Transformations of power functions with positive integer index

Sunday, 18 February 2018 5:54 pm

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Understand what functions with positive integer indexes are
- Understand what functions with odd positive integer indexes are and what they look like
- Understand what functions with odd positive integer indexes are and what they look like
- Know how to transform all the above graphs

RECAP:

We have spent a lot of time working with transformations.
We have been given a series of transformations and then been expected to use whatever means we can to find the equations of the transformed function.
We should, with practice, now be in a position to look at a function and know:

- What the base function was
- How it has been transformed

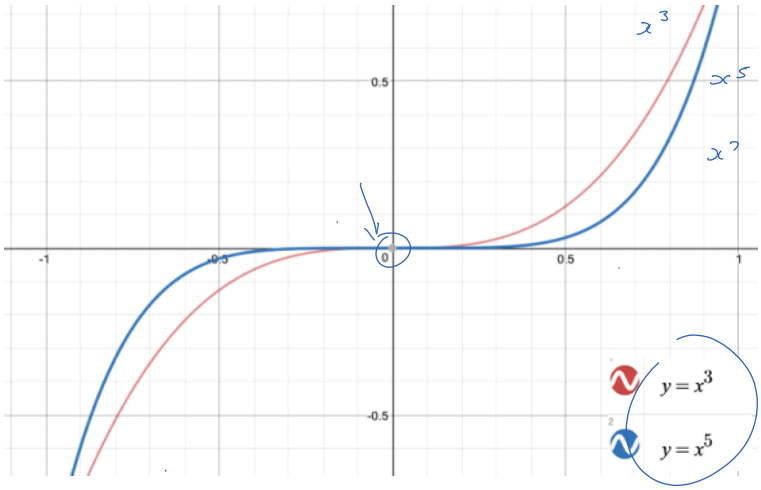
Now we are going to look at Power Functions.
These are functions with positive integer indexes

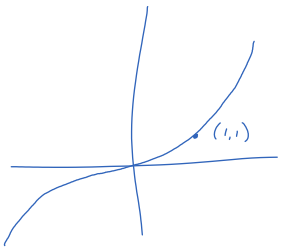
What's an index and integer?

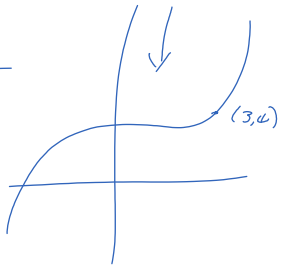
An index is another word for a power!
An integer is a whole number.
Let's continue!

Graphs with positive odd integer indexes

If we compare the graphs of $y = x^3$ and $y = x^5$ we see the following







They have the same basic shape.
They share some of the same characteristic:

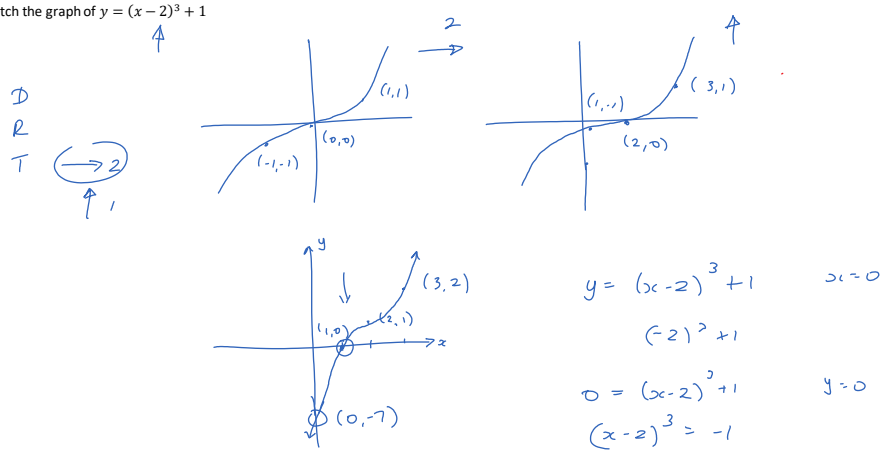
- They are odd functions
- They have common coordinates as $(-1,-1)$ and $(1,1)$ and $(0,0)$

This is awesome! Knowing what the graph looks like, will help us transform it.
To be fair, this is true for all graphs!

Examples of transformations

Example:
Example extracted from *Cambridge Mathematical Methods Units 3 and 4 Textbook*

Sketch the graph of $y = (x-2)^3 + 1$



$y = (x-2)^3 + 1$ $x = 0$

$(-2)^3 + 1$

$0 = (x-2)^3 + 1$ $y = 0$

$(x-2)^3 = -1$

$x - 2 = \sqrt[3]{-1}$

$x = 1$

Remember:
When drawing graphs (at any point in Methods), you must always ensure to label the following:

- x-axis intercept(s) **in co-ordinate form**
- y-axis intercept(s) **in co-ordinate form**
- Turning points (maxima and minima)
- Endpoints (when the domain is limited)
- Equations of any asymptotes

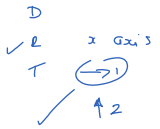
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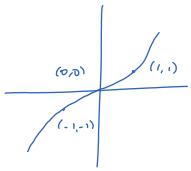
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- y-axis intercept(s) **in co-ordinate form**
- Turning points (maxima and minima)
- Endpoints (when the domain is limited)
- Equations of any asymptotes
- At least one co-ordinate value

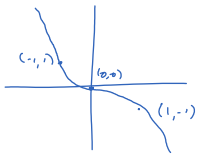
Hence, we can use the CAS (or hand) to find intercepts!

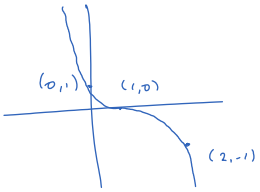
Example 2:
Example extracted from *Cambridge Mathematical Methods Units 3 and 4 Textbook*

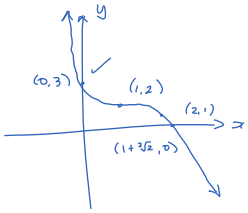
Sketch the graph of $y = -(x-1)^3 + 2$











$y = -(x-1)^3 + 2$

$0 = -(x-1)^3 + 2$

$(x-1)^3 = 2$

$(x-1) = \sqrt[3]{2}$

$x = 1 + \sqrt[3]{2}$

Remember:
When drawing graphs (at any point in Methods), you must always ensure to label the following:

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- Equations of any asymptotes
- At least one co-ordinate value

Hence, we can use the CAS (or hand) to find intercepts!

Turning Mathematics on its head!

Just when we think we are getting our heads around this stuff … they change the game.
Sometimes, they will give you the general form of a graph and some key points and expect you to find the equation of the curv e.

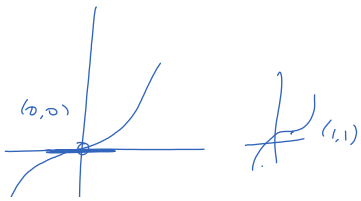
Example:
Example extracted from *Cambridge Mathematical Methods Units 3 and 4 Textbook*

The graph of $y = a(x-h)^3 + k$ has a point of **zero gradient** at $(1,1)$ and passes through the point $(0,4)$. Find the values of a, h and k.

$y = a(x-h)^3 + k$

$h = 1$

$k = 1$



$h = 1$

$k = 1$

$y = a(x-1)^3 + 1$

$x = 0$

$y = 4$

$4 = a(0-1)^3 + 1$

$3 = -a$

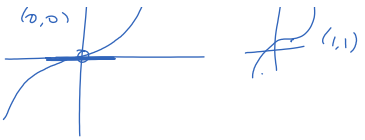
$a = -3$

$a = -3$

$h = 1$

$k = 1$

$\therefore y = -3(x-1)^3 + 1$



Back to the stuff we know!

Then they can go right back and give us transformations as a list again!

Example:
Example extracted from *Cambridge Mathematical Methods Units 3 and 4 Textbook*

Find the rule for the image of the graph of $y = x^5$ under the following sequence of transformations:

- ~~reflection in the y-axis~~
- dilation of factor 2 from the y-axis
- translation 2 units in the positive direction of the x-axis and 3 units in the positive direction of the y-axis.

$y = x^5$

$y = (-x)^5$

$y = -x^5$

$y = -\left(\frac{x}{2}\right)^5$

$\frac{x}{a}$

$x \to \frac{x}{2}$ → 2

$x \to x-2$ $\quad y = -\left(\frac{x-2}{2}\right)^5$

$y \to y-3$ $\quad y-3 = -\left(\frac{x-2}{2}\right)^5$

$y = -\left(\frac{x-2}{2}\right)^5 + 3$

$y = x^5$

$(x, y) \to (-2x+2, y+3)$

$x' \quad y'$

$\therefore x' = -2x+2 \qquad y' = y+3$

$y = x^5$

$y = y' - 3$ $\qquad 2x = -x+2$

$x = \frac{-x'+2}{2}$

$y = x^5$

$y' - 3 = \left(\frac{-x'+2}{2}\right)^5$

$y' = \left(\frac{(-1)x'+2}{2}\right)^5 + 3$

$y = \left[\frac{(-1)(x-2)}{2}\right]^5 + 3$

$y = -\left(\frac{x-2}{2}\right)^5 + 3$