Standard scores

Thursday, 21 February 2019 6:32 pm

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Know what it means to be a "standard score"
- How to calculate standard scores
- How to use standard scores to compare performance.

RECAP:

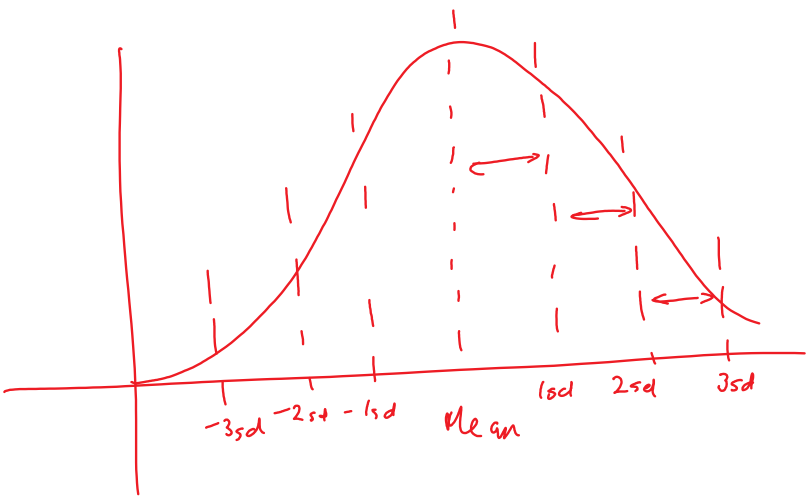
This section of the course has been dealing with measures of centre and measures of spread.
In the last lesson we looked at the normal distribution and the 68-95-99.7% rule.
We have also looked at the idea of standard deviations and how they are used to split populations into sections.
We can use the sections to help us describe data.

This lesson we're going to look at something called a Standard Score.
These are also called "Z" Scores.
I have no idea why!

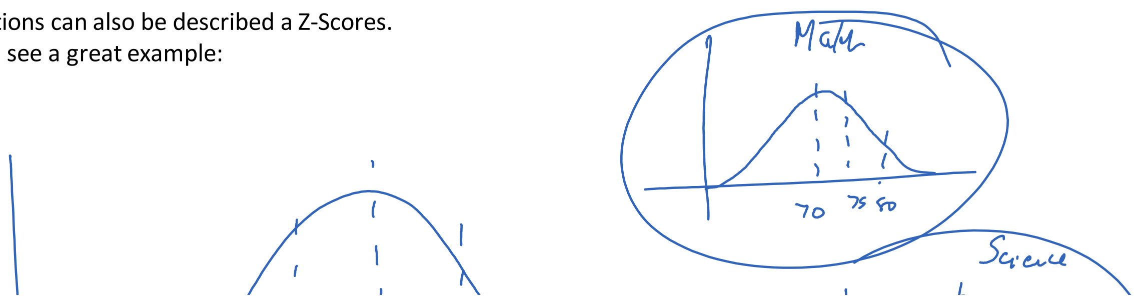
The good news is ... you have already met them!

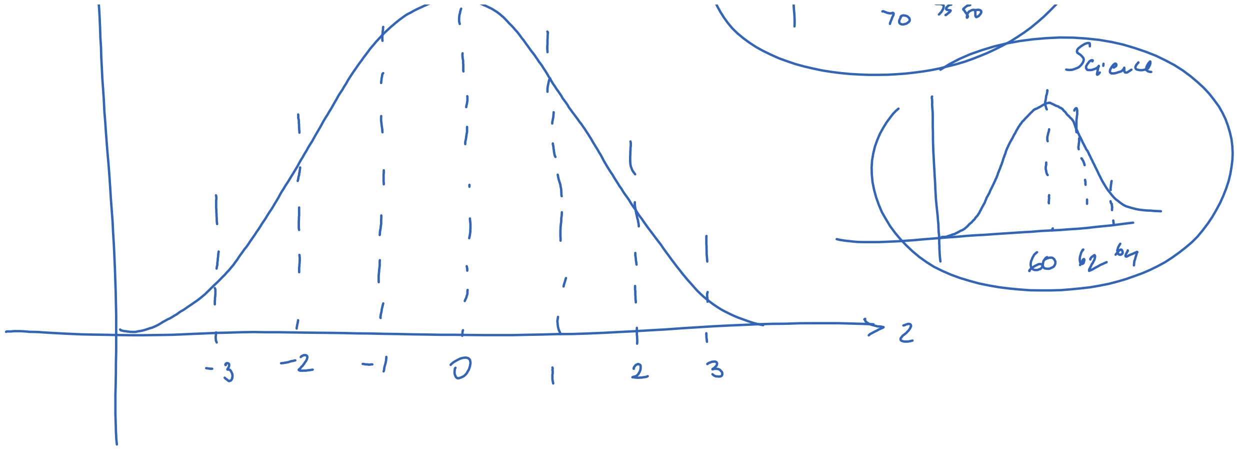
RECAP: Normal distribution.

Remember, the shape of the Normal Distribution?
We took it and then split it into sections.
We said these sections related to standard deviations about the mean score.



These standard deviations can also be described a Z-Scores.
Looking below we can see a great example:





We need a way to translate **standard deviations** into **z-scores.**
Let's look at a previous example from the *Cambridge Further Mathematics Textbook Series*.

The distribution of delivery times for pizzas made by House of Pizza is approximately normal, with a mean of 25 minutes and a standard deviation of 5 minutes.

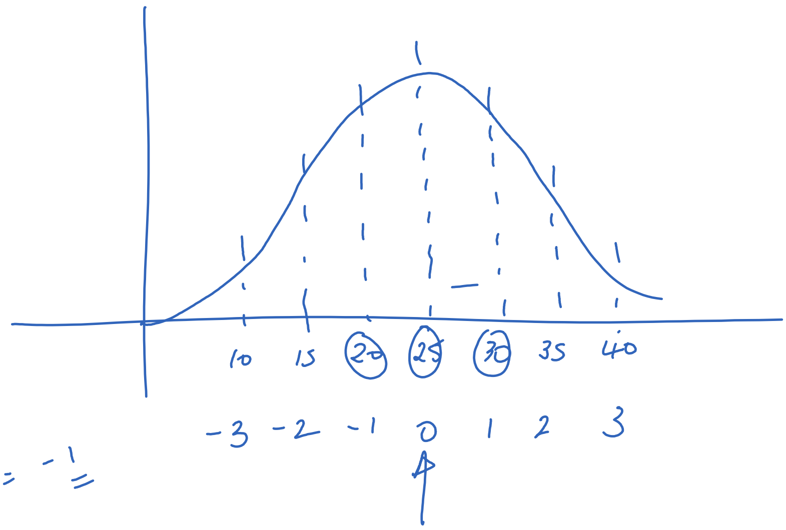
a What percentage of pizzas have delivery times of between 15 and 35 minutes?

b What percentage of pizzas have delivery times of greater than 30 minutes?

c In 1 month, House of Pizza delivers 2000 pizzas. How many of these pizzas are delivered in less than 10 minutes?

Notice how they gave us the mean and the standard deviation.
We would normally draw the following graphs marking on the points!

$$z = \frac{20-25}{5} = \frac{-5}{5} = \underline{\underline{-1}}$$



$$z = \frac{40-25}{5} = \frac{15}{5} = \underline{\underline{3}}$$

We need to find a way, when given the mean and the standard deviation to be able to convert this into a Z Score (and the other way around!).

Barry has been kind to us, and come up with a great little formula:

$$\underset{z}{Standard\ Score} = \frac{actual\ score - mean}{standard\ deviation}$$

$$Standard\ Score = \frac{actual\ score - mean}{standard\ deviation}$$

Whilst he's been kind in one way, he's also thrown in a different version of the formula too:

$\bar{x}$ =

$$z = \frac{x - \bar{x}}{s}$$

Where z is the Z Score
Where x is the score we are looking to convert
Where $\bar{x}$ is the **mean**
Where s is the **standard deviation**.

The words in bold will generally be given to you in the question!

Sometimes, they might give you the z score and ask you to find the actual score.
This means we need to transpose the formula.

$$x = z \times s + \bar{x}$$

Or:

$$Actual\ score = (z\ score) \times (standard\ deviation) + mean$$

Putting it into practice

Let's do an example:
Extracted from the Cambridge Further Mathematics Textbook Series

The heights of a group of young women have a mean of $\bar{x}$=160 cm and a standard deviation of s=8 cm.
Determine the standard or z-scores of a woman who is:

a) *172 cm tall*
b) *150 cm tall*
c) *160 cm tall.*

$$z = \frac{x - \bar{x}}{s}$$

a) $z = \frac{172 - 160}{8}$

$= \frac{12}{8}$

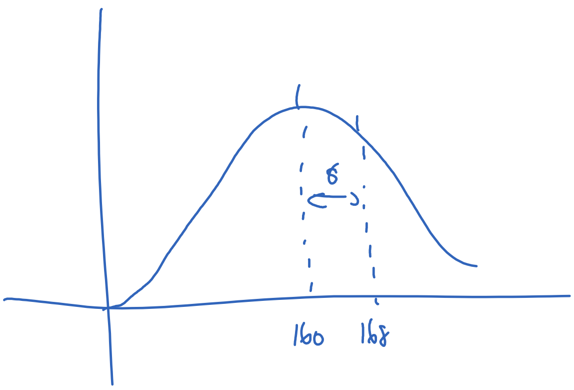
$= \underline{\underline{1.5}}$

b) $z = \frac{150 - 160}{8}$

$= \underline{\underline{-1.25}}$

c) $z = \frac{160 - 160}{8}$

$z = \underline{\underline{0}}$



Why do we need standard scores?

Standard scores help us compare results across different groups (if we are looking in a school).
Imagine there are two students **Kylie** and **Jason**.
They are having a debate about who is the most intelligent.

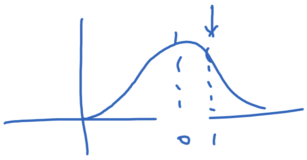
They decided to make the decision based on their next two test scores.

Sadly, Kylie gained a score in her Physics Exam and Jason a score in his Further Mathematics Exam.

Putting the results into a table we see:

$\bar{x}$ s

Name	Subject	Score	Mean	Standard Deviation
Jason	Further Mathematics	75	65	10
Kylie	Physics	70	60	5

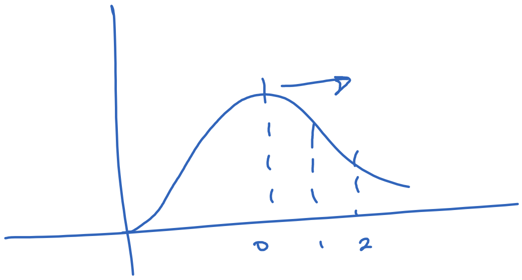


It is really hard to work out who did better!
We can look at percentages, but this is only part of the picture.
If we looked at where they sat in the class by looking at their position on the Normal Distribution we would have a much better indication of who won the competition!

The way to compare them using the bell-curve (or Normal Distribution) is to covert each of their scores to z-scores.

Jason:

$$z = \frac{75 - 65}{10} = \underline{\underline{1}}$$



Kylie:

$$z = \frac{70 - 60}{5} = \underline{\underline{2}}$$

What do the results suggest?

Example of converting from z-scores to a "normal score"

At the start of the video we said that we can do things forwards and backwards. Here is an example of doing it backwards:

Example:
Extracted, with permission, from the Cambridge Further Mathematics Textbook

A class test (out of 50) has a mean mark of $\bar{x} = 34$ and a standard deviation of $s = 4$. Joe's standardised test mark was $z = -1.5$. What was Joe's actual mark?

$$z = \frac{x - \bar{x}}{s}$$

$\bar{x} = 34$

$s = 4$

$z = -1.5$

$$zs = x - \bar{x}$$

$$\boxed{x = z.s + \bar{x}}$$

$$x = (-1.5) \times 4 + 34$$
$$= \underline{\underline{28}}$$