Inverses

Saturday, 2 June 2018 5:29 PM

By the end of the teaching of this module I would hope that the following has been covered and understood. Hopefully you will be able to apply the learning to any exam questions:

- Know how to find the inverse of exponential functions
- Know how to find the inverse of logarithmic functions
- Know how to transpose both exponential and logarithmic functions
- Know how to find the intercepts of inverse functions

RECAP:

In the past lessons we have spent a lot of time looking at how to use the rule for exponentials and logarithms. This lesson we are going to combine two areas of mathematics together by looking at how to find the inverse of functions.

Finding the inverse

We need to ensure that we understand the difference between an inverse relation and an inverse function.

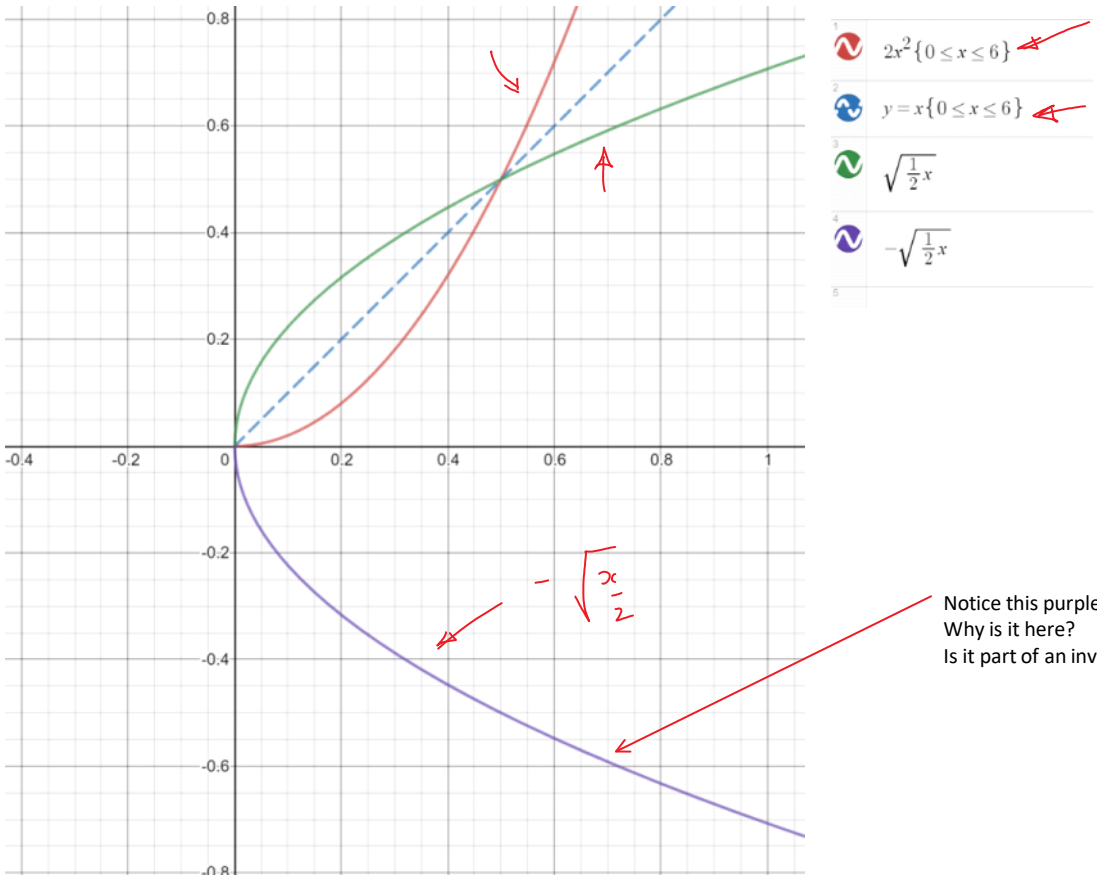
A **function** must pass the vertical line test.
All equations are relations.

All **relations** can have an inverse.
Only **one to one functions** can have an **inverse function**.

One to one functions are those which pass the vertical and horizontal line test.

What is the inverse of a function?

When we are taking the inverse of a function we are looking at reflecting a one to one function in the line $y = x$
This is really important.
An example of a function and it's inverse is shown below.
Note how they are both one-to-one.



$y = 2x^2$

$x = 2y^2$

$\frac{x}{2} = y^2$

$y = \pm\sqrt{\frac{x}{2}}$

$-\sqrt{\frac{x}{2}}$

Notice this purple line.
Why is it here?
Is it part of an inverse?

Knowing which function is the inverse is really important.
So, when taking the inverse we are swapping the x-value and the y-value.

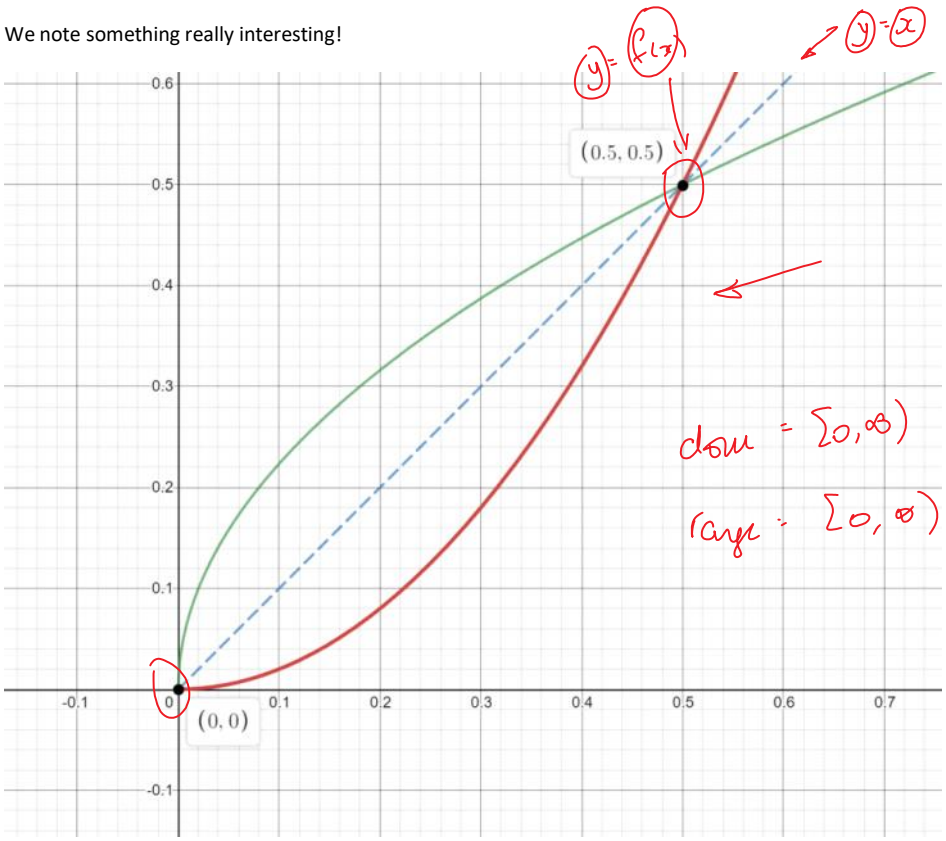
This would then suggest that the domain and range of an inverse if the range and domain of the original function.

Hence: It's important to find the domain and range of an original function to then quickly find the domain and range of its inverse.

Finding the points of intersection

Because the inverse and it's original function might intersect, it's important to know where these intersections are.

We note something really interesting!



Not only does the graph of $f(x) = f^{-1}(x)$
But they seem to meet on the line $y = x$
This means that we can also solve for:

$f(x) = x$

And

$f^{-1}(x) = x$

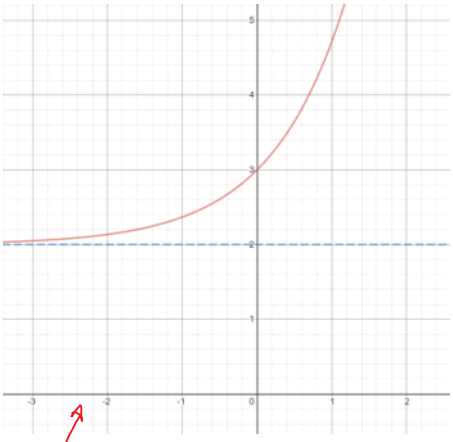
In most cases we would choose to solve $f(x) = x$ as this would generally give the nicest answers.

Finding inverses using exponentials and logs

Finally .. I move onto the exciting stuff!
We have already done the theory for most of the questions either in the RECAP or the previous videos, so let's go straight to the examples.

The following examples have been extracted, with permission, from the Cambridge Mathematical Methods Units 3 and 4 textbook.

Example 1:
Find the inverse of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = e^x + 2$ and state the domain and range of the inverse function.



Dom = $\mathbb{R}$
Ran = $(2, \infty)$

$y = e^x + 2$
$x = e^y + 2$
$x - 2 = e^y$
$\log_e(x - 2) = y$

$f^{-1}: (2, \infty) \to \mathbb{R}, f^{-1}(x) = \log_e(x - 2)$
Dom: $(2, \infty)$
Rage = $\mathbb{R}$

Example 2:
Rewrite the equation $y = 2\log_e(x) + 3$ with x as the subject.

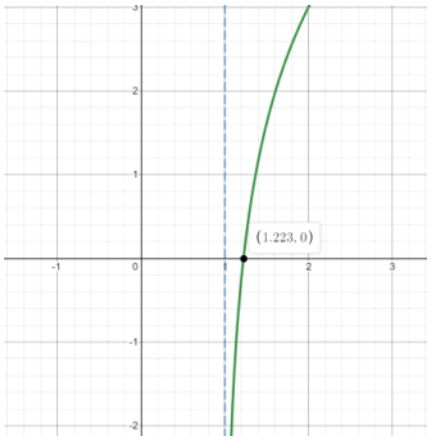
$y = 2\log_e(x) + 3$

$y - 3 = 2\log_e(x)$

$\frac{y-3}{2} = \log_e(x)$

$e^{\left(\frac{y-3}{2}\right)} = x$

Example 3:
Find the inverse of the function $f: (1, \infty) \to \mathbb{R}, f(x) = 2\log_e(x-1) + 3$. State the domain and range of the inverse.



Dom $= (1, \infty)$

Ran $= \mathbb{R}$

$y = 2\log_e(x-1) + 3$

$x = 2\log_e(y-1) + 3$

$x - 3 = 2\log_e(y-1)$

$\frac{x-3}{2} = \log_e(y-1)$

$e^{\left(\frac{x-3}{2}\right)} = y - 1$

$y = e^{\left(\frac{x-3}{2}\right)} + 1$

$x \in \mathbb{R}$

$y \in (1, \infty)$

$f^{-1}: \mathbb{R} \to \mathbb{R}, f^{-1}(x) = e^{\left(\frac{x-3}{2}\right)} + 1$

Example 4:
Rewrite the equation $P = Ae^{kt}$ with t as the subject.

$P = Ae^{kt}$

$\frac{P}{A} = e^{kt}$

$\log_e\left(\frac{P}{A}\right) = kt$

$\log_e\left(\frac{P}{A}\right) = t$

k

$\frac{1}{k} \cdot \log_e\left(\frac{P}{A}\right) = t$

$t = \frac{1}{k} \log_e\left(\frac{P}{A}\right)$

Using the CAS to help you!

Remember that this is a CAS course and there is a good chance the CAS can do this for you!

E.g. Find the inverse of the function $f: (1, \infty) \to \mathbb{R}, f(x) = 2\log_e(x-1) + 3$. State the domain and range of the inverse.

Just use the menu commands: **Interactive** -> **Equation/Inequality** -> **Solve**
Remember to highlight the function first and to change the x and y in the equation and make y the subject!