Translations

Sunday, 11 February 2018 7:22 pm

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Know what it means to translate something
- Know how to use the correct notation
- Apply transformations to sketches of graphs
- Know how to use transformation algebra to go from a base graph to a transformed graph

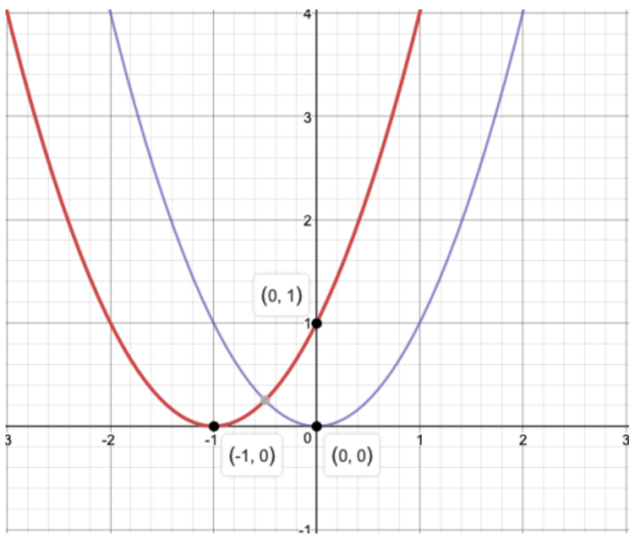
RECAP

This is all very exciting and builds on the work which was covered in Methods 1 and 2.
The first two-thirds of the Cambridge Essentials Textbook for Methods 3 and 4 is graph sketching and transformations.
There is a lot to cover.
This lesson builds on the work from last year but goes deeper.

TRANSLATIONS: A new perspective

Translation is a movement of a point (or a series of points) in a horizontal and/or vertical direction.
This can be described as parallel to the x-axis or parallel to the y-axis.

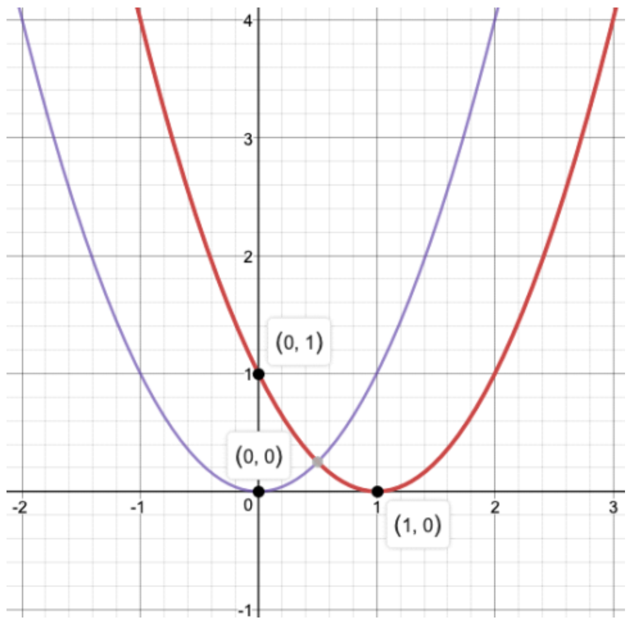
Looking at this from a graphical perspective first:



Movement one unit to the **left**

$y = x^2$

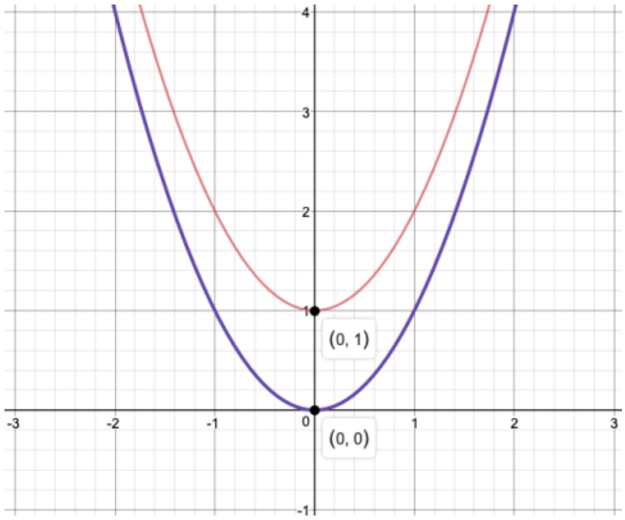
$y = (x+1)^2$



Movement one unit to the **right**

$y = x^2$

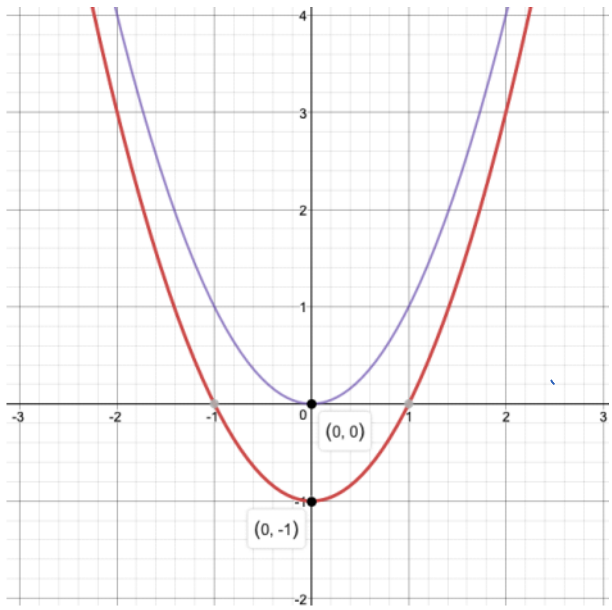
$y = (x-1)^2$



Movement one unit in the positive direction of the y-axis

$y = x^2$

$y = x^2 + 1$



Movement one unit in the negative direction of the y-axis

$y = x^2$

$y = x^2 - 1$

There are a number of **short cuts** we can use to remember how these all work.
We can also use algebra to help us.

USING ALGERBRA to do TRANSFORMATIONS

A coordinate and it's image can be expressed using algebra: (x, y) maps onto (x', y')

(x, y) (x', y')

If we have a translation of 4 units to the right and 2 units down then we can express this as: (x, y) maps onto $(x + 4, y - 2)$

(x, y) $(x + 4, y - 2)$

More generally: (x, y) maps onto $(x + h, y + k)$

Looking at one-co-ordinate:
Find the image of the point $(3, -4)$ after a translation of 3 units in the negative direction of the x-axis and a translation of 3 units in the positive direction of the y-axis.

$(x, y) \longrightarrow (x - 3, y + 3)$

$(3, -4)$ $(0, -1)$

Extending this to a function (hence a collection of ordered pairs)
Using the idea of algebraic manipulations, find the equations of the transformed graph of $y = x^2$ under a translations of 4 units in the positive direction of the x-axis and a translation of 2 units in the negative direction of the ~~x~~ y-axis

$y = x^2 \xrightarrow{4} \downarrow 2$

$(x, y) \rightarrow (x+4, y-2)$

(x', y')

$x' = x + 4 \qquad y' = y - 2$

$x = x' - 4 \qquad y = y' + 2$

$y = x^2$

$x = y$

$x' \quad y'$

$y' + 2 = (x' - 4)^2$

$y + 2 = (x - 4)^2$

$y = (x-4)^2 - 2$

Example 2
Find the equations for the image of the graph of $y = (x-3)^2 + 2$ with a translation described as $(x, y) \rightarrow (x-2, y+3)$

$(x, y) \rightarrow (x-2, y+3)$

$x', \quad y'$

$x' = x - 2$

$y' = y + 3$

$x = x' + 2$

$y = y' - 3$

$y = (x-3)^2 + 2$

$y' - 3 = (x' + 2 - 3)^2 + 2$

$y - 3 = (x-1)^2 + 2$

$y = (x-1)^2 + 5$

Another Short cut?

When we have the graph of $y = f(x)$ and we subject it to a translation of h units in the positive x-direction and k units in the positive y-direction we can turn $f(x)$ into:

$\Downarrow$

$$y - k = f(x - h)$$

Where you see an 'x' in the function replace it with $x - h$.
Where you see a 'y', replace it with $y - k$

NOTE: Remember that k is a positive movement and h is a positive movement. If there are negative movements, then replace the k or h with a negative value and ensure you take account of the double negatives!

Example:

Find the equation for the image of the curve $f(x) = \frac{1}{x}$ under a translation of 2 units in the negative direction of the x-axis and 3 units in the positive direction of the y-axis.

$\leftarrow$ 2 $\quad \uparrow$ 3

$x \quad x - h$

$x - (-2)$

$x + 2$

$y - k$

$y - 3$

$f(x) = \frac{1}{x}$

$f(x) = \frac{1}{x+2}$

$f(x) - 3 = \frac{1}{x+2}$

$$f(x) = \frac{1}{x+2} + 3$$