The Area under a graph

Sunday, 17 June 2018 6:21 pm

By the end of the teaching I would ask that you complete the following work:

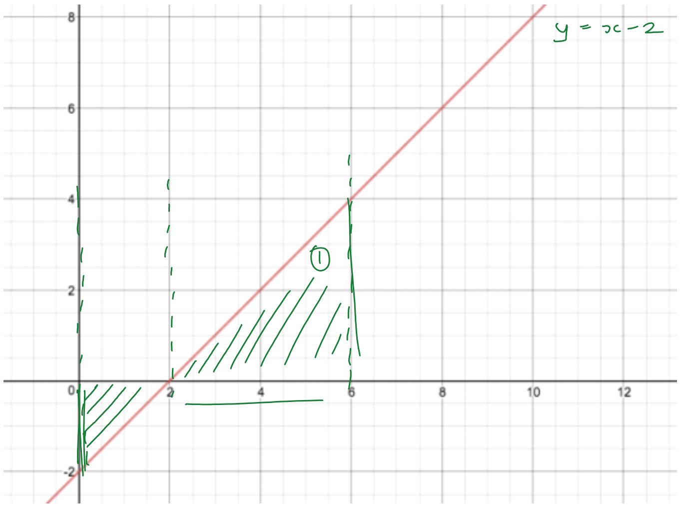
Approximating regions	11A	1ac, 2, 4, 6, 7

By the end of the lesson you should understand the following terms and know how to find the area under a graph:

- Left-endpoint estimate
- Right-end point estimate

RECAP: Finding basic areas

How do you find the area under the function $y = x - 2$ between the points where $x = 2$ and $x = 6$?



$A_1 = \frac{1}{2}hb$

$= \frac{1}{2} \cdot 4\ 4$

$= 8$ units^2

$A_2 = \frac{1}{2} b.h$

$= \frac{1}{2}.2.2$

$= 2$ units^2

Tot $= 8 + 2$

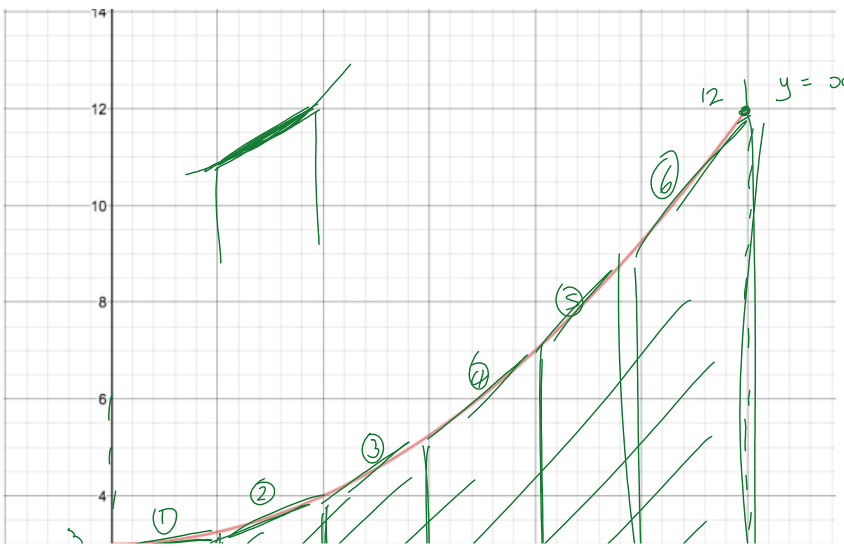
$= 10$ units^2

How about if we wanted to find the area between the points where $x = 0$ and $x = 6$?

Finding areas under graphs with straight line functions is simple really.
We use the information from Year 6 to 10 Mathematics.
What if we need to find the area under a curve?

Areas under curves

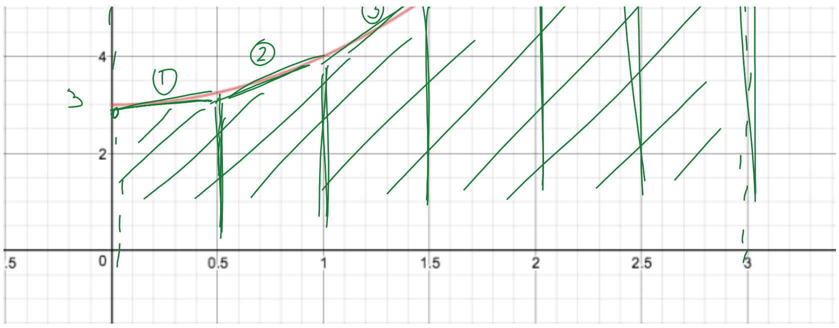
How would we find the area under the curve with equation $f\colon [0,3] \to \mathbb{R}, f(x) = x^2 + 3$

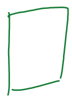


$A = \frac{1}{2}(h_1 + h_2).w$

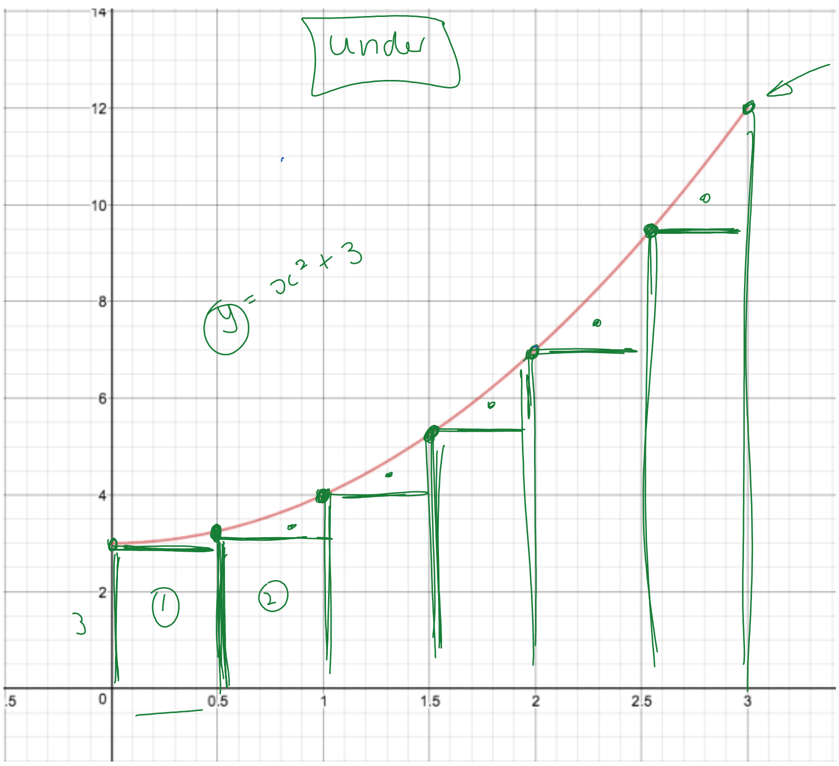
$= \frac{1}{2}(12 + 3).3$

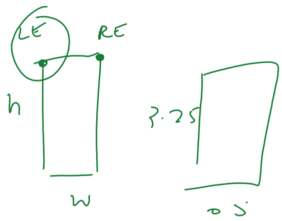
$A_1 = \frac{1}{2}(3 + 3.25).0.5$





The left-endpoint estimate





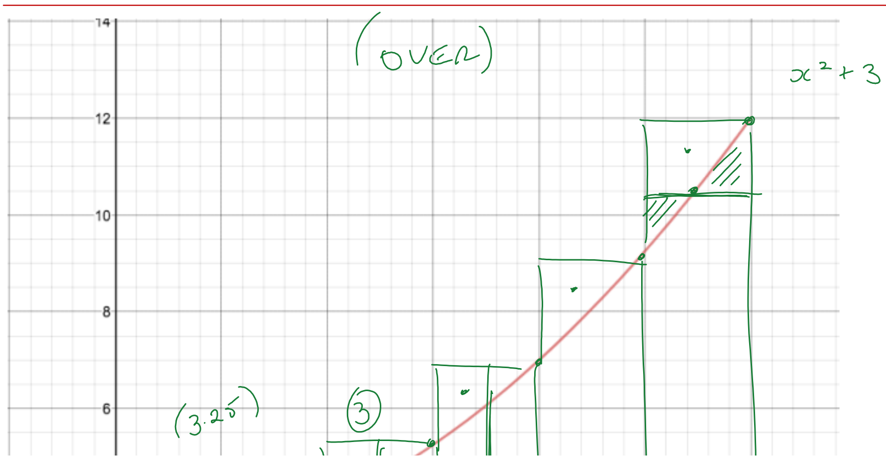
① = 3 × 0.5

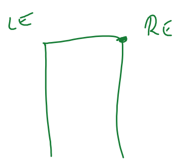
② = 3.25 × 0.5

③ = 4 × 0.5

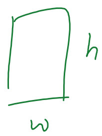
④ = 5.25 × 0.5

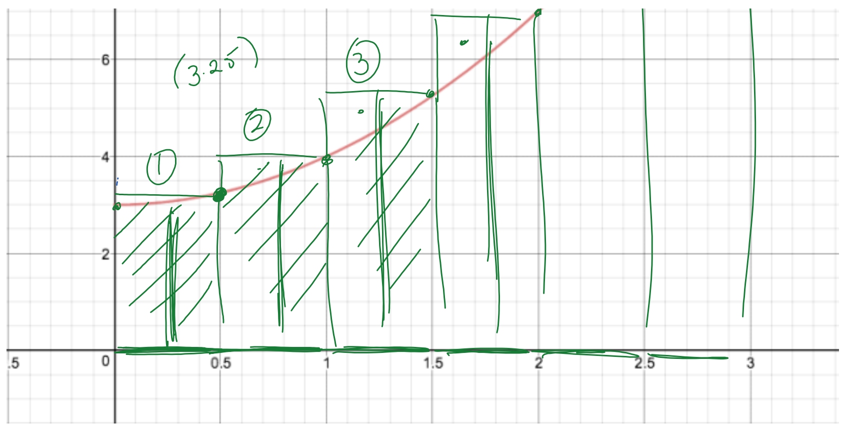
The right-endpoint estimate

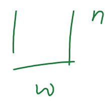




$\frac{LE + RE}{2}$







① = 3.25×0.5 = 1.625

② = 4×0.5 = 2

③ = 5.25×0.5 = 2.625

④ = 7×0.5 = 3.5

⑤ = 9.25×0.5 = 4.625

⑥ = 12×0.5 = 6

Total Area = 20.375 units^2

Let's work out the areas and see what happens then!

18 units^2

Things to note:

It would make sense that, the smaller the strips, the closer you would get to the actual area under the graph!

Exact Area

To find the exact value of the area under a continuous function you use something called the **definite integral.**
This is expressed using the following notation:

$$\boxed{\int_a^b f(x).dx}$$

$$\int_0^3 (x^2+3)\,dx = 18 \text{ units}^2$$

$f(x) = x^2 + 3$