

Compound interest investments with regular additions to the principal (annuity investment)

Sunday, 24 March 2019

1:34 pm

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Know how to use the Financial Solver on your CAS to answer questions relating to compound interest investments with regular additions to the principal
- Understand the language used for perpetuities
 - V_0 as the principal
 - r as the rate of interest
 - D as the regular payment made

RECAP:

This is the last lesson in this series on financial mathematics and it's one of the best!

Well ... not really ... but I need to say this to keep you all watching.

In previous videos we have been looking at the following financial mathematics and how we can solve problems relating to questions on each section:

- Reducing balance loans
- Interest only loans
- Annuities
- Perpetuities

Now it's time to see what happens when we decide to add money to our investments.

For example, imagine you are left \$500,000 from someone's estate. You decide to invest this, and add some of your monthly income to become richer quicker ...

Let's look at an example of how you can become a millionaire before you retire ... with some savvy saving ...

EXCEL EXAMPLE: Who wants to be a millionaire?

This uses what is known as an amortisation table!

Recurrence relations (again!)

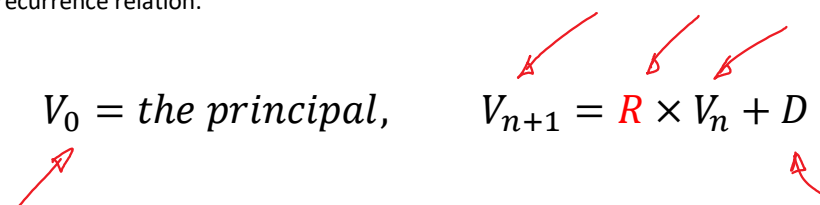
Don't you just love these things!!!

We can model compound interest investments with regular additions to the principal in the following way ...

Language first ..

r : the rate of interest being paid
 D : the payment being added each period
 V_0 : the principal

We can use the following recurrence relation:

$$V_0 = \text{the principal}, \quad V_{n+1} = R \times V_n + D$$
The diagram shows the recurrence relation equation with red arrows pointing to specific variables. One arrow points to V_0 in the first part of the equation. Another arrow points to R in the second part. A third arrow points to V_n in the second part. A fourth arrow points to D in the second part.

$$v_0 = \text{the principal}, \quad v_{n+1} = R \times v_n + D$$

$$R = 1 + \frac{r}{100}$$

Let's look an example:

All examples used have been taken, with permission, from the Cambridge Further Mathematics (Units 3 and 4) Textbook:

Example:

Nor plans to travel overseas when she finishes her VCE. She has already saved \$1200 and thinks that she can save an additional \$50 each month that she plans to add to her savings account. The account pays interest at a rate of 3% per annum, compounding monthly.

Model this investment using a recurrence relation of the form

$$V_0 = \text{the principal}, \quad V_{n+1} = R \times V_n + D$$

3% p.a

where V_n is the value of the investment after n payments (additions to the principal) have been made.

$$V_0 = 1200, \quad V_{n+1} = 1.0025 \times V_n + 50$$

$$R = 1 + \frac{r}{100}$$

$$R = 1 + \frac{0.25}{100} = 1.0025$$

Example 2:

Nor's investment can be modelled by the recurrence relation

$$V_0 = 1200, \quad V_{n+1} = 1.0025 \times V_n + 50$$

where V_n is the value of the investment after n payments have been received.

- Use your calculator to determine recursively the value of the investment after Nor has made three additional payments to her investment.
- What will be the value of her investment after 1 year?

a) \$1359.40

$$b) \quad \$1844.82$$

Amortisation table for Nor

Let's switch back to Nor to see how her money increases using an amortisation table

WOT ABOUT THE FINANCIAL SOLVER?

I know ... once again ... we can use the financial solver to help us solve these types of problems

Example:

Lars invests \$500000 at 5.5% per annum, compounding monthly. He makes a regular deposit of \$500 per month into the account. What is the value of his investment after 5 years?

$$\$692\,292.30$$