The geometry of simultaneous linear equations with two variables

Sunday, 11 February 2018 6:11 pm

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- Know what it means for two simultaneous equations to have zero, one and infinite solutions
- Know how to graph simultaneous equations
- Know how to find if simultaneous equations have one, two or infinite solutions
- Use the CAS to find solutions to simultaneous equations

RECAP

And, we're back to these old simultaneous equations again!
This time we're going to be using a different form of algebra to see if two lines cross in one, infinite, or no places.
We remember that where two lines cross we call it a solution. We can find this solutions using a number of methods:

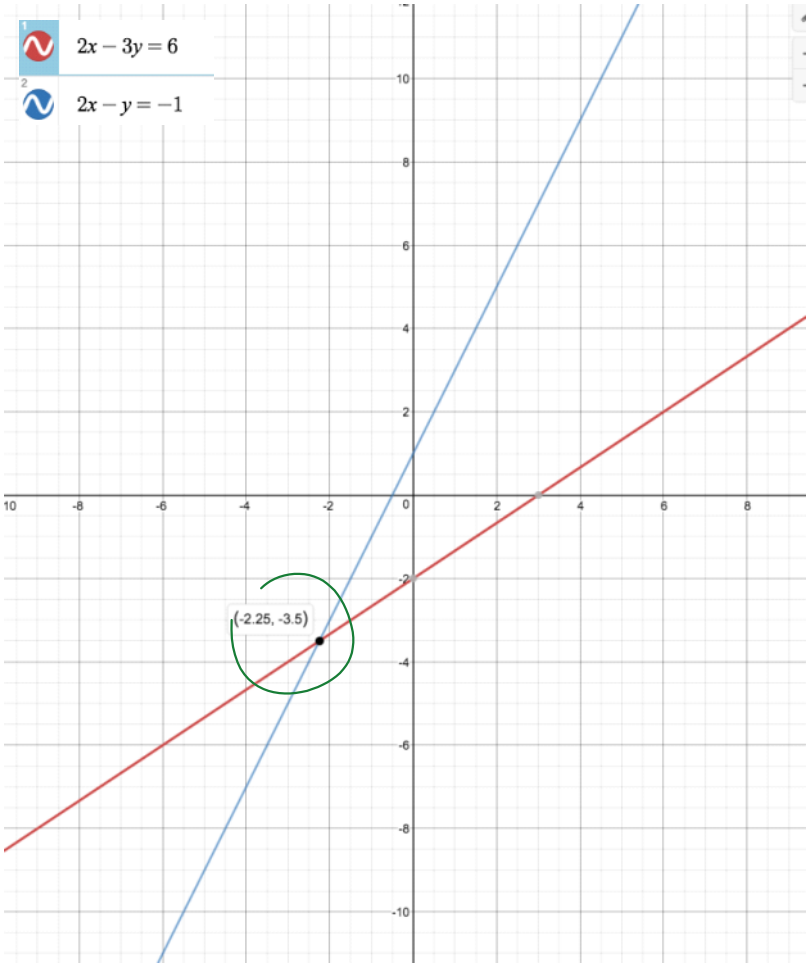
- Elimination
- Substitution
- Graphing
- Matrices
- CAS

Solving simultaneous equations

Three cases:

- Meet at only one points (only one solution)
- Never meet (no solution)
- Meet at an infinite number of points (infinitely many solutions)

Meet at only one point



When the coefficients of x and y are not the same (and they are not multiples of one another) there will be only **one solution**.

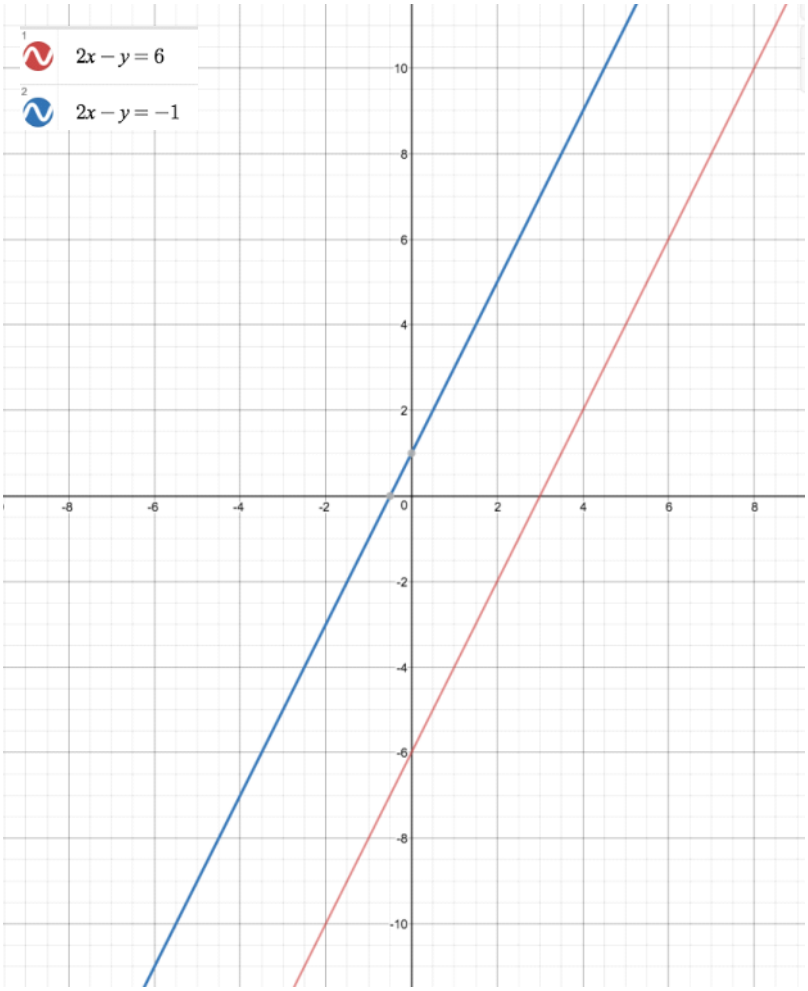
$2x - 3y = 6$

$2x - y = -1$

$x = -2.25$

$y = -3.5$

Never meet (no solution)

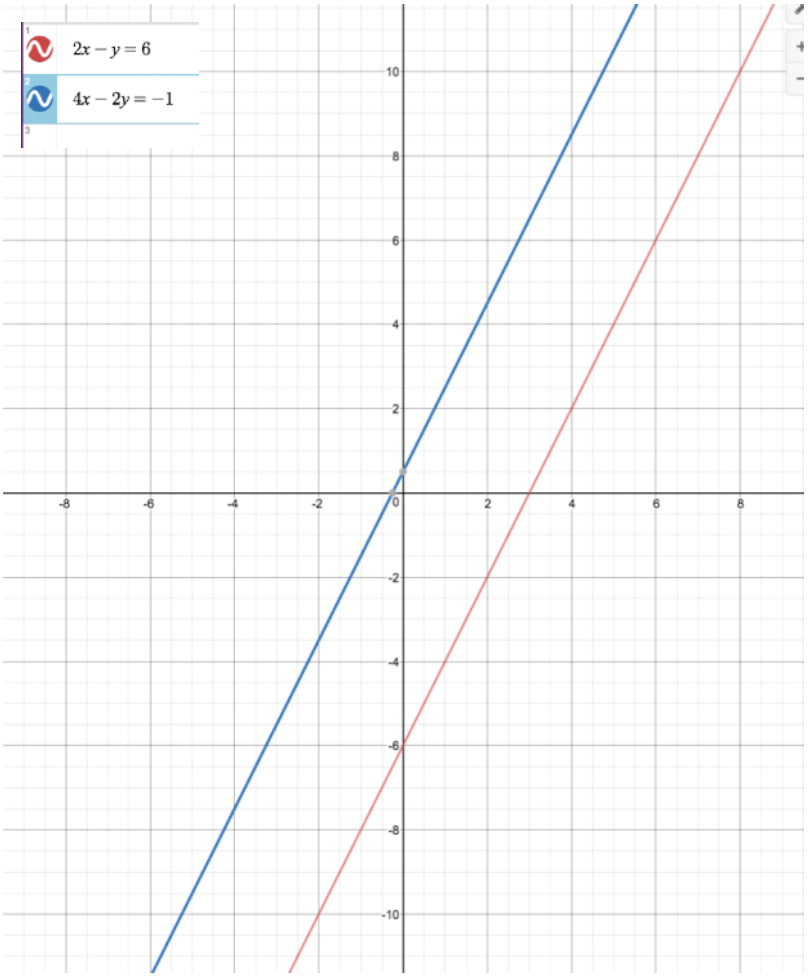


When the coefficients of x and y are the same, they are parallel and hence there will be **no solution**

$2x - y = 6$

$2x - y = -1$

Never meet (no solution) where the coefficients of x and y in one equation are multiplied by a scalar to get to the next equation.



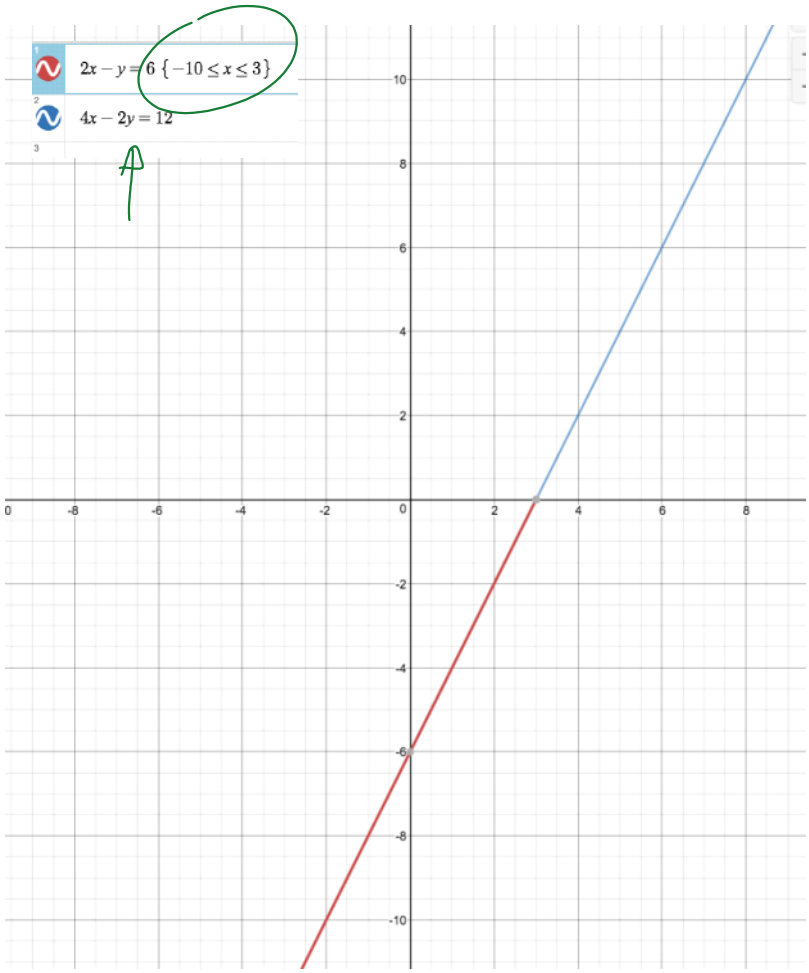
The second equation is simply the first half of the equation multiplied by 2. Note that the integer after the equals sign isn't multiplied by two. This means the y-intercept will be different, but the gradient will stay the same.

$2x - y = 6$

$4x - 2y = -1$

$2x - y = \frac{1}{2}$

The equations are the same hence there are infinite number of solutions



When all terms are multiplied by the same number to get from one equation to the other, they are obviously the same line! Hence, sitting on top of one another, they have **infinite solutions.**

$\times 2 \rightarrow$ $2x - y = 6$

$4x - 2y = 12$

***Note**: The domain has been limited for the red line to show that it fits exactly over the blue line.*

Parameters

We can use parameters to describe a series of solutions. Generally used for where there are infinite number of solutions.

E.g. The simultaneous equations $3x + 4y = 12$ and $6x + 8y = 24$ have infinitely many solutions. Describe these solutions through the use of a parameter.

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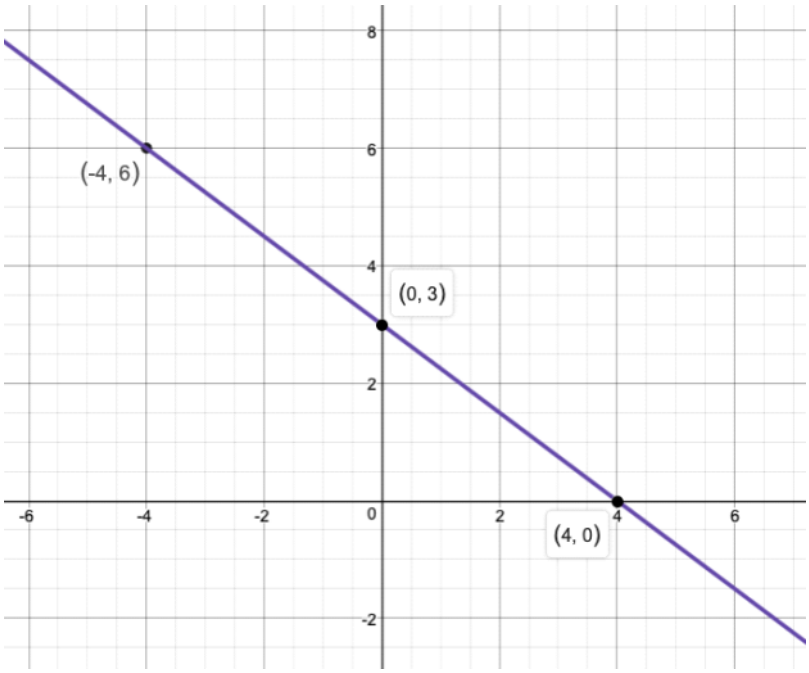

$3x + 4y = 12$

$3x = 12 - 4y$

$x = \frac{12 - 4y}{3}$

$y = \lambda$

$\left(\frac{12 - 4\lambda}{3}, \lambda\right)$

$\lambda = 0$

$\lambda = 1$

$\lambda = 2$

Using Matrices to find if there is one solution or no/infinite solutions

Example: $3x + 2y = 6$
$x - y = 7$

$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$ad - bc$

$3(-1) - (2)(1)$

$= -3 - 2$

$= -5$

No / Infinite solns.

Example $x + 2y = 6$
$2x + 4y = 12$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

$(1)(4) - (2)(2)$
$= 4 - 4$
$= 0$

Example:
Find the value of m for which the following simultaneous equations have no solution.

$(m+3)x + my = 12$
$(m-1)x + (m-3)y = 7$

$$\begin{bmatrix} (m+3) & m \\ (m-1) & (m-3) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

$(m+3)(m-3) - m(m-1) = 0$
$m^2 - 3m + 3m - 9 - m^2 + m = 0$
$-9 + m = 0$
$m = 9$

$\times 3$

$12x + 9y = 12$
$8x + by = 7$