

Power functions

Monday, 29 January 2018 8:08 pm

★ By the end of the lesson I would hope that you have an understanding of the concepts below which you can apply to a number of complex questions:

- What it means for a function to be strictly increasing and decreasing
- What common functions with positive integer indexes look like
- What functions look like with an odd positive integer index
- What functions look like with an even positive integer index
- What functions look like with negative integer indexes



RECAP

In this chapter we have been looking at a range of functions. Whilst we can't possibly be expected to remember them all, we need to know that many of the functions have similar traits.

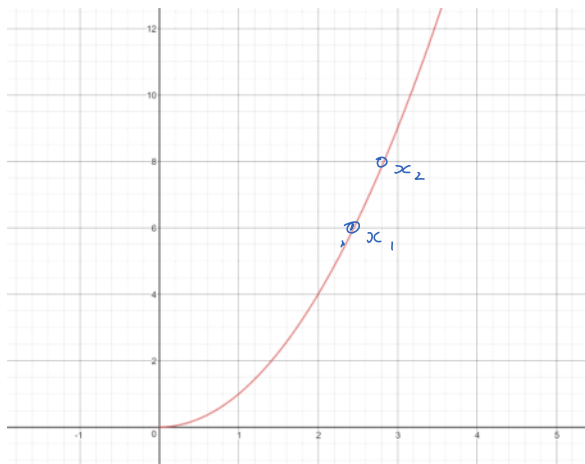
Strictly increasing and decreasing



$$\text{If } x_2 > x_1 \text{ then } f(x_2) < f(x_1)$$

This function is strictly decreasing

Strictly increasing



$$\text{If } x_2 > x_1 \text{ then } f(x_2) > f(x_1)$$

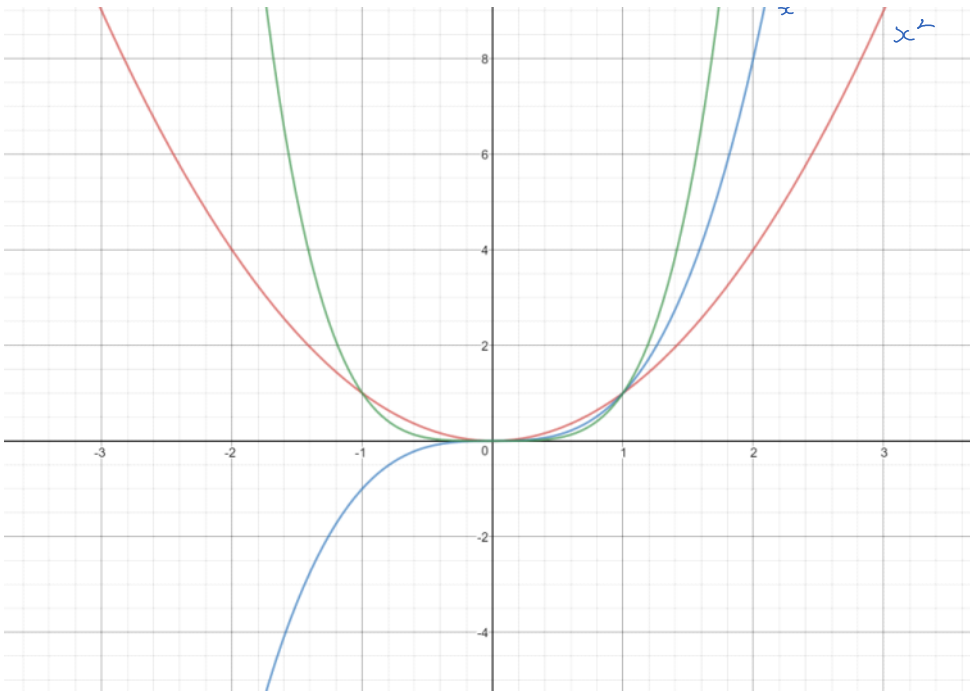
This is a strictly increasing function

Power functions - Positive integers



Not the similarities between some of the graphs.

$$\begin{aligned} y &= x^2 \\ y &= x^3 \\ y &= x^4 \end{aligned}$$



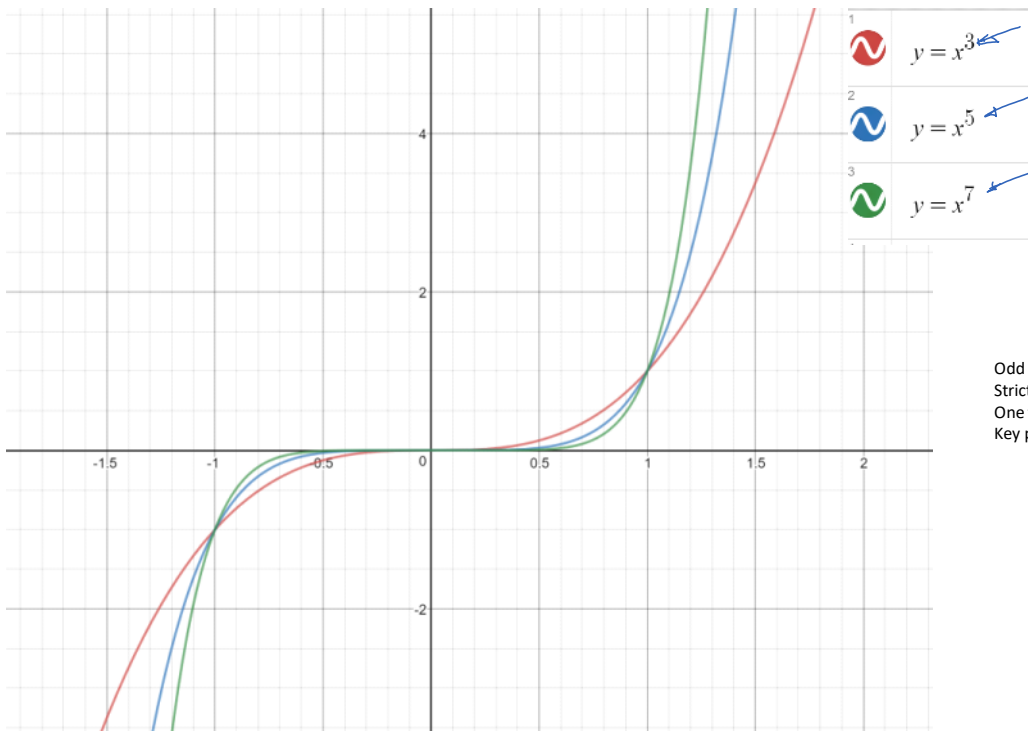
Not the similarities between some of the graphs.

$$y = x^2$$

$$y = x^3$$

$$y = x^4$$

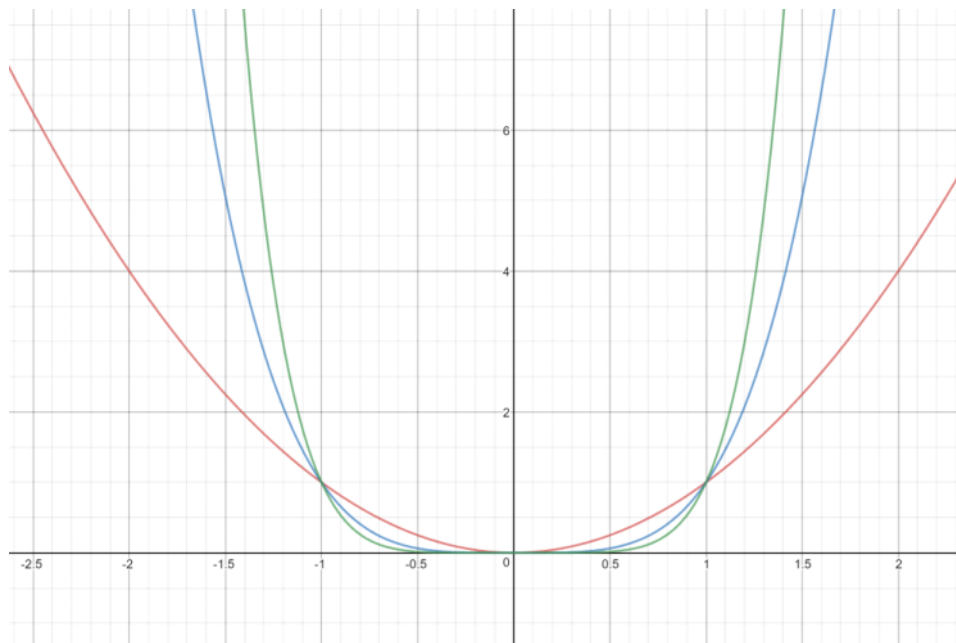
Positive odd powers



$$f(-x) = -f(x)$$

Odd function
Strictly increasing
One to one
Key point (0,0) and (1,1) and (-1, -1)

Positive even numbers

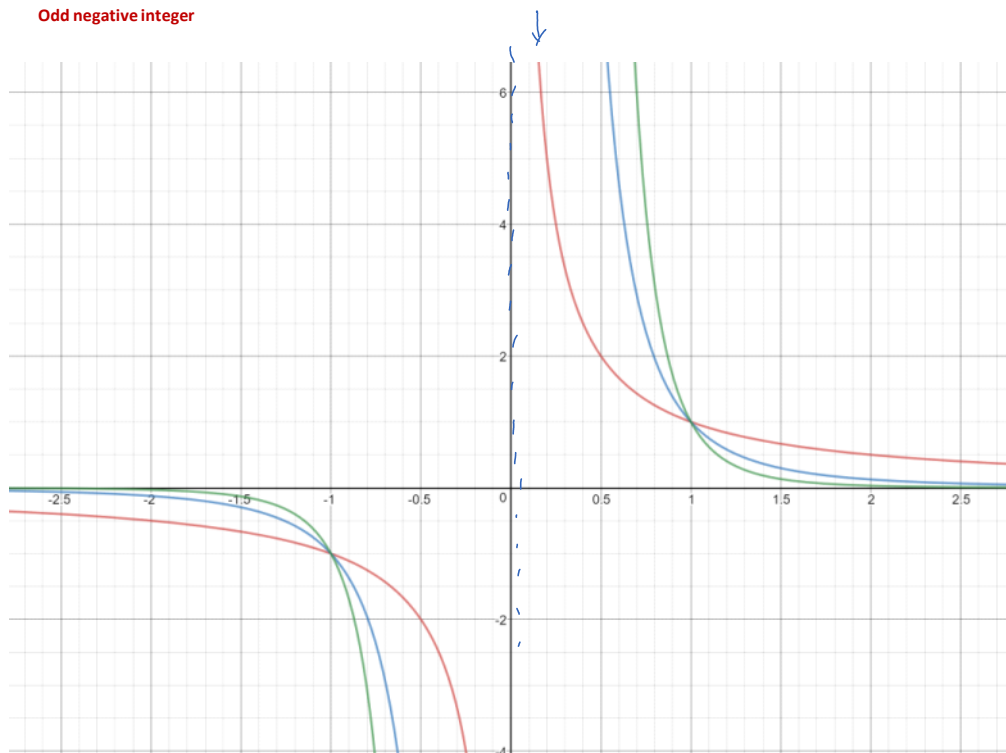


1		$y = x^2$	↖
2		$y = x^4$	↖
3		$y = x^6$	↖
4			

$$f(-x) = f(x)$$

Even function
 Strictly increasing for $x > 0$
 Strictly decreasing for $x < 0$
 Key points!

Odd negative integer



1		$y = x^{-1}$	↖
2		$y = x^{-3}$	↖
3		$y = x^{-5}$	↖
4			

$$y = \frac{1}{x}$$

$$y = \frac{1}{x^3}$$

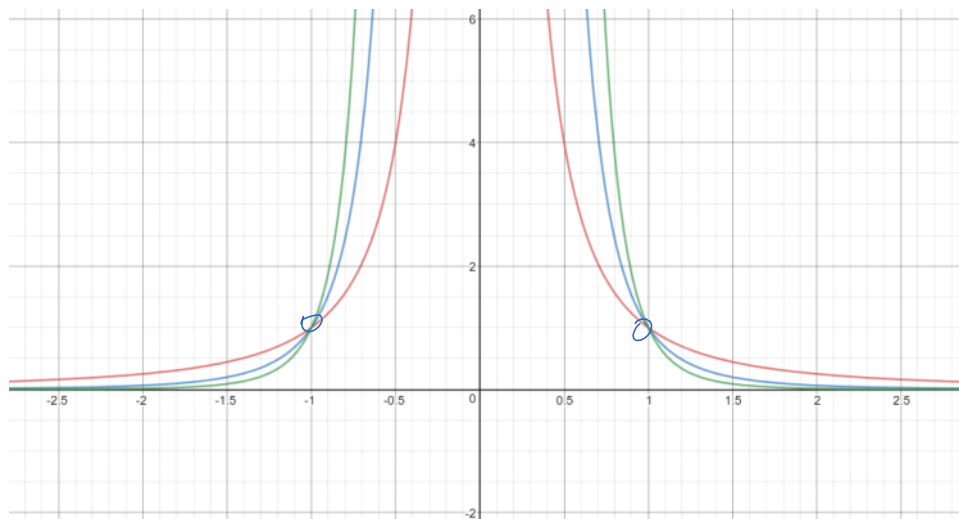
Odd function
 Horizontal asymptote
 Vertical asymptote
 Maximal domain

$$y = \frac{1}{x}$$

$$x \neq 0 \quad y \neq 0$$

$$x = -1 \quad y = -1$$

Even negative integer

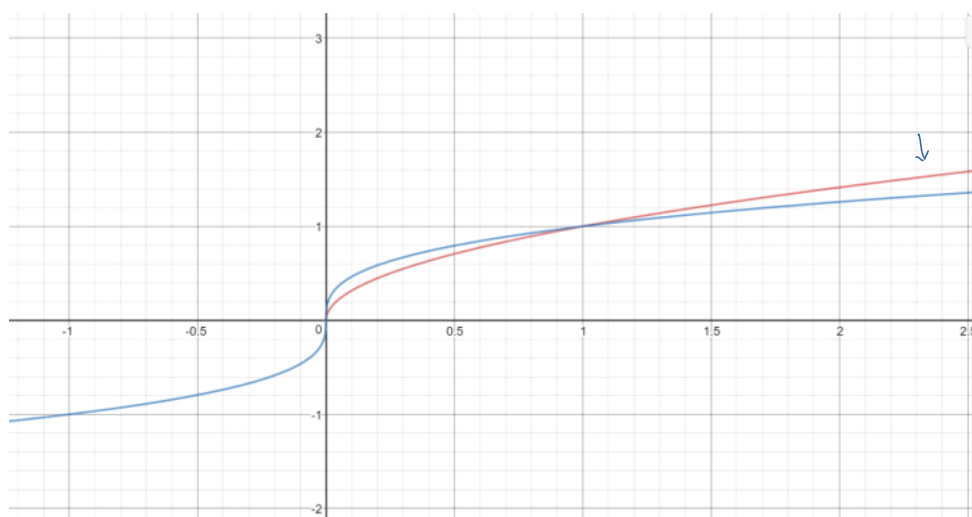


1	 $y = x^{-2}$
2	 $y = x^{-4}$
3	 $y = x^{-6}$
4	

$$y = \frac{1}{x^2}$$

Maximal domain ✓
 Even function ✓
 Horizontal asymptote ✓
 Vertical asymptote ✓

Fractional integers



1	 $y = x^{\frac{1}{2}}$ ✓
2	 $y = x^{\frac{1}{3}}$ ✓
3	$y = x^{\frac{1}{4}}$

$$y = x^{\frac{1}{2}}$$

$$y = \sqrt{x}$$

$$[0, \infty)$$