Graphs and networks

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Specialist Mathematics course.

- Understand how to represent a connections using a graph
- Know what Edge, Vertex, Degree of a vertex and Loop mean
- Know how to describe a graph
    - Simple graph
    - Isolated vertex
    - Degenerate graph
    - Connected graphs and bridges
    - Complete graphs
    - Subgraphs
- Understand what equivalent (or isomorphic) graphs are
- Know what planar graphs are
- Understand what Euler's rule is and how it applied to shapes with face, vertices and edges

Recap of past learning

This is the start of a new topic in the Further Mathematics course looking at graphs, networks and trees. And, as is usual, there is a lot of language which we need to understand first. As the chapter progresses, we are going to be using the language in more and more complex ways.

The good news is, for the most part, you can have this on hand in a summary book (or just remember it!).

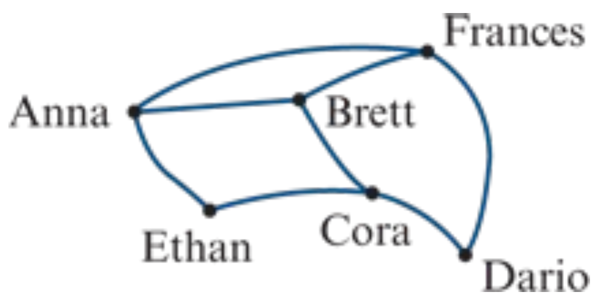
What is a graph?

We live in a connected world. Perhaps it's too connected! Social media keeps those connections alive (and growing) for good and for bad.

When we represent these connections using a diagram we call it a graph!

Note: Barry is, once again, trying to confuse us by using the same word to mean different things. No wonder people find Maths confusing!

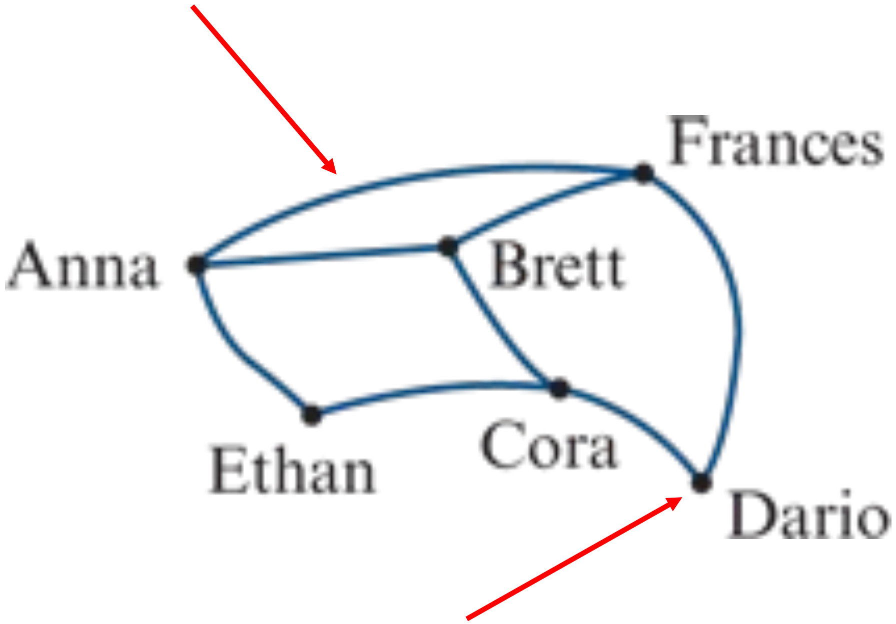
This is an example of a **graph**.



Language used to describe a graph

Ready for the language! For now, you need to remember the following two terms: Edge and vertex

Edge: A line wot connects two vertices

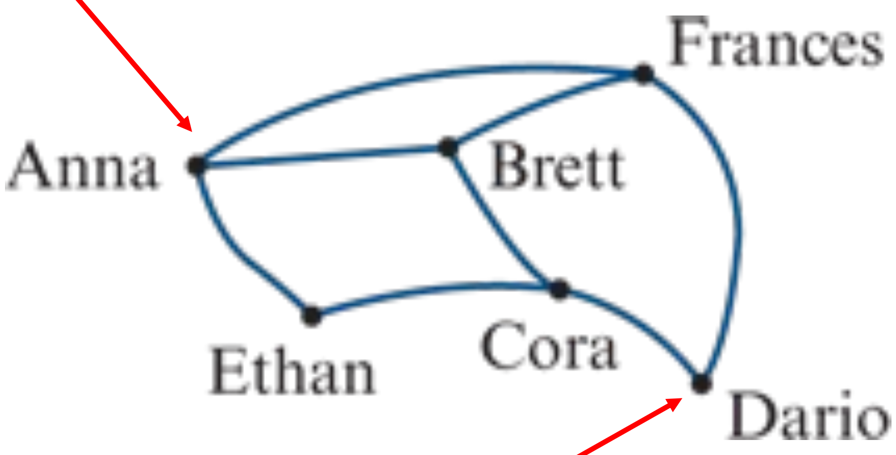


Vertex: A point on the graph

The degree of a vertex

The degree is the number of connections a vertex has coming into it (or going away from it).

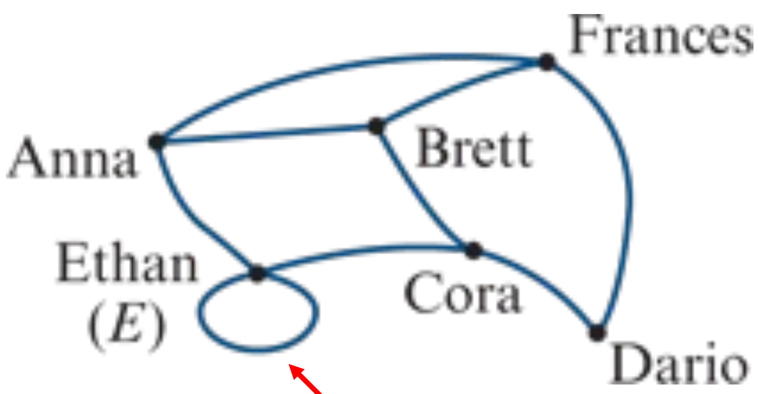
Anna has three friend. Hence, she has three connections. The degree of the vertex is **3.** We can express this as deg(A)=3



Dario has two friend. Hence, he has three connections. The degree of the vertex is **2.** This would be written as deg(2)

Going loopy!

Ethan like to talk to himself, so he has himself as a friend.
So, we draw a loop (because he's loopy lol).
This might be better explained with a road network!

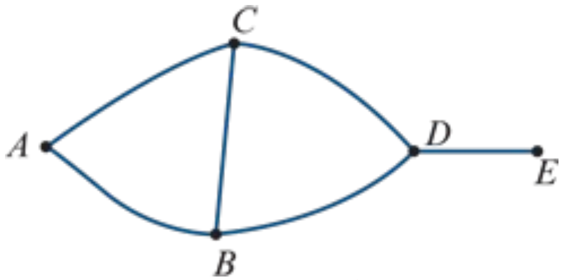


Here is a loop. It describes when there is a connection back to the same vertex

Simple graphs

Different graphs have different properties and so it makes sense to give them names so they can be identified and hence described.

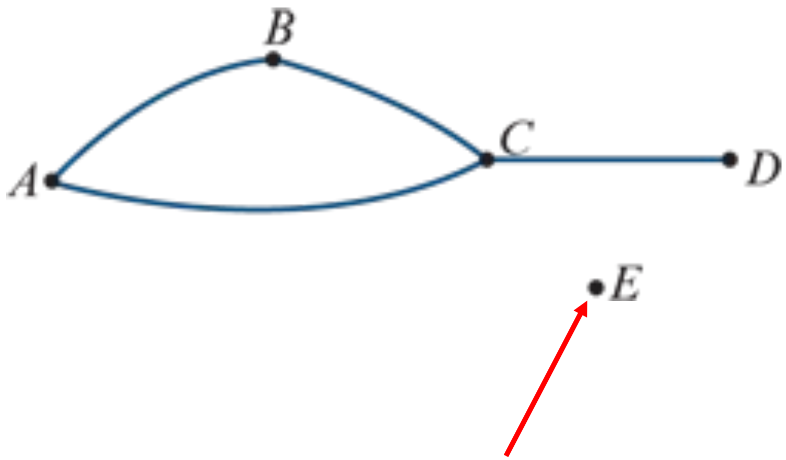
A simple graph doesn't have any loops!



There are no loops; so, it looks pretty easy.

Isolated vertex

When we are feeling isolated we are feeling alone. This is a good way to remember what an **isolated vertex** is. It's a point (or vertex) which has no connections going to it.

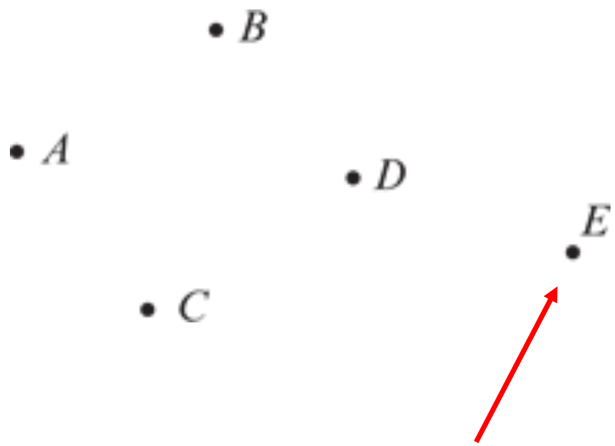


Point E is out there on its own.

Degenerate graphs

I do NOT know where they get the language for this topic as the word degenerate isn't a very nice word … but … in this context it means that all vertices are isolated.

Much like me at a nightclub!



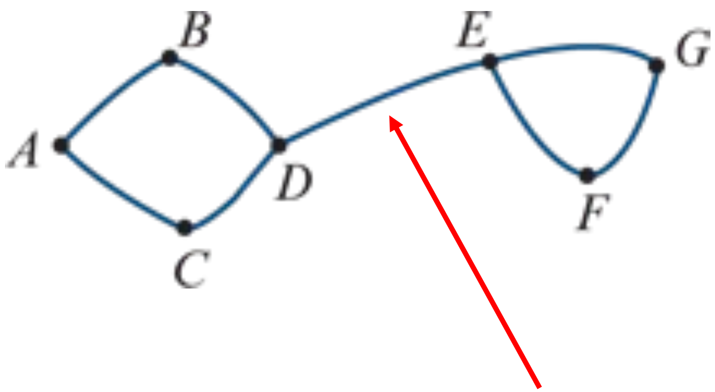
This is me.

Connected graphs and bridges

This is really important! It comes up a lot in exams and tests.

A **connected graph** is one where every vertex is connected.

Between D and E there is a **bridge**.

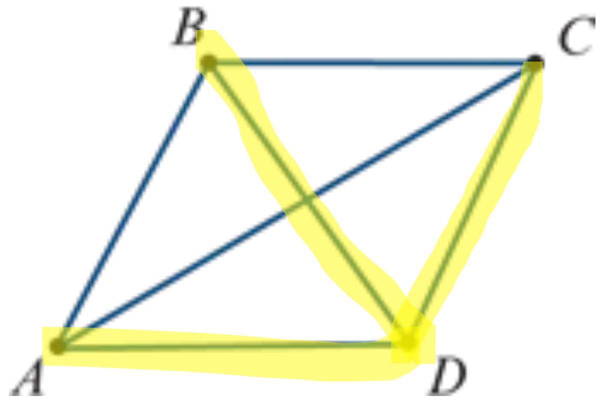


This is a bridge. If the bridge was not there, the graph would be disconnected

Complete graphs

A **complete graph** is where every vertex of a graph is connected to every other vertex using an edge.

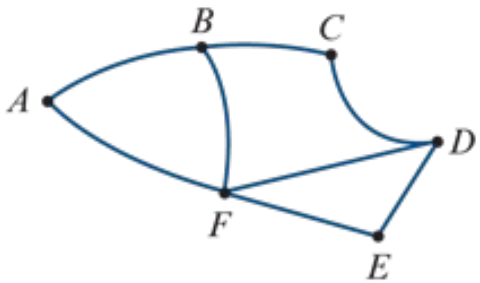
A good example is shown below:



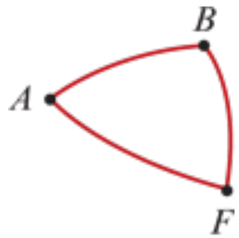
Point D is connected to all other vertices using an edge.

Subgraphs

A subgraph is part of a larger graph. All of the edges and vertices in the subgraph must exist in the original graph. If there are extra edges or vertices, the graph will not be a subgraph of the larger graph

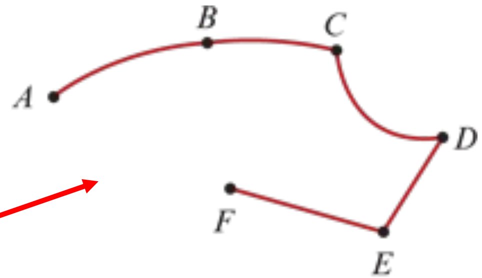


Graph 1



Graph 2

If this is an original graph …



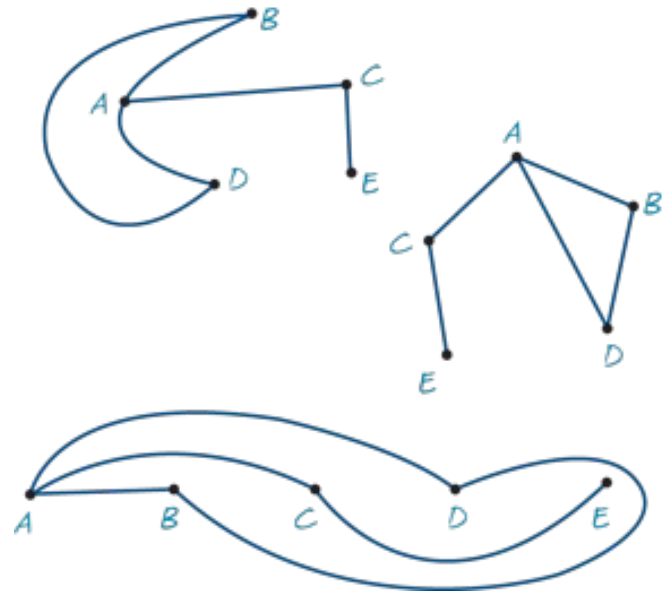
Graph 3

These are subgraphs

Isomorphic (equivalent) graphs

It might seem obvious that certain graphs could be drawn in a number of different ways. They would have the same number of vertices and the **exact** same edges, but seem to look different.

The following are all examples of the same graph, just drawn in a different way.

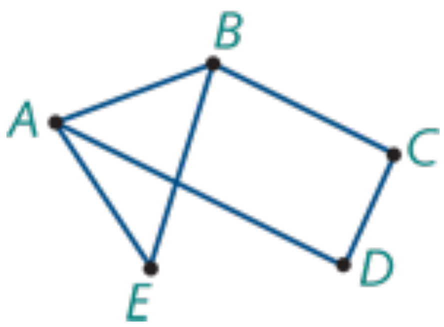


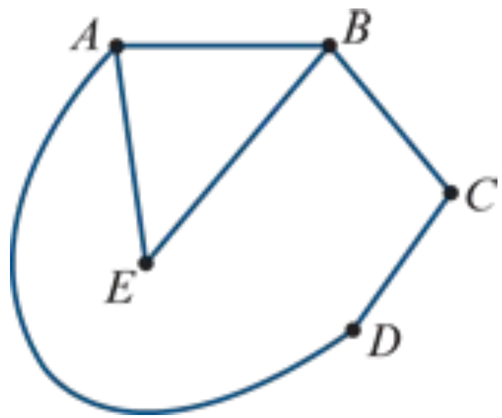
Planar graphs

For some reason, we don't seem to like graphs to have overlapping edges. It might lead us to make mistakes interpreting the data.

Hence, where possible, we try and draw the graphs without having edges which overlap. Graphs like this are called **planar graphs**.

It's not always going to be possible to do this and so we call these graphs, non-planar graphs.





Euler's rule

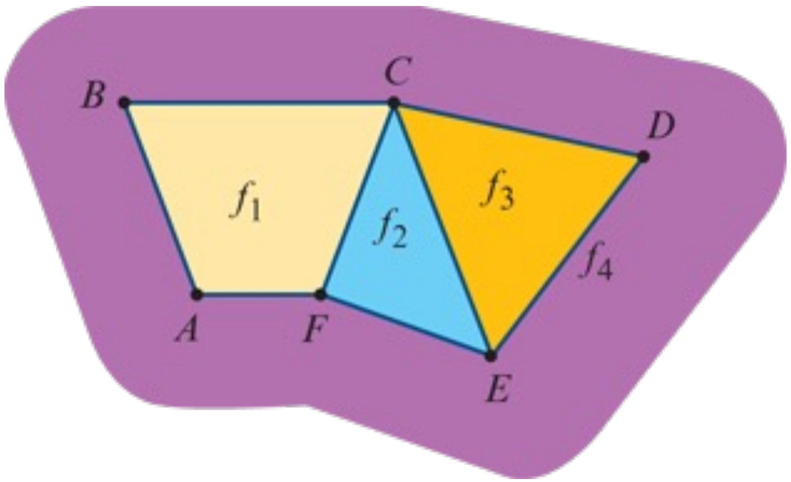
We'll gloss right over who Euler is (for now!).

He basically came up with the following rule:

$$v + f - e = 2$$

I remember this using very fat elephants, but this isn't always the best!

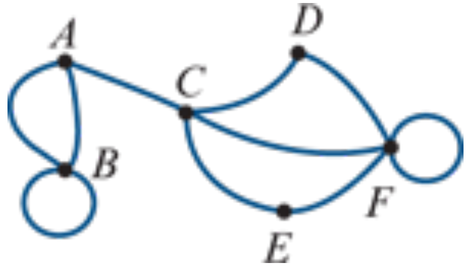
It involves **v**ertices, **f**aces and **e**dges.



Example

A connected graph is shown on the right.

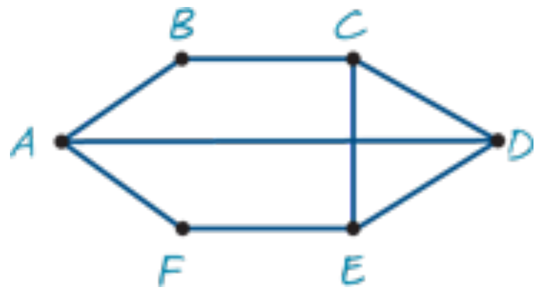
- What is the degree of vertex C?
- Which vertices have a loop?
- What is the degree of vertex F?
- A bridge exists between two vertices. Which vertices are they?
- Draw a subgraph of this graph that involves only vertices A, B and C.



Note: A loop counts as two edges!

Example

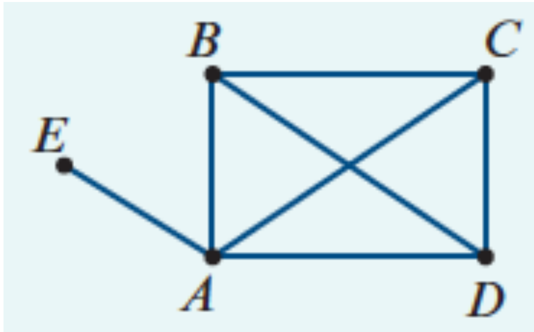
Show that this graph is planar by redrawing it so that no edges cross



Example

For the graph shown on the right:

- redraw the graph into planar form
- verify Euler's rule for this graph.



Example

A connected planar graph has six vertices and nine edges. How many faces does the graph have? Draw a connected planar graph with six vertices and nine edges.

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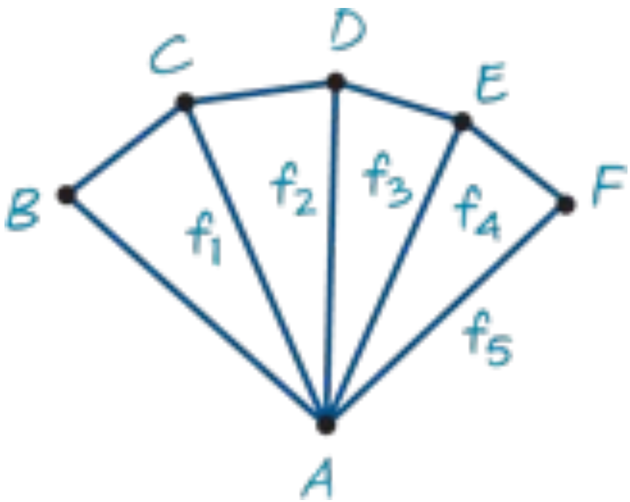


There are other examples which might also work

VCAA Question from 2024 Paper 1

Note: You can choose the hard way or the easy way to do this question!

Question 35

Consider the following graph.

The number of faces is

A. 5

B. 6

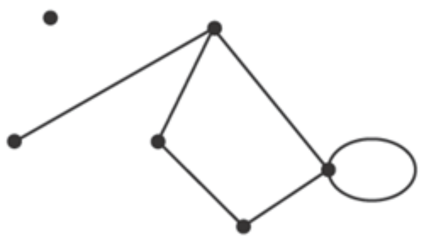
C. 7

D. 8

VCAA Question from 2024 Paper 1

Question 33

Consider the following graph.



The sum of the degrees of the vertices is

A. 10

B. 11

C. 12

D. 13