# Solution of exponential equations using logarithms

Saturday, 2 June 2018 5:28 PM

By the end of the teaching of this module I would hope that the following has been covered and understood. Hopefully you will be able to apply the learning to any exam questions:

- Know how to solve exponential equations using logarithms

## RECAP:

We have spent a long time looking at what logs are.
In short, they are a way of turning large numbers into a more sensible scale by using powers.
Logs have bases which effectively means "the power of".

Hence, $\log_{10} 100 = 2$

We know we can simplify logs using a number of rules:

Law 1: $\log_a(mn) = \log_a m + \log_a n$

Law 2: $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

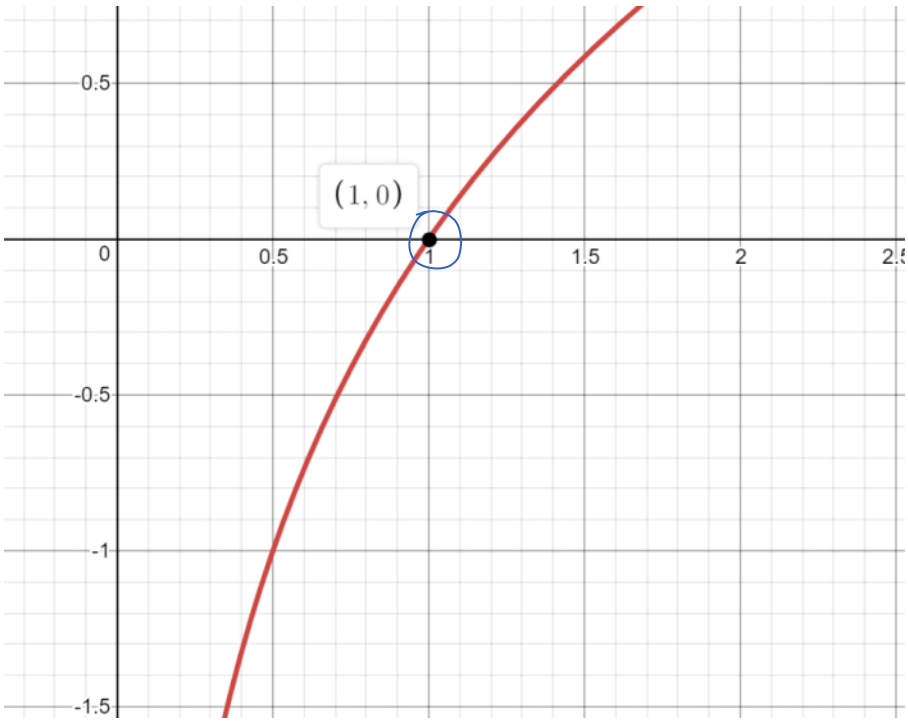
Law 3: $\log_a(m^p) = p \log_a m$

Law 4: $\log_a\left(m^{-1}\right) = -\log_a m$

Law 5: $\log_a 1 = 0$ and $\log_a a = 1$

We know we can graph logs and determine the graphs for logarithmic functions when given coordinates (and the general form).

We know that log graphs (generally) have crossing points at (1,0)




What is really important it to know that there is an asymptote at $x = 0$
And … that the values to the left of the point (1,0) are negative.
This is going to come in very useful when dealing with inequalities!

## Let's get solving.

Using the information from the lessons we can now use the log laws and information to help us solve exponential equations.

The following examples are taken, with permission, from the Cambridge Mathematical Methods Units 3 and 4 textbook.

**Example 1**
If $\log_2 6 = k\log_2 3 + 1$, find the value of k.

$\log_2 \frac{2}{} = 1$

$\log_2 6 = \log_2 3 + \log_2 2$

$\log_2 6 = \log_2 3^k + \log_2 2$

$\log_2 6 = \log_2 (2 \times 3^k)$

$6 = 2 \times 3^k$

$3^k = 3$

$k = 1$

**Example 2:**
Solve for $x$ if $2^x = 11$, express your answer correct to 2 decimal places.

$2^x = 11$

$\log_2 2^x = \log_2 11$

$x = \log_2 11$

$x = 3.46$

**Example 3:**
Solve $3^{2x-1} = 28$ expressing the answer correct to three decimal places

$3^{2x-1} = 28$

$\log_3 3^{(2x-1)} = \log_3 28$

$\Rightarrow (2x-1)\log_3 3 = \log_3 28$

$2x = \log_3 28 + 1$

$x = \frac{\log_3 28 + 1}{2} = 2.017$

**Beware the tricks!**

The Methods exam is testing your understanding and not regurgitation of content.

Hence, the following example looks easy enough … but it's a trick!

**It is always advised that you sketch graphs of functions and understand what it is they are asking you to do in the question.**

**Example 4:**
Solve the inequality, $0.7^x \geq 0.3$

Remember, $\log_{10} x$ has a crossing point at $(1,0)$.

$$0.7^x \geqslant 0.3$$

$$\log_{10} 0.7^x \geqslant \log_{10} 0.3$$

$$x(\log_{10} 0.7) \geqslant \log_{10} 0.3$$

$$x \leqslant \frac{\log_{10} 0.3}{\log_{10} 0.7} \quad \text{!!}$$

$$x \leqslant 3.38$$

