Determining Transformations

Wednesday, 14 February 2018 7:40 pm

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Understand that what you can do forwards you can do backwards too!
- Know how to take a transformed function and find the sequence of transformations which led from a base graph to the transformed function

RECAP:

In previous videos we have worked in a way that we have been given a series of transformations and an "original function" and have used algebra, or short cuts or graphs to help us work out the equation of the image after the transformations have all been applied.

Well, what can be done forwards, can be done backwards.
This is more likely to be in an exam or SAC than something which you have to do all the time.
This can be done using either algebra, or short cuts. Or, with a CAS!
I (personally) prefer to the use the algebra.
But there is no given way of being able to do them.

Doing it backwards ...

Example 1
Find a sequence of transformations which takes the graph from $y = x^2$ to the graph of $y = 3(x+3)^2 - 2$

The trick here is to make the transformed function look as close to the original function as you can.

$y = x^2$ orig

$(x, y) \rightarrow (x - 3, 3y - 2)$

x' y'

$y = 3(x+3)^2 - 2$

image (x', y')

$y' = 3(x'+3)^2 - 2$

$y = x^2$

$y' + 2 = 3(x'+3)^2$

$\frac{y'+2}{3} = (x'+3)^2$

$y = x^2$

$y = \frac{y'+2}{3}$ $\quad x = x' + 3$

$3y = y' + 2$ $\quad x' = x - 3$

$3y - 2 = y'$

$(x - 3, 3y - 2)$

Then write down the sequence of transformations. Remember, when the order isn't given you must express them in the order of DR T.

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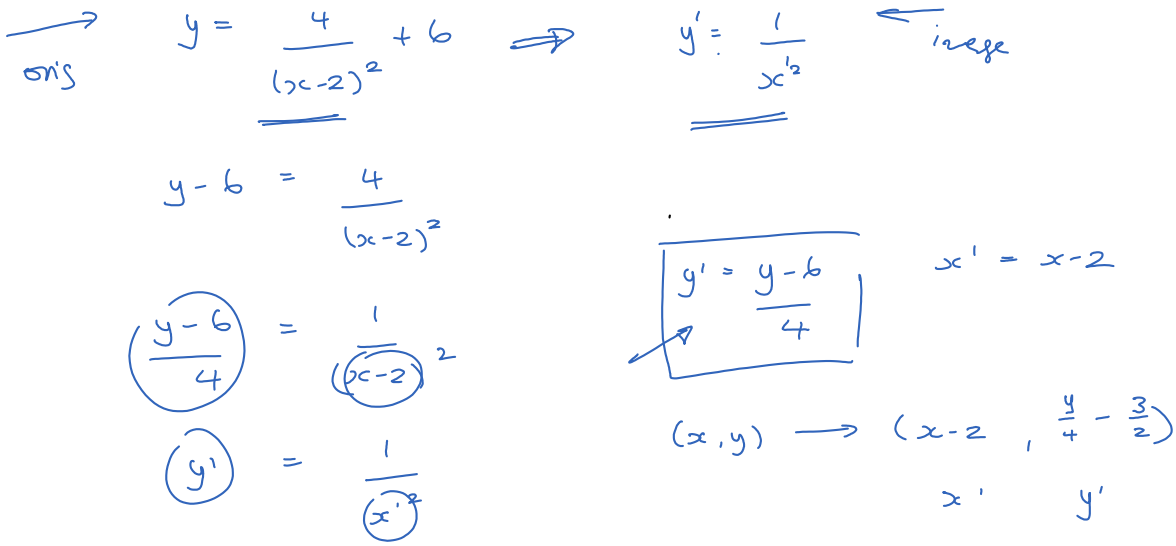
Answer:

Dilation factor 3 from x-axis
Translation of 3 units in the negative direction of the x axis
Translation of 2 units in the negative direction of the y-axis

Example 2

Find a sequence of transformations which takes the graph of $y = \frac{4}{(x-2)^2} + 6$ to $y = \frac{1}{x^2}$



Then write down the sequence of transformations. Remember, when the order isn't given you must express them in the order of DR T.

Answer:

Dilation factor $\frac{1}{4}$ from x axis
Trans. 2 units neg direction of x-axis
Trans $\frac{3}{2}$ units neg direction of y-axis

Example:

Extracted from the Cambridge Mathematical Methods Units 3 and 4 Textbook

Find a sequence of transformations which takes the graph of $y = (5x - 1)^2 + 6$ to the graph of $y = x^2$.

$y = (5x - 1)^2 + 6$ $\quad y' = x'^2$

$y - 6 = (5x - 1)^2$

$y' = (x')^2$

$y' = y - 6 \qquad x' = 5x - 1$

$(5x - 1, \; y - 6)$

Dil factor 5 from y axis

Trans ← 1

Trans ↓ 6

Example:

Find a sequence of transformations which takes the graph of $y = \frac{4}{(x-2)^2} - 6$ *to* $y = \frac{1}{x^2}$

Note:

There are LOTS of ways of doing this!

- Using algebra
- Using logic
- Using reverse transformations (but need to be careful here!)

$y \to 4y$

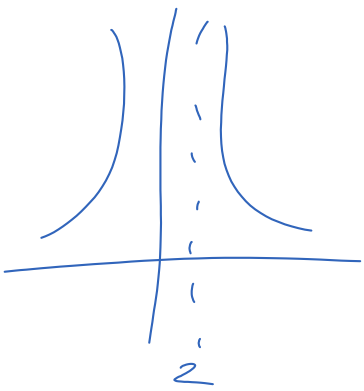
$y \to \frac{y}{a}$

$y \to 4y \to \frac{y}{\frac{1}{4}}$

$4y = \frac{1}{(x-2)^2} - \frac{6}{4} \longrightarrow y = \frac{1}{x^2}$

$y = \frac{1}{(x-2)^2} \left(-\frac{6}{4}\right)$

$\times 4$



Dilation factor $\frac{1}{4}$ from x axis

Trans 2 ←

Trans $\frac{3}{2}$ ↑

$$y = \frac{4}{(x-2)^2} - 6$$