

Determining the rule for a function from its graph

Tuesday, 20 February 2018 11:45 AM

★ By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Understand how to find the equation from a graph by reading the key points
- Understand that graphs have common forms of expression which can help us with this.

RECAP

We are nearing the end of this section.

We have been looking at how to draw graphs and how to move them around by combining:

- Dilations
- Reflections
- Translations

We have also looked at a number of ways of doing this:

- Algebraically
- Graphically
- Using short cuts

Sometimes, they will give us a graph and expect us, using key points given, to find the equation of the graph. This isn't as hard as it seems as they will generally give us the form of the equation.

It then becomes a trivial matter of substituting in some numbers and using the CAS to solve.

Example

Example extracted from *Cambridge Mathematical Methods Units 3 and 4 Textbook*

The points (1, 5) and (4, 2) lie on a curve with equation $y = \frac{a}{x} + b$. Find the values of a and b .

$$y = \frac{a}{x} + b$$

$x \ y$
(1, 5)

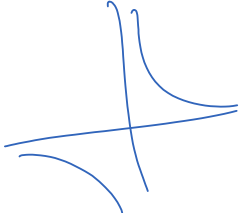
$$5 = \frac{a}{1} + b \quad (1)$$

$x \ y$
(4, 2)

$$2 = \frac{a}{4} + b$$

$$8 = a + 4b \quad (2)$$

$$y = \frac{a}{x} + b$$

$$y = a(x+b)(x+c)$$


$$\begin{aligned} a+b &= 5 \\ -a+4b &= 8 \\ \hline 3b &= 3 \\ b &= 1 \\ a &= 4 \end{aligned}$$

$a = 4$
 $b = 1$

Example

Example extracted from *Cambridge Mathematical Methods Units 3 and 4 Textbook*

The points (2, 1) and (10, 6) lie on a curve with equation $y = a\sqrt{x-1} + b$. Find the values of a and b .

(x, y)
(2, 1)

$$1 = a\sqrt{2-1} + b$$

$$1 = a + b \quad (1)$$

(x, y)
(10, 6)

$$6 = a\sqrt{10-1} + b$$

$$6 = 3a + b \quad (2)$$



$$\begin{aligned} a+b &= 1 \\ -3a+b &= 6 \\ \hline 2a &= -5 \\ a &= -\frac{5}{2} \\ b &= \frac{3}{2} \end{aligned}$$

$a = -\frac{5}{2}$
 $b = \frac{3}{2}$

$$\frac{15}{2} - \frac{3}{2} = \frac{12}{2}$$

$$a = \frac{5}{2}$$

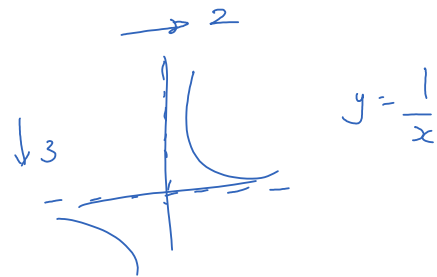
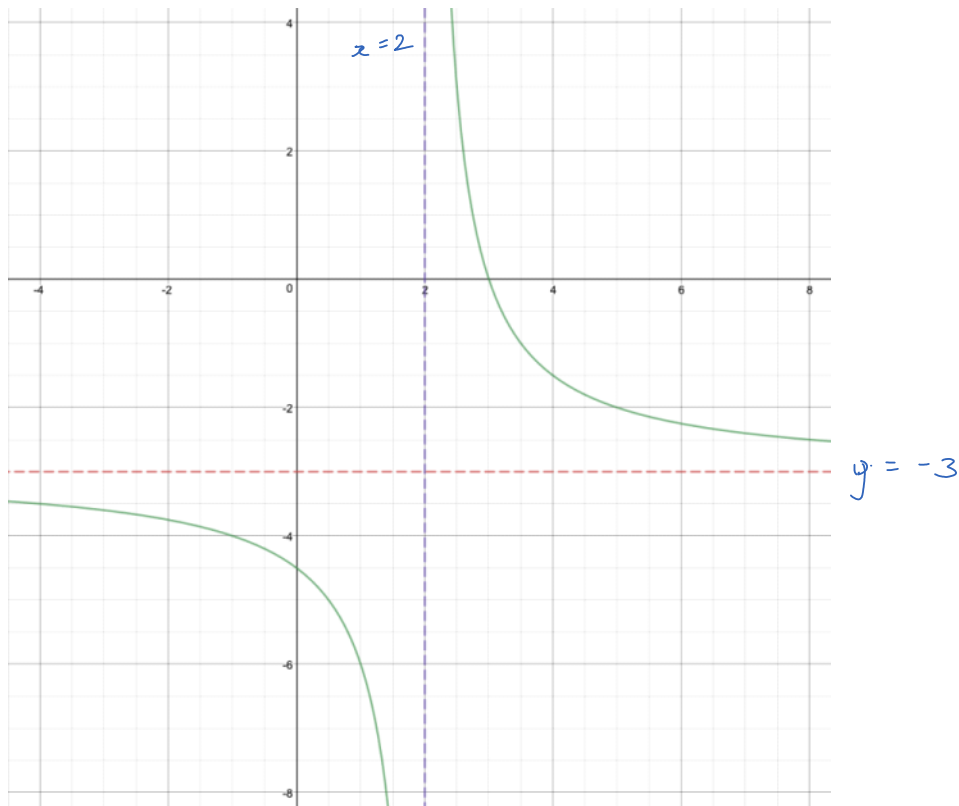
$$a + b = 1 \quad \frac{5}{2} + b = 1$$

Asymptotes!

Those pesky asymptotes can cause some headaches!

However, they can also point us in the right direction to make life easier for ourselves.

Look at the following graph:



If we know it's of the form:

$$f(x) = \frac{A}{x+B} + C$$

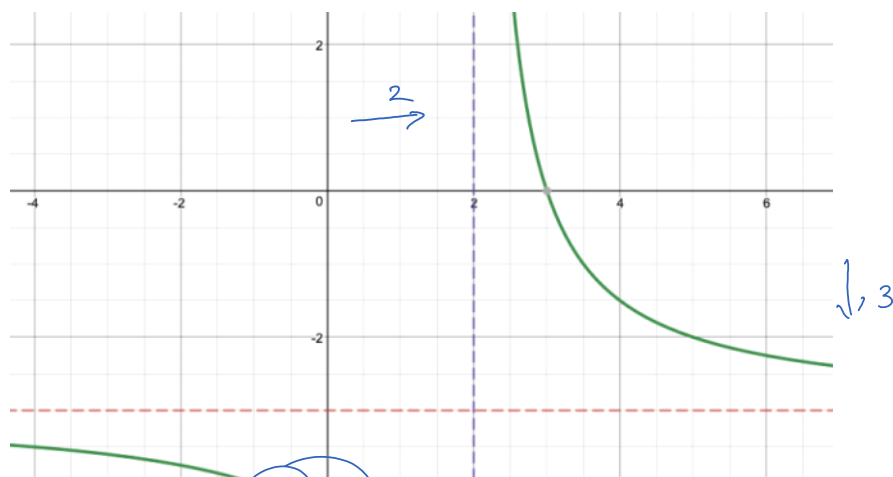
$$f(x) = \frac{A}{x+B} + C$$

What can the graph help us with?

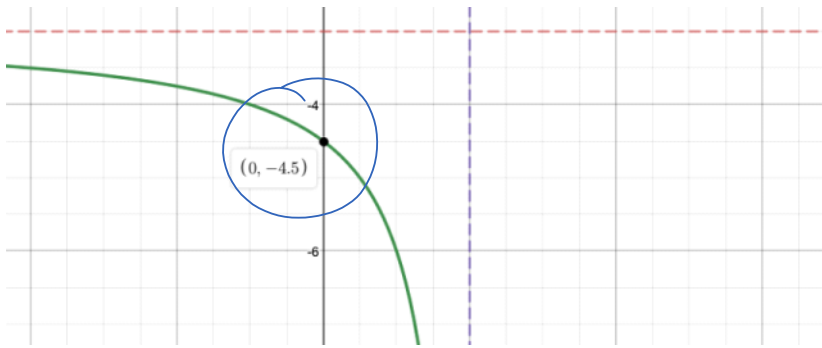
I'm thinking the value of B for a start!

Then the value of C

The last thing I need is the value of C ... and for this I need a point.



$$y = \frac{A}{x-2} - 3$$



Always use the knowledge of what the "base" graphs looks like and where the asymptotes have moved to.