Stationary Points

Wednesday, 9 May 2018 6:52 pm

By the end of the lesson I would ask that the work shown below is completed:

Stationary Points	10C	1acgh,2abcf,3,5,6,10

RECAP:

We have, over the course of the modules we have been studying, been dealing with lots of different functions.
We have been asked to draw them and move them
In all cases we have been asked to ensure that, when we sketch the graphs of the functions, we include the following things on the sketch:

- End points (really important when the domain has been limited)
- Axes labels
- Function label
- $x - axis$ intercepts
- $y - axis$ intercepts
- Equations of asymptotes
- At least one co-ordinate outside of those shown above

$f: [2,4] \to R,\ f(x) = x^2+2$

$(2,4)$

$y = x^2+2$ $(3,11)$

$y = x^2 - 2$

$0 = x^2 - 2$

Limited domain

$(\sqrt{2},0)$ $(0,-2)$

The one thing we have been unable to do, until today, is find the co-ordinates of any turning points.

Welcome then to this lesson on stationary points.

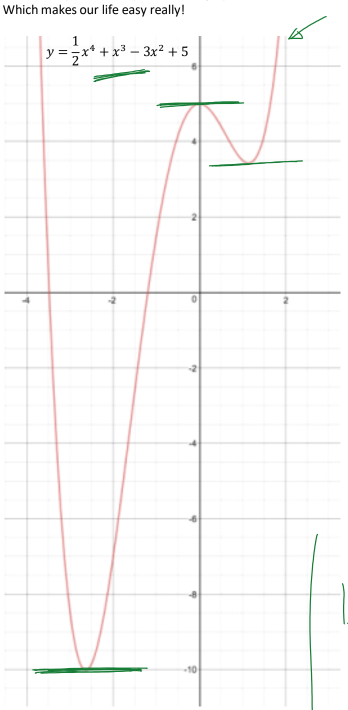
What is a stationary point?

This is basically a point on a graph where then differential of a point is equal to zero.
Or, in a simpler form, a point on the graph where there is **zero** gradient.
Which makes our life easy really!

minimum grad = zero

Max $m = 0$

$x = 0$ $y = 0$



In this graph there are three stationary points.
Three points where there is a gradient of zero.

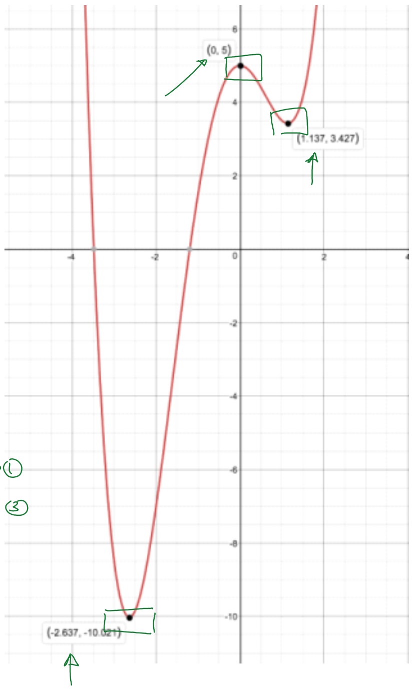
We can identify these visually.

We can also use algebra.

Remember:
The stationary points (otherwise called turning points) are where the gradient is equal to zero.
Hence, we can differentiate the function and then solve it for zero.
This will give me all points on the graph where the is a gradient of zero.
We will expect more than one point.

This will give me only the x co-ordinates.
We would then substitute them back into original equations to find the y values.

Note:
All coordinates much be expressed in coordinate form on the sketch.



$y = x^3 + \frac{1}{2}x^2$ $x^2 \Rightarrow 1$ $x^4 \Rightarrow 3$

$y' = 3x^2 + x$

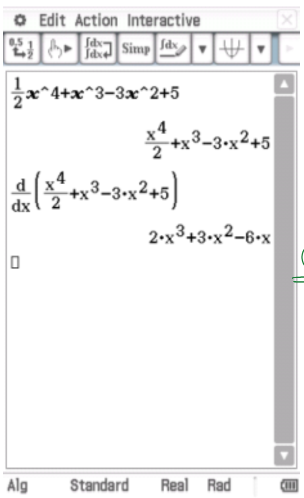
$m = 0$

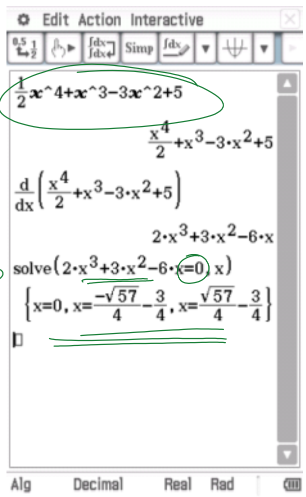
$3x^2 + x = 0$

$x(3x+1) = 0$

$x = 0$ or $x = -\frac{1}{3}$

Using the CAS:





In these two screen shots I worked out the differential of the function and then solved this by putting it equal to zero.

$y = \frac{1}{2}x^4 + x^3 - 3x^2 + 5$

$y' = 2x^3 + 3x^2 - 6x$

S.P $\Rightarrow m = 0$ $\boxed{y' = 0}$

$2x^3 + 3x^2 - 6x = 0$

$x(2x^2 + 3x - 6) = 0$

$x = 0$ $x = \frac{-\sqrt{57}}{4} - \frac{3}{4}$ $x = \frac{\sqrt{57}}{4} - \frac{3}{4}$

BIG FAT TRICKS

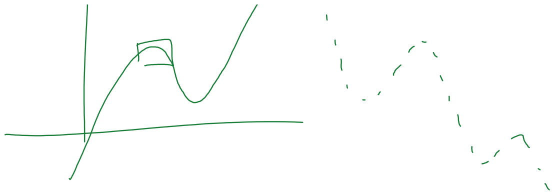
Questions will always try and trick you.
The best way to do this is to change the language.

Stationary points are also called:

- Maximum
- Minimum
- Maxima
- Minima
- Local Maximum

Stationary points are also called:

- Maximum
- Minimum
- Maxima
- Minima
- Local Maximum
- Local Minimum
- Turning points



Any of these used in a question are asking you to find stationary points.
Differentiate, solve by putting equal to zero, substitute back into the original equation and find each of the y values.

Combining other areas of the course:

You can now be given information in a number of ways and asked to find equations of curves.
For example: Given a curve with equation $y = x^3 + ax^2 + bx + c$, if you know that it passes through the point $(0, 5)$ and has a stationary point at $(2, 7)$ find the values of a, b and c.

Diff.

Remember: **This is a CAS course**
So learn how to use it.

Define the first function as f(x)
Find the differential and define it as g(x)
Remember that you can express coordinates in lots of ways.

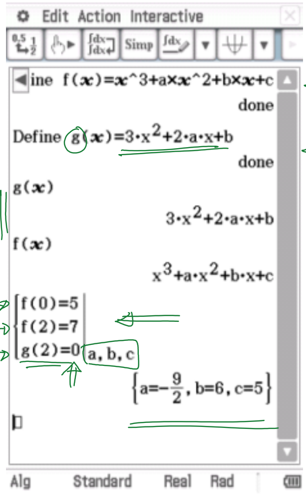
(0, 5) -> f(0)=5
(2, 7) -> f(2)=7

How do we know the above?
The question was clear and told us that there was a stationary point at (2, 7). This point must be on the line therefore.

It also means that g(2)=0

Remember, g(x) is the same as the gradient of the function.
We know at (2, 7) the gradient is zero.

Hence, g(2)=0



$f(x) =$

$g(x) =$

$g(x) = f'(x)$

$f(x)!$ $x^3 + ax^2 + bx + c$ 3 unknowns!

$f(0) = 5$

$f(2) = 7$

Ch 1

Ch 2

Ch 3

B.F.T

$x = 2$ $m = 0$