Differentiation of e^x

Monday, 29 April 2019 8:56 PM

By the end of the teaching of this module I would hope that the following has been covered and understood. Hopefully you will be able to apply the learning to any exam questions:

- Know how to differentiate $y = e^x$
- Know how to use the chain rule to differentiate more complex exponential powers
- Know how to apply the above to questions relating to gradient of lines

RECAP:

We have, in previous videos, looked at how to differentiate a range of polynomials.
We looked at how to do it from first principles and then, how to use "short cut" methods to differentiate.
We understood that the process of differentiating was to find the value of the **gradient of a tangent to a point**.
We also looked at how to use the chain rule to solve more complex differential problems.

We now return to this topic and build on the knowledge we have of exponentials and logarithms.

RECAP: Basic Differentiation

$f(x) = x^n$

$f'(x) = y' = \frac{dy}{dx} = n x x^{n-1}$

$y = x^3$

$y' = 3x^2$

$y = 4x^3 - 3x^2 + x + 2$

$y' = 12x^2 - 6x + 1$

x^1

$1x^0$

RECAP: The Chain Rule

$y = (x^2+3)^4$

$u = x^2 + 3$

$y = u^4$

$\frac{du}{dx} = 2x$

$\frac{dy}{du} = 4u^3$

$y' = 4(x^2+3)^3 \times 2x$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 4(x^2+3)^3(2x)$

$= 8x(x^2+3)^3$

The differential of $y = e^x$

$y = e^x$

$y' = e^x$

Using the chain rule with more complex exponential functions

It's always good to learn using examples.
The following examples have been taken, with permission, from the *Cambridge Mathematical Methods Units 3 and 4 Textbook* series.

Example 1

Find the derivative of each of the following with respect to x:

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a e^{3x}

b e^{-2x}

c e^{2x+1}

d $\frac{1}{e^{2x}} + e^{3x}$

a) $y = e^{3x}$

$y = e^u \quad u = 3x$

$\frac{dy}{du} = e^u \quad \frac{du}{dx} = 3$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{3x} \times 3 = 3e^{3x}$

b) $y = e^{-2x}$

$y' = -2.e^{-2x}$

c) $y = e^{2x+1}$

$y' = 2e^{2x+1}$

d) $\frac{1}{e^{2x}} + e^{3x}$

$y = e^{-2x} + e^{3x}$

$y' = -2e^{-2x} + 3e^{3x}$

$y' = \frac{-2}{e^{2x}} + 3e^{3x}$

Example 2:

Find the derivative of each of the following with respect to x:

a e^{x^2}

b e^{x^2+4x}

a) $y = e^{x^2}$

$y' = 2x.e^{x^2}$

b) $y = e^{(x^2+4x)}$

$y' = (2x+4)e^{(x^2+4x)}$

Example 3:

Find the gradient of the tangent to the curve $y = e^{2x} + 4$ at the point:

a $(0, 5)$

b $(1, e^2 + 4)$

a) $y = e^{2x} + 4$

$y' = 2e^{2x}$

$\therefore x = 0 \quad \therefore y' = m = 2e^0$

$= 2$

b) $y' = 2e^{2x}$

$y' = 2e^2$

Example 4:

For each of the following, first find the derivative with respect to x. Then evaluate the derivative at $x = 2$, given that $f(2) = 0$, $f'(2) = 4$ and $f'(e^2) = 5$.

a $e^{f(x)}$

b $f(e^x)$

a) $y = e^{f(x)}$

$y' = \frac{dy}{dx} = f'(x) . e^{f(x)}$

$x = 2$ $= f'(2) . e^{f(2)}$

$= 4 . \cancel{e^0}\ 1$

$= \underline{\underline{4}}$

b) $y = f(e^x)$ $u = e^x$

$y = f(u)$ $\frac{du}{dx} = e^x$

$\frac{dy}{du} = f'(u)$

$\frac{dy}{dx} = f'(e^x) . e^x$

$x = 2$ $\frac{dy}{dx} = f'(e^2) . e^2$

$= 5 e^2$

$= \underline{\underline{5e^2}}$