Rational Indices

Year 11
Mathematical Methods

www.maffsguru.com

Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be a rational index.
- Know how to manipulate rational indices.
- Know how to apply the learnings to a range of complex problems.

RECAP

Having looked, in the previous lesson, at the index laws, we can now start to look at rational indices. These have, once again, been covered in previous years, but it doesn't hurt to do a reminder lesson.

What is a rational index?

A rational number is one which can be expressed as a fraction. Or, to borrow a definition from somewhere else:

A rational number is one that is expressed as the ratio of two integers, where the denominator should not be equal to zero.

Examples are:

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\sqrt[2]{x} = x^{1/2}$$

$$\sqrt[3]{x} = x^{1/3}$$

$$\sqrt[4]{x} = x^{1/4}$$

Tricks and more tricks

This might not really be a trick, but it's an interesting thing to note!

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$$

This can also be expressed in another way too!

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$$

$x^a \times x^b = x^{a+b}$

$2 \times \frac{1}{3}$

$a^{2/3} = \left(a^{1/3}\right)^2$ or $\left(a^2\right)^{1/3}$

$(a^b)^c = a^{bc}$

Example 1

Evaluate:

a $(-64)^{\frac{1}{3}}$ **b** $9^{-\frac{1}{2}}$ **c** $16^{\frac{5}{2}}$ **d** $64^{-\frac{2}{3}}$

a. $(-64)^{1/3} = \sqrt[3]{-64} = -4$

b. $9^{-1/2} = \frac{1}{9^{1/2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

c. $16^{5/2} = (16^{1/2})^5$ or ~~$(16^{5})^{1/2}$~~

$= 4^5 = 1024$

d. $64^{-2/3} = \frac{1}{64^{2/3}} = \frac{1}{(64^{1/3})^2} = \frac{1}{4^2} = \frac{1}{16}$

Example 2

Simplify:

a $\dfrac{3^{\frac{1}{4}} \times \sqrt{6} \times \sqrt[4]{2}}{16^{\frac{3}{4}}}$

b $(x^{-2}y)^{\frac{1}{2}} \times \left(\dfrac{x}{y^{-3}}\right)^4$

a. $\dfrac{3^{1/4} \cdot 6^{1/2} \cdot 2^{1/4}}{2^3} = \dfrac{3^{1/4} \cdot 3^{1/2} \cdot 2^{1/2} \cdot 2^{1/4}}{2^3} = \dfrac{3^{3/4} \cdot 2^{3/4}}{2^3}$

$= \dfrac{3^{3/4}}{2^{12/4} \cdot 2^{-3/4}} = \dfrac{3^{3/4}}{2^{9/4}}$

b. $\dfrac{x^{-1} \cdot y^{1/2} \cdot x^4}{y^{-12}}$

$= \dfrac{y^{1/2} \cdot y^{12} \cdot x^3}{x} = y^{25/2} \cdot x^3$

Learning Objectives: Revisited

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be a rational index.
- Know how to manipulate rational indices.
- Know how to apply the learnings to a range of complex problems.

$\sqrt{6} = \sqrt{3 \times 2} = 3^{1/2} \times 2^{1/2}$

Questions to complete

The following are the minimum number of questions you are expected to answer. There is nothing wrong with answering more!

Ex 13B
Questions: TBA

www.maffsguru.com