Using transformations to sketch graphs

Wednesday, 14 February 2018 8:21 pm

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Understand that what you can do forwards you can do backwards too!
- Know how to take a transformed function and find the sequence of transformations which led from a base graph to the transformed function

RECAP:

We have spent a lot of time working with transformations.
We have been given a series of transformations and then been expected to use whatever means we can to find the equations of the transformed function.
We should, with practice, now be in a position to be able to look at a function and know:

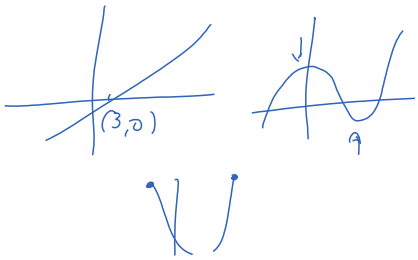
- What the base function was
- How it has been transformed

This lesson is going to take some time to look at how to sketch graphs from functions without having to do the algebra

IMPORTANT NOTES ABOUT DRAWING GRAPHS

When drawing graphs (at any point in Methods), you must always ensure to label the following:

- x-axis intercept(s) **in co-ordinate form**
- y-axis intercept(s) **in co-ordinate form**
- Turning points (maxima and minima)
- Endpoints (when the domain is limited)
- Equations of any asymptotes
- At least one co-ordinate value



Starting simple:

Example 1:

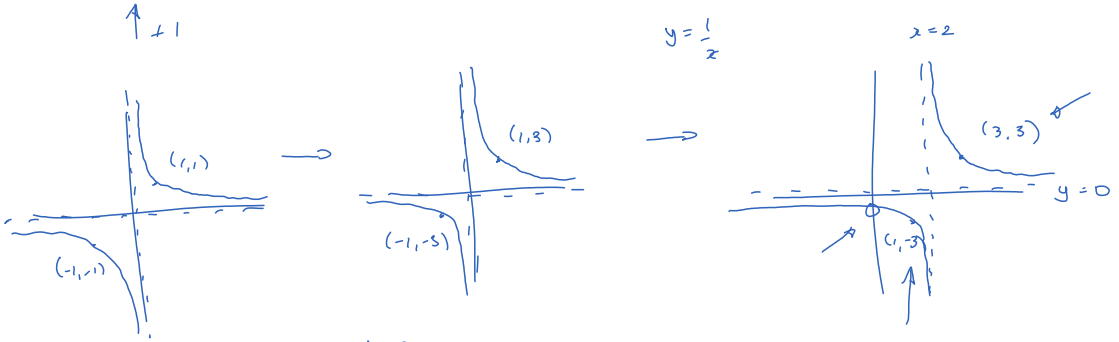
Identify a sequence of transformations which will have mapped from $y = \frac{1}{x}$ to $y = \frac{3}{x-2} + 1$
Use the information to sketch the equation labelling all key points.

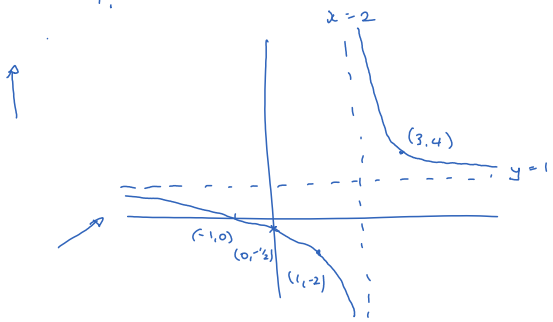
Dilations: ×3

Reflections:

Translations: → 2, ↑ 1

$y = \frac{3}{x-2} + 1$





$y = \frac{3}{x-2} + 1$

$y = \frac{3}{-2} + 1$

$y = -\frac{1}{2}$

@ $x = 0$

$y = \frac{3}{x-2} + 1$

$0 = \frac{3}{x-2} + 1$

$-1 = \frac{3}{x-2}$

$-(x-2) = 3$

$-x + 2 = 3$

$-x = 1$

$x = -1$

Example 2:
Mathematics, and in particular Methods, tries to throw curve balls at you.
This is more of a problem for people who learn a whole range of short cuts.
Remember: Short cuts only work if you completely understand the theory behind how they came to be

$(ab)^2 = a^2b^2$

Identify a sequence of transformations which have mapped from $f(x) = \frac{1}{x^2}$ to $f(x) = \frac{2}{(2x-4)^2} - 3$.
Sketch the final function labelling all key points.

Dilations: Factor ½ away from x-axis

Reflections:

Translations: → 2

$y = \frac{2}{(2x-4)^2} - 3$

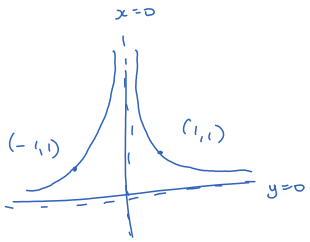
$(2x-4)^2$

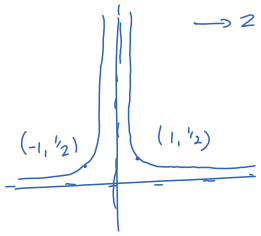
$\{2(x-2)\}^2$

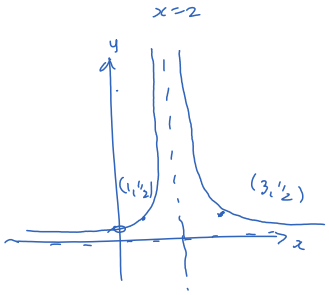
Dilations:

Reflections:

Translations: → 2, ↓ 3







$(2x-4)^2$

$y = \frac{2^1}{4(x-2)^2} - 3$

$y = \frac{1}{2(x-2)^2} - 3$

$[2(x-2)]^2$

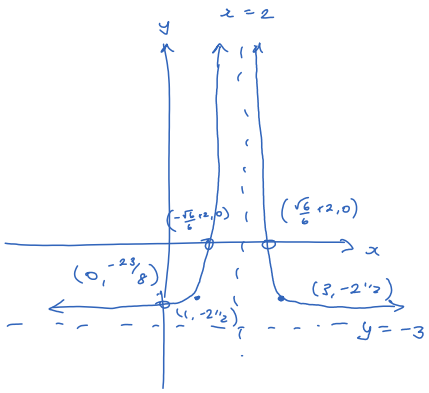
$2^2(x-2)^2$

$4(x-2)^2$

$y = \frac{1}{2(x-2)^2} - 3$

$x = 0$

$y = 0$



Mathematics is more than just a collection of boxes

The study of Mathematics is complex.
Maths isn't complex (well not really!) but we tend to learn it in discrete boxes. Once we have completed the learning for that box, we close it, we put it in the back of our brains, and we only bring it out when we are threatened with a test.

This is the worst way to learn Mathematics!

We need to keep building and remembering all which has gone before.
This is the difference between learning and understanding.

Example:

Show that $\frac{2x+3}{x-1} = 5 + \frac{2}{x-1}$.

Hence, without using a calculator, sketch the graph of $f: \mathbb{R} \setminus \{+1\} \to \mathbb{R}, f(x) = \frac{2x+3}{x-1}$

$\frac{2x+3}{x-1} = 5 + \frac{2}{x-1}$

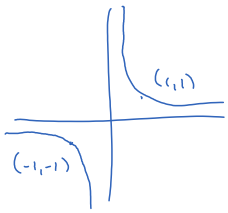
$x-1 \overline{)\ 2x+3}$ gives 2 remainder 5

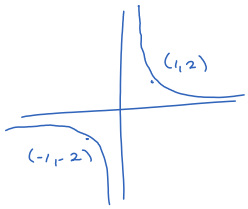
$\therefore \frac{2x+3}{x-1} = 5 + \frac{2}{x-1}$

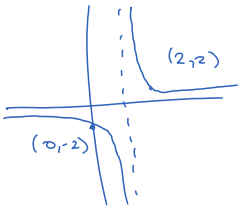
$= \frac{2}{x-1} + 5$

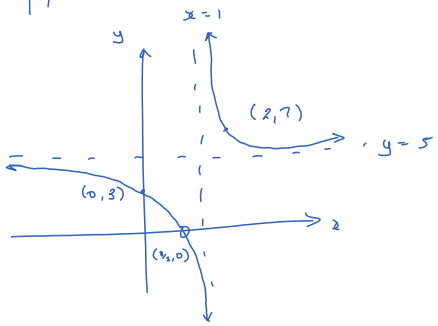
$y = \frac{1}{x}$ ↑ ×2 → +1 ↑ +5

$x = 1$









$$y = \frac{2}{x-1} + 5$$

$$0 = \frac{2}{x-1} + 5$$

$$\frac{2}{x-1} = -5$$

$$2 = -5(x-1)$$

$$2 = -5x + 5$$

$$-3 = -5x$$

$$x = \frac{3}{5}$$