

Simultaneous linear equations with more than two variables

Friday, 4 January 2019 11:43 am

- ★ By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:
- Know how to use your CAS and pencil and paper methods to find solutions to simultaneous line equations with more than two variables

RECAP

Just when you thought it was safe to get back into the water ...

In a previous lesson we looked at the ways you can solve simultaneous equations with two variables.

Guess what ... you can solve them when they have more than two variables too!

If you've not had time to look back and learn the methods previously used, I would recommend doing so.

If you're up to speed. Let's start with the easiest way!

The easiest way is using the CAS.

Find the solution to the following system of three equations with three unknowns:

$$\begin{aligned}2x + y + z &= -1 \\3y + 4z &= -7 \\6x + z &= 8\end{aligned}$$

$$2x + 3y = 6$$

Now, let's do the same question using pencil and paper methods

The trick is to ensure that you can eliminate one of the variables from two of the equations and then solve them simultaneously!

$$\begin{aligned}2x + y + z &= -1 \\3y + 4z &= -7 \\6x + z &= 8\end{aligned}$$

$$\begin{array}{r}2x + y + \cancel{z} = -1 \\- 6x \quad + \cancel{z} = 8 \\ \hline * \quad -4x + y = -9 \\ \Rightarrow \quad \underline{4x - y = 9}\end{array}$$

$$\begin{array}{r}4x - y = 9 \\+ \quad 8x + y = 3 \\ \hline 12x = 12 \\ \underline{x = 1}\end{array}$$

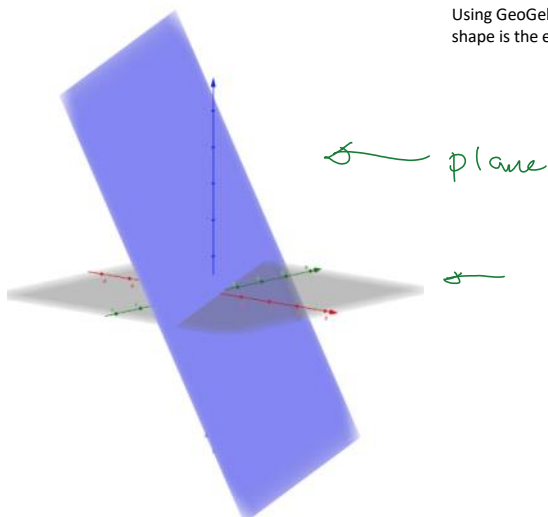
$$\begin{array}{r}2x + y + z = -1 \quad \times 4 \\3y + 4z = -7 \\ \hline 8x + 4y + 4\cancel{z} = -4 \\- \quad 3y + 4\cancel{z} = -7 \\ \hline 8x + y = 3\end{array}$$

$$\begin{aligned}6x + z &= 8 \\6 + z &= 8 \\z &= \underline{2}\end{aligned}$$

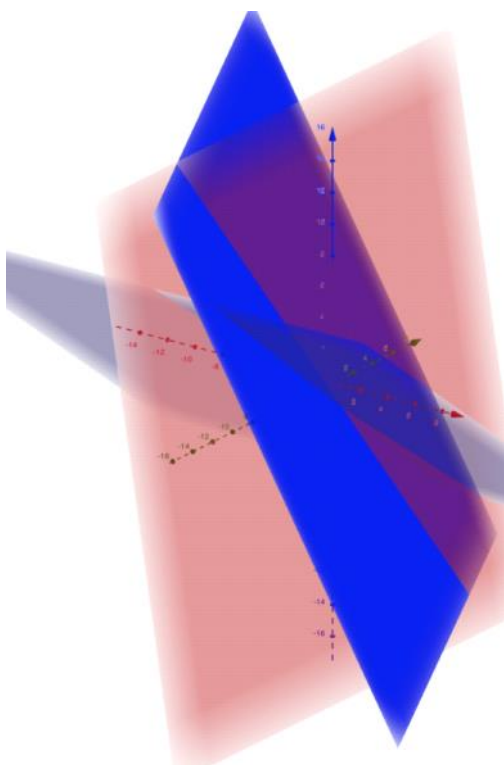
$$\begin{aligned}2x + y + z &= -1 \\2 + y + 2 &= -1 \\y &= \underline{-5}\end{aligned}$$

HOLD ON! What is $2x + y + z = -1$

Using GeoGebra (online and free) we can see that the blue shaded shape is the equation shown above. This is a plane.



If we look at what the equations look like above using GeoGebra:



Whilst it might not be immediately obvious, the three planes all meet at one single point.

Think of the three walls of a classroom.
Where do they meet?
Most times, they meet at a point; the corner of the classroom.

There are other occasions when planes meet and intersect at a line.
Or they might not meet at all.

The following diagram has been taken from the Cambridge Essentials Series and depicts what is possible with planes:

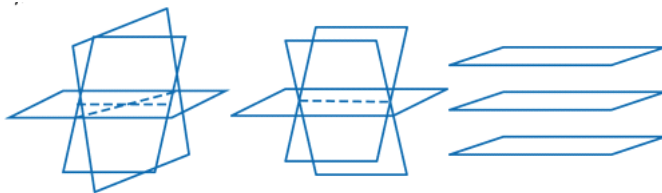


Diagram 1:
Intersection at a point

Diagram 2:
Intersection in a line

Diagram 3:
No intersection

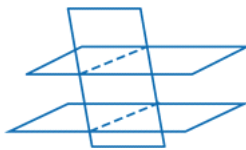


Diagram 4:
No common intersection



Diagram 5:
No common intersection

Finally, using your CAS with parameters

Trying to solve the following on the CAS presents some interesting notation!

Solve the equations shown below.

$$\begin{aligned} x + 2y + 3z &= 13 \\ -x - 3y + 2z &= 2 \\ -x - 4y + 7z &= 17 \end{aligned}$$

$$x = -13z + 43$$

$$y = 5z - 15$$

$$z = 2$$

$$\left(-13\lambda + 43, 5\lambda - 15, \lambda \right)$$

$$\lambda = 0 \quad \left(43, -15, 0 \right)$$

$$\lambda = 1 \quad \left(30, -10, 1 \right)$$