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 3^x$ and $g : \mathbb{R} \to \mathbb{R}$, $g(x) = 2 \times 3^x + 1$

c $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 3^x$ and $g : \mathbb{R} \to \mathbb{R}$, $g(x) = -3^x + 2$



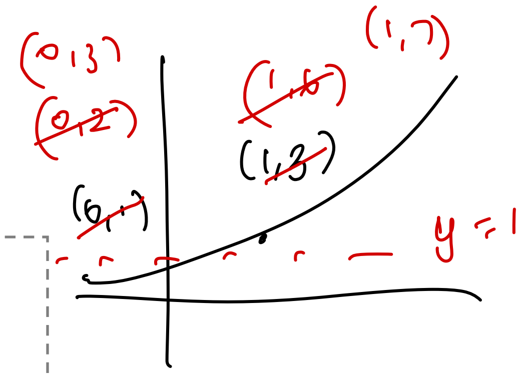
Example 1

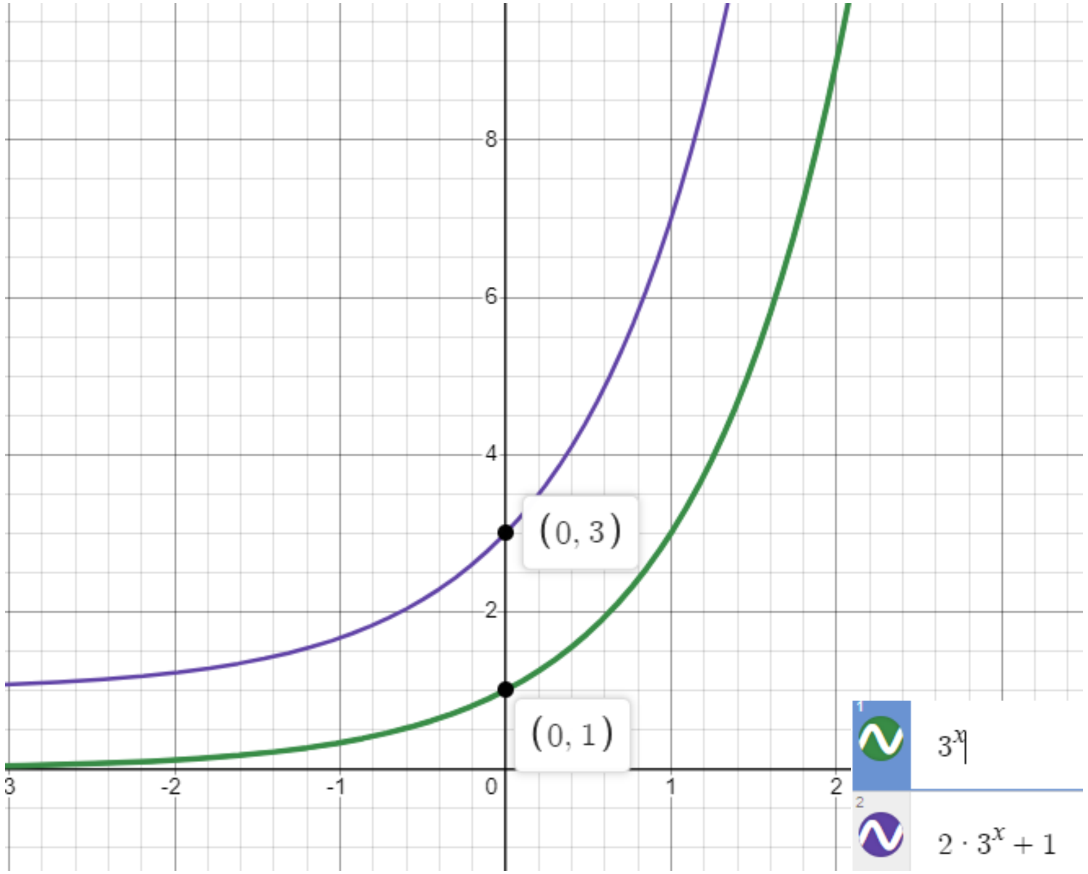
Sketch the graphs of each of the following pairs of functions. For the second function in each pair, state the equation of the asymptote, the y-axis intercept and the range. (The x-axis intercepts need not be given.)

a $f : \mathbb{R} \to \mathbb{R}, f(x) = 2^x$ and $g : \mathbb{R} \to \mathbb{R}, g(x) = 2^x + 3$

b $f : \mathbb{R} \to \mathbb{R}, f(x) = 3^x$ and $g : \mathbb{R} \to \mathbb{R}, g(x) = 2 \times 3^x + 1$

c $f : \mathbb{R} \to \mathbb{R}, f(x) = 3^x$ and $g : \mathbb{R} \to \mathbb{R}, g(x) = -3^x + 2$





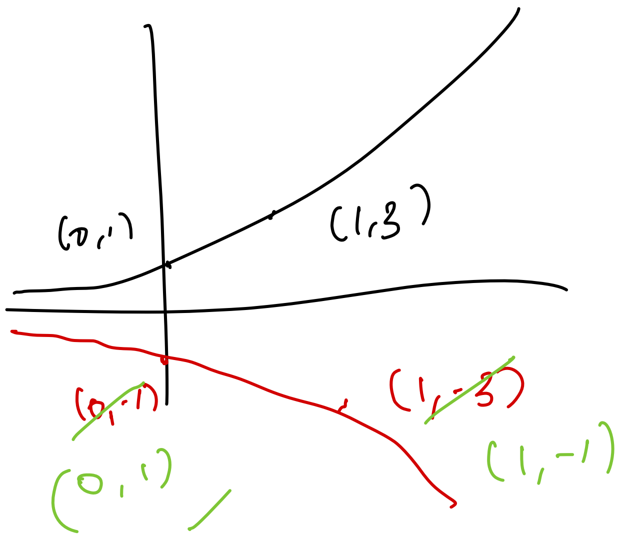
Example 1

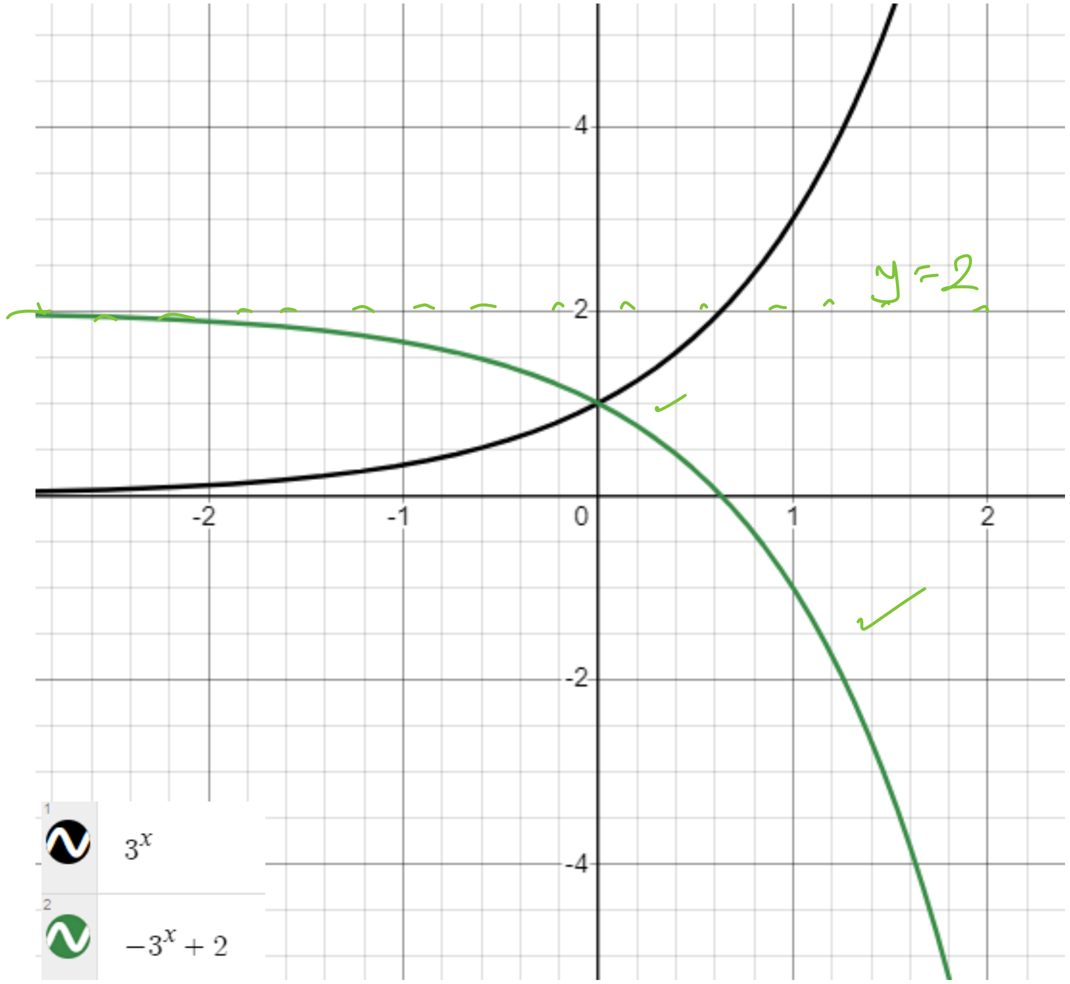
Sketch the graphs of each of the following pairs of functions. For the second function in each pair, state the equation of the asymptote, the y-axis intercept and the range. (The x-axis intercepts need not be given.)

a $f: \mathbb{R} \to \mathbb{R}, f(x) = 2^x$ and $g: \mathbb{R} \to \mathbb{R}, g(x) = 2^x + 3$

b $f: \mathbb{R} \to \mathbb{R}, f(x) = 3^x$ and $g: \mathbb{R} \to \mathbb{R}, g(x) = 2 \times 3^x + 1$

c $f: \mathbb{R} \to \mathbb{R}, f(x) = 3^x$ and $g: \mathbb{R} \to \mathbb{R}, g(x) = -3^x + 2$



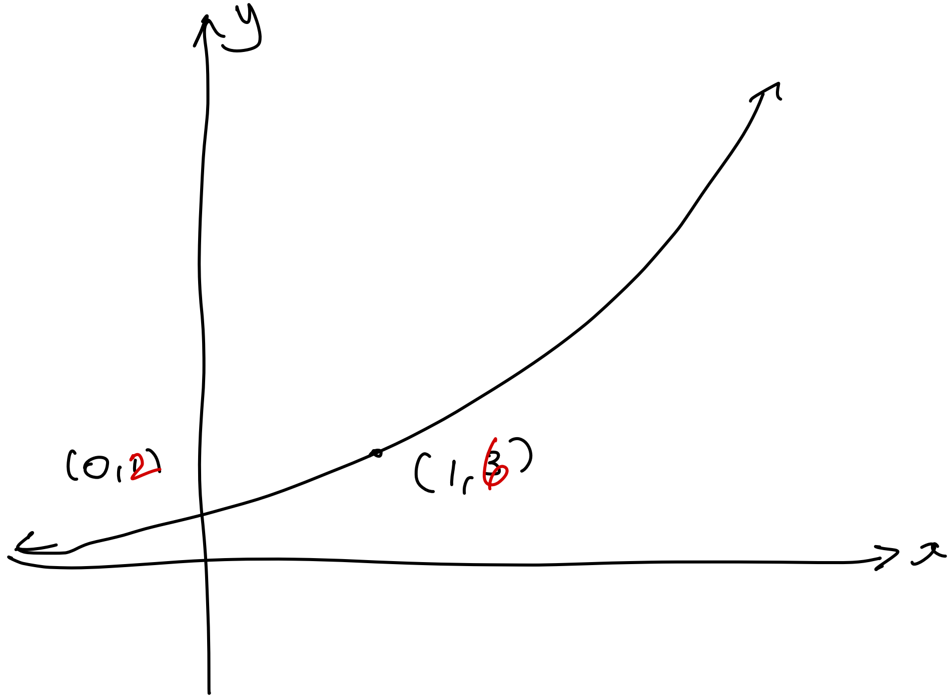


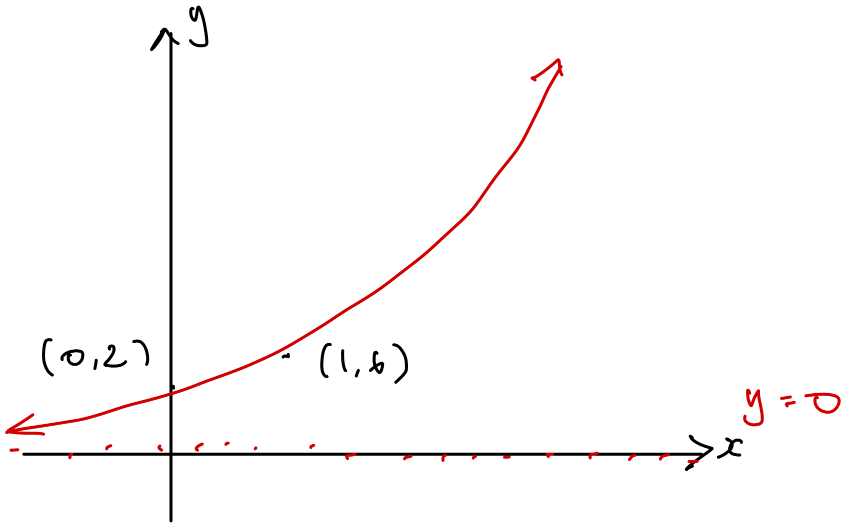
Example 2

Sketch the graph of each of the following:

a $y = 2 \times 3^x$ **b** $y = 3^{2x}$ **c** $y = -3^{-2x} + 4$

a.



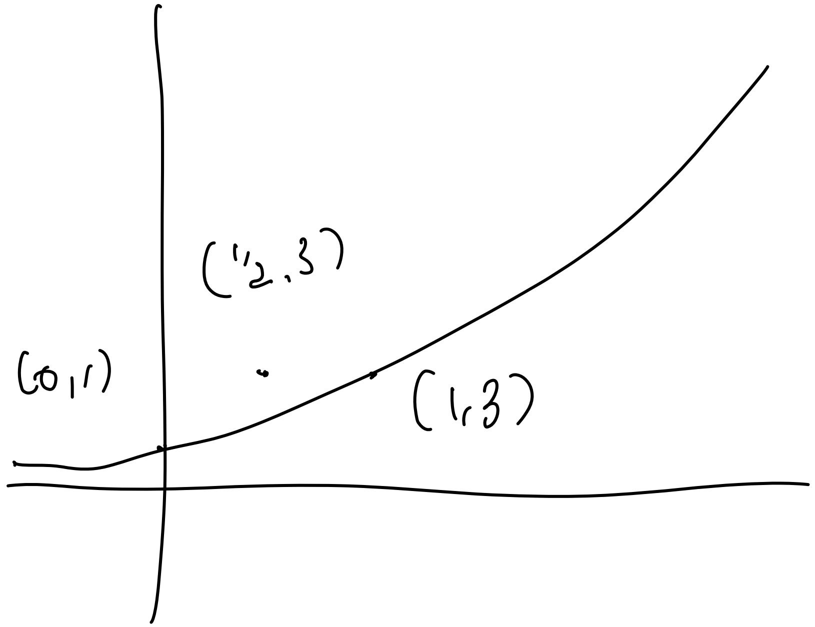


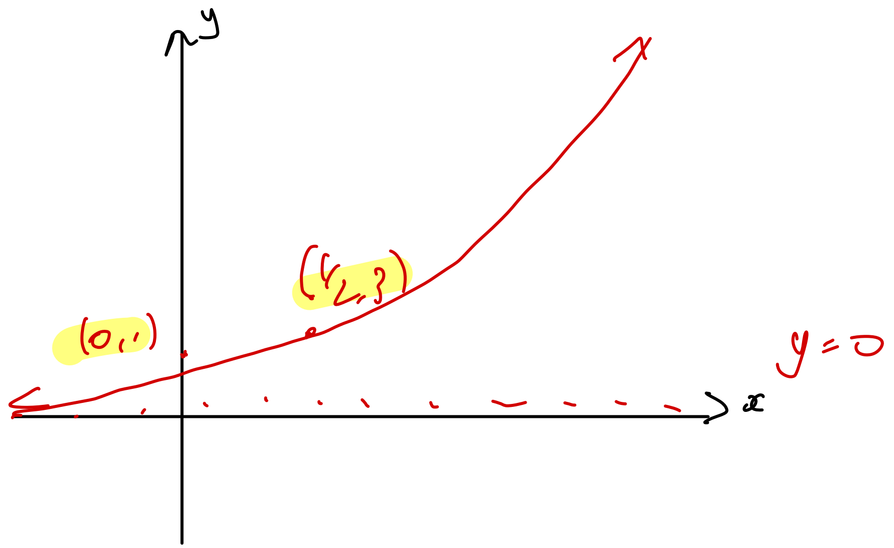
Example 2

Sketch the graph of each of the following:

a $y = 2 \times 3^x$ **b** $y = 3^{2x}$ **c** $y = -3^{-2x} + 4$

$y = 3^{\frac{x}{1/2}}$





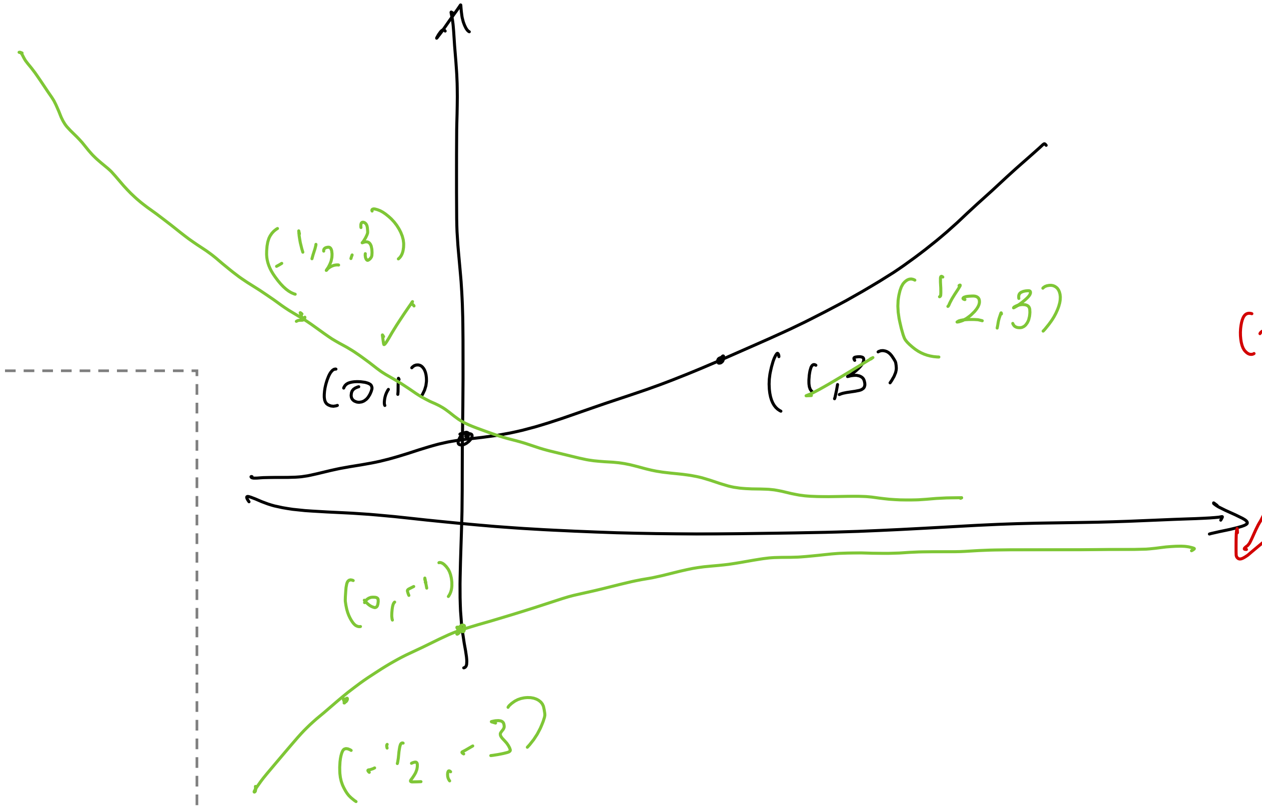
Example 2

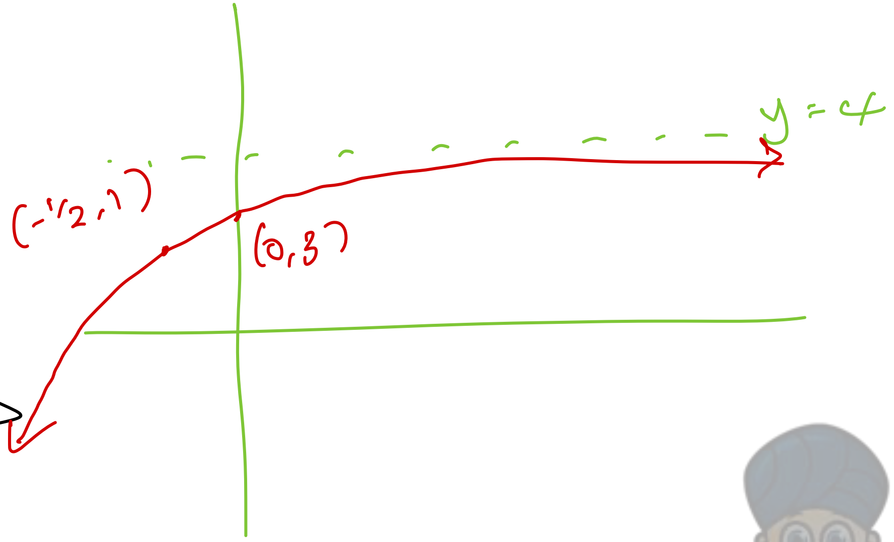
D
R
T

✓ dilate ½ from y

Sketch the graph of each of the following:

a $y = 2 \times 3^x$ **b** $y = 3^{2x}$ **c** $y = -3^{-2x} + 4$





Learning Objectives: Revisited

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be growing exponentially.
- Understand what a graph of an exponential function looks like.
- Understand the important parts of an exponential function.
- Know how to apply transformations to exponential functions

Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 13C
Questions: TBA

www.maffsguru.com