Reflections

Sunday, 11 February 2018 7:56 pm

By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

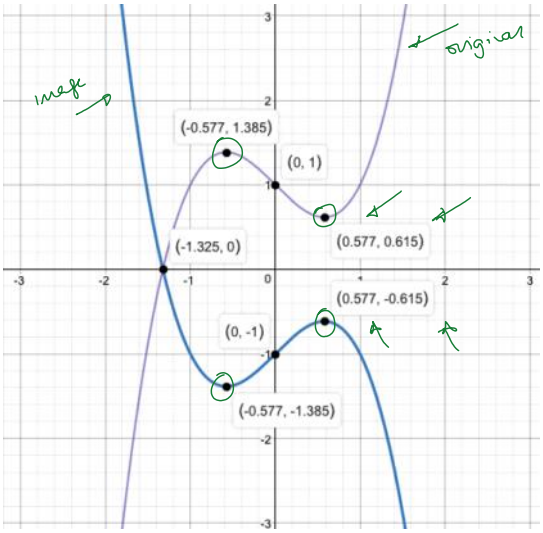
- Know what it means to reflect something
- Know how to use the correct notation
- Apply transformations to sketches of graphs
- Know how to use transformation algebra to go from a base graph to a transformed graph

RECAP:

In the previous two lessons we have looked at what transformations are and how they can be used to change the shape of functions or move functions around the cartesian plane. Translations and dilations form two of the three main transformations we use in Mathematical Methods Units 3 and 4.
This lesson will look at the final transformation. We will look at it in isolation before looking at how we can combine them all!

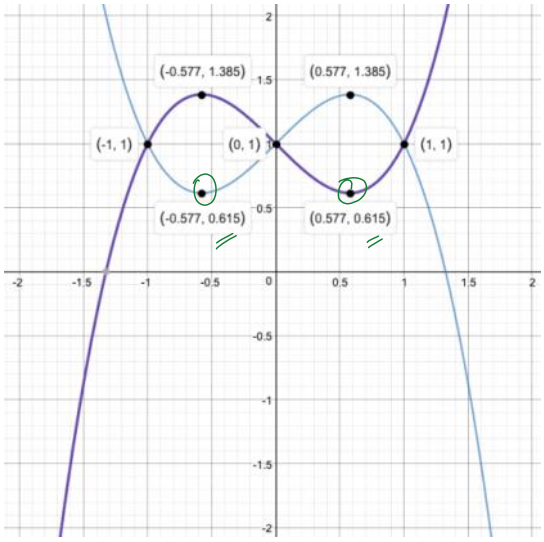
SAME SAME, but DIFFERENT

The theory with reflections is exactly the same as that already covered with **Translations** and **Dilations**.
I think reflections is a lot easier, as your merely swap the y values to be negative y, and the x-values for negative x.
Shown below are some examples of some graphs which have been reflected:



This is an example where the graphs have been reflected in the x-axis.

Notice how the y-values have swapped sign.



This is an example where the graphs have been reflected in the y-axis.

Notice how the x-values have swapped sign.

Reflections in both the x- and y-axes using ALGEBRA

A coordinate and its image can be expressed using algebra: (x, y) maps onto (x', y')

Reflection in the x-axis

If we have a reflection in the x-axis, we are effectively making all the y-values negative

(x, y) maps onto $(x, -y)$

Reflection in the y-axis

If we have a reflection in the y-axis, we are effectively making all the x-values negative

(x, y) maps onto $(-x, y)$

Examples using an ordered-pair

$(2,-3)$ Reflect in x-axis $(x,y) \longrightarrow (x,-y)$

$(2,3)$

$(-3,6)$ Reflect in y-axis $(x,y) \longrightarrow (-x,y)$

$(3,6)$

Example using algebra:

Find the equation of the image of the function, $f(x) = (x-3)^2 - 2$, after a reflection in:

A) The x-axis

B) The y-axis

$y = (x-3)^2 - 2$

$(x,y) \longrightarrow (x,-y)$

(x',y')

$x' = x$

$y' = -y$

$x = x'$

$y = -y'$

$y = (x-3)^2 - 2$

$-y' = (x'-3)^2 - 2$

$-y = (x-3)^2 - 2$

$y = -(x-3)^2 + 2$

b) $f(x) = (x-3)^2 - 2$

$(x,y) \longrightarrow (-x,y)$

(x',y')

$x' = -x$

$y' = y$

$y = y'$

$x = -x'$

$y = (x-3)^2 - 2$

$y' = (-x'-3)^2 - 2$

$y = (-x-3)^2 - 2$

$y = [(-1)(x+3)]^2 - 2$

$y = (-1)^2(x+3)^2 - 2$

$y = (x+3)^2 - 2$

Short cuts

Reflection in the x-axis	e.g. $y = f(x) becomes -y = f(x)$	Replace y with $-y$ in the equation.
Reflection in the y-axis	e.g. $y = f(x) becomes y = f(-x)$	Replace x with -x in the equation

$f(x) = \frac{1}{x-3} + 2$

$-f(x) = \frac{1}{x-3} + 2$

$f(x) = \frac{-1}{x-3} - 2$

$f(x) = \frac{1}{x-3} + 2$

$f(x) = \frac{1}{-x-3} + 2$

$f(x) = \frac{1}{-1(x+3)} + 2$

$f(x) = \frac{-1}{x+3} + 2$