Types of functions and implied domains

Tuesday, 20 November 2018 12:47 PM

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- How to test and classify functions
  - One-to-one
  - Many-to-one
  - Odd functions
  - Even functions
- Know what is meant by an implied domain
- Know what a piecewise function is

RECAP:

In previous lessons we have looked at what a function is. We have looked at how to write things in function notation.
We even looked at what the difference between a function and a relation is. So much fun.

We continue looking at functions as, when we understand the language, we can make sure that we answer questions correctly.

Important Advice:

Everything you are going to deal with in this course is related to and dealing with graphs.
You **must, must, must** know what each of the common graphs look like.
This is no joke!

Given the following you must have an idea of what the graph will look like.
This has, for the most part, been covered in Year 10 and Year 11.

$$y = -2(x+3)^2 - 3$$

$$g: [0, 2\pi] \to \mathbb{R}, g(x) = 3\cos 2\left(x + \frac{3\pi}{2}\right) - 3$$

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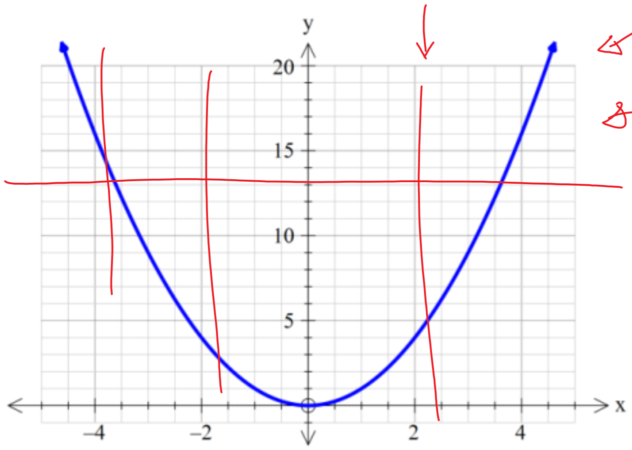
Note how we are now starting to build on the knowledge we have already gained from the previous lessons.

If you can see the graphs in your head, you are far more likely to be able to answer any question which are thrown at you

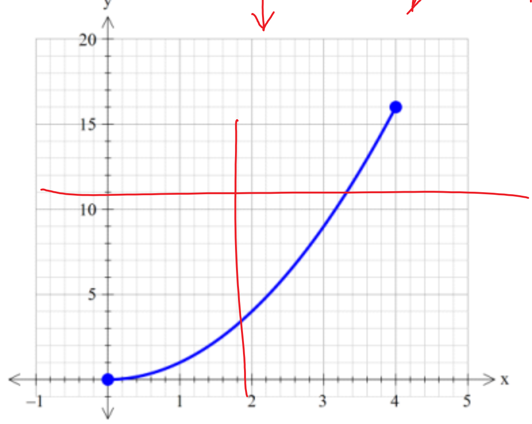
RECAP: One to One Functions

In the last lesson we looked at what it means to restrict a function.
The primary reason we might do this is to turn a function into a one-to-one function.
If we can draw a **horizontal line** through the function and it only cuts once, then the function is a one-to-one function



This is **NOT** a one-to-one function.

[0,4]



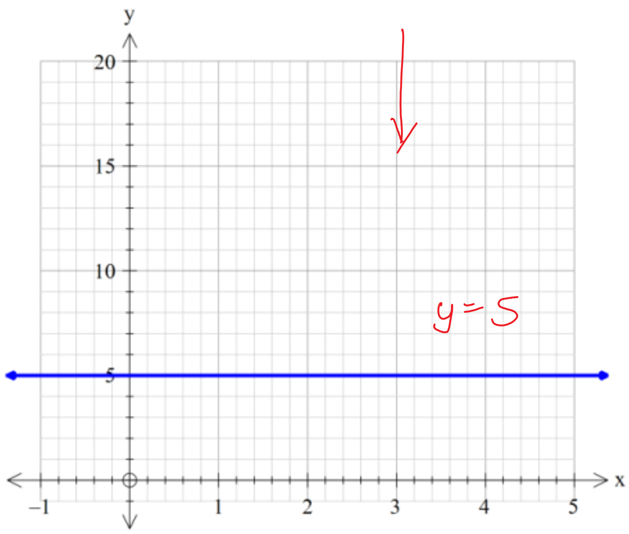
This IS a **one-to-one** function

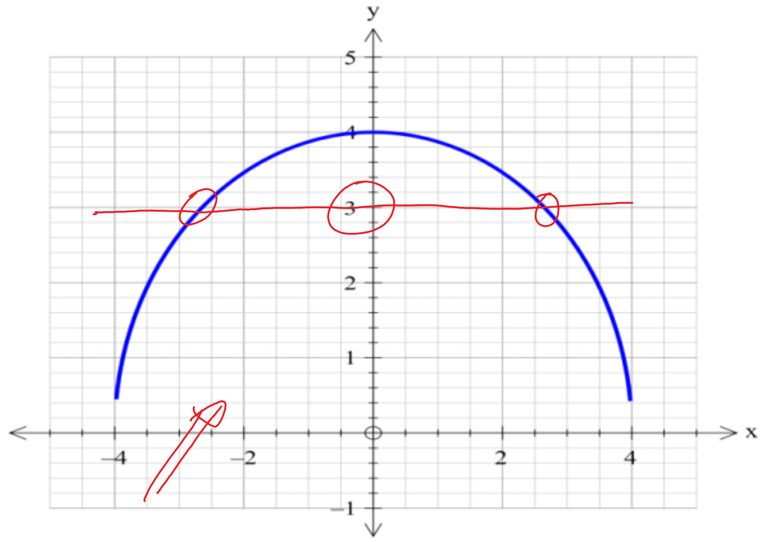
Why would we need to know about one-to-one functions

Later, in Methods, we spend a lot of time looking at **inverse functions.**
An function can only have an inverse if it is a **one-to-one** function.

Important note: All relations can have inverses.

Are the following one-to-one functions?





Many-to-one functions

A one-to-one functions is defined that for any value of y there is only one value of x.
Hence, why the horizontal line test.
A many-to-one function is defined such that a y value belongs to many values of x.

The examples above show many-to-one functions

Implied domains

Barry has been at it again.
He's decided that we need two terms to describe the same thing.
They are **implied domains** and **maximal domains**.

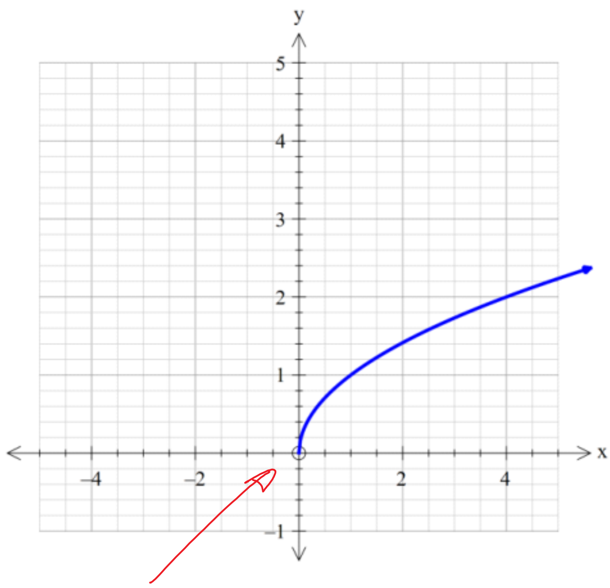
If I write the relation $y = x^2$ you will note that I have not told you the domain.

$y = x^2$

In fact there is no reference to the domain at all.

$\mathbb{R}$ $(-\infty, \infty)$

Hence, I imply what the domain would be. I use common sense and know that all REAL values for the x are possible and so the implied (or maximal) domain is $x \in \mathbb{R}$



This graph is for the function $f(x) = \sqrt{x}$

You must learn that the implied domain for this function is $x \in [0, \infty)$

We could then re-express the function as:

$x \in [0, \infty)$

$$f : [0, \infty) \to \mathbb{R}, f(x) = \sqrt{x}$$

Piecewise functions

I don't know why ... but whenever I see the word "Piecewise" I think of Pennywise the clown from IT.
Thankfully it has nothing to do with murderous clowns who hide in the sewers.
It has everything to do with having the ability to draw more than one functions on a set of axes.

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Example:

$$f(x)=\begin{cases} -x-1, & x<0 \\ 2x-1, & 0\le x\le 1 \\ \frac{1}{2}x+\frac{1}{2}, & x>1 \end{cases}$$

$y = -x - 1 \qquad x < 0$

$y = 2x - 1 \qquad 0 \le x \le 1$

$y = \frac{1}{2}x + \frac{1}{2} \qquad x > 1$

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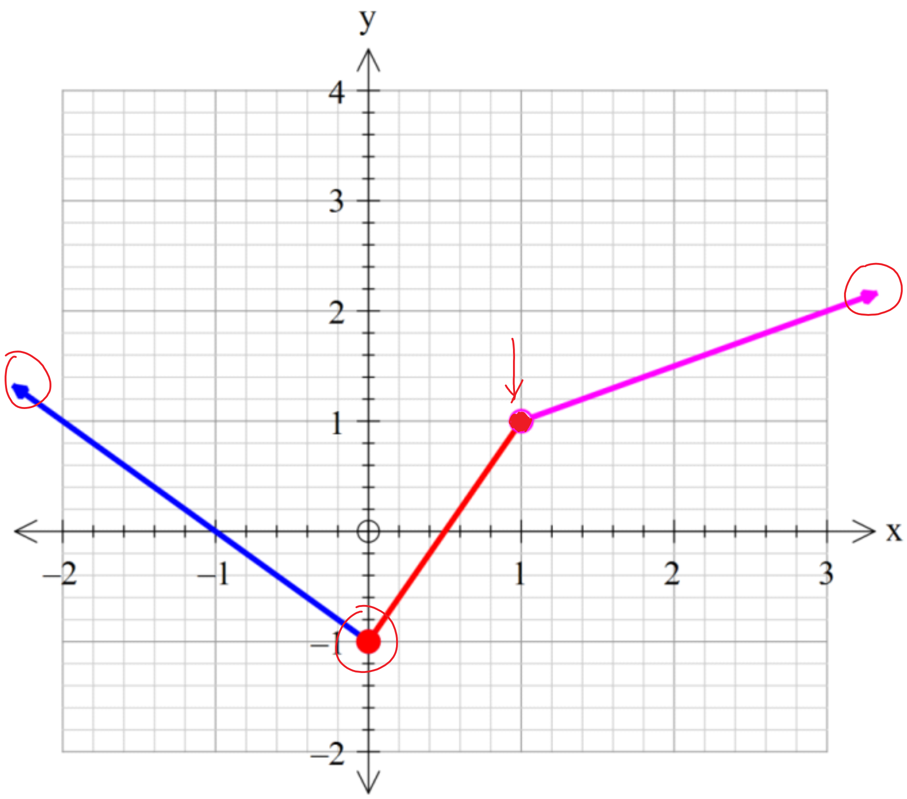
Notice how there are a number of graphs which defined for certain values of x.
This is called a piecewise function.

We can draw this using a graphing program such as DESMOS.com or using your CAS



Odd and Even Functions

Now, I know a lot of odd people! I know there was a TV program called the "Odd Couple".
But I have little reason to know why we would be interested in knowing if a function is odd or even.

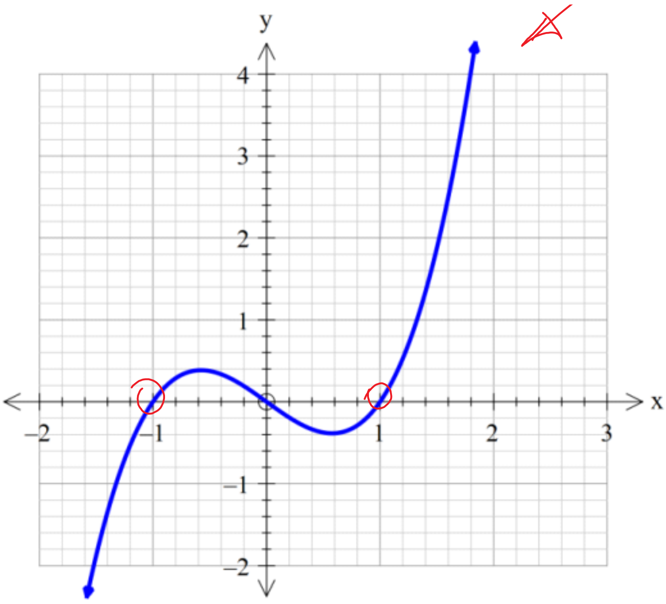
At this moment, I think we are going to learn it as "something I might be asked a question on".

Odd functions
An odd function is one where you can rotate it through 180 degrees and it looks the same.
Hence, it has Order 2 Rotational Symmetry.
The more technical understanding is the following:

$$f(-x) = -f(x)$$

$f(-x) = -f(x)$

$f(-1) = -f(1)$

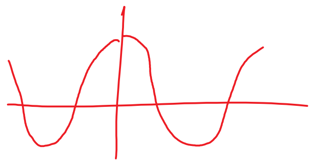


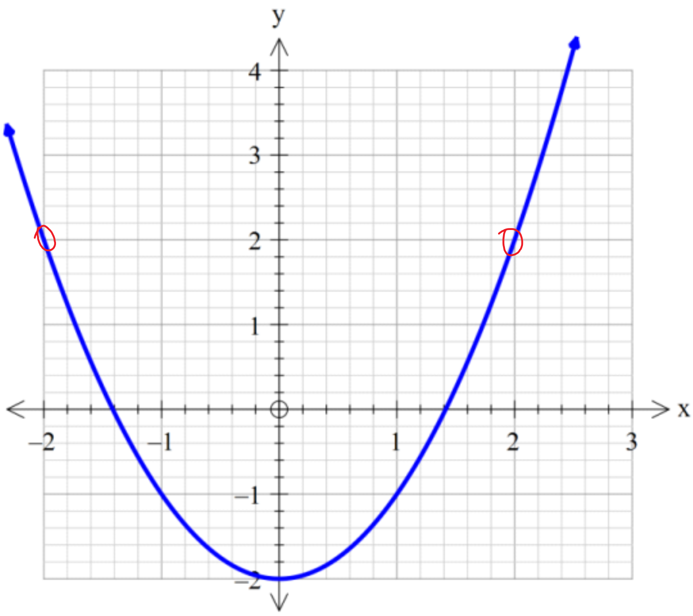
An example is shown with the graph of $y = x^3 - x$

Even functions
An even function is one which has a mirror line (or line of symmetry) in the y-axis.
Again, this has a more formal definition:

$$f(-x) = f(x)$$

$f(-2) = f(2)$





An example of an even function is shown.
It has the equation $y = x^2 - 2$

Understanding the tricks of function notation

Maths, being a BIG FAT TRICK, will try and confuse you!
It took a whole year for one of my Maths groups to understand the following question:

Example:
If $f(x) = 3x^2 - 2x + 1$, find $f(3)$

$f(x) = 3x^2 - 2x + 1$ $\quad f(3)$

$f(3) = 3(3)^2 - 2(3) + 1$

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